Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 14, Number 6 (2018), pp. 819-826

http://www.ripublication.com

Complementary Inverse Signed Domination of Corona of a Cycle with a Complete Graph

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Abstract

Let $G(V, E)$ be a graph. A function$f: V \rightarrow \{-1,1\}$ is said to be a complementary signed dominating function (CSDF) of G if $\sum_{u \notin N[v]} f(u) \geq 1$ for all $v \in V$ with $\deg(v) \neq n-1$. The weight of a complementary signed dominating function f is defined as $W(f) = \sum_{v \in V} f(v)$. The complementary signed domination number of G is the minimum weight of a CSDF of G and it is denoted by $\gamma_{cs}(G)$. In this paper the concepts of complementary inverse signed dominating function of G are developed and the complementary inverse signed domination numbers of corona of a cycle with a complete graph are obtained.

Keyword: signed dominating function, complementary signed dominating function, complementary inverse signed dominating function, complementary inverse signed domination number.

1. INTRODUCTION

Dunbal et al[1] introduced the concept of signed dominating function of a graph and there is a variety of possible applications for signed domination. By assigning the values -1 or +1 to the vertices of a graph we can model such things as networks of positive and negative electrical charges, networks of positive and negative spins of electrons, and networks of people or organizations in which global decisions must be made. The concept of complementary signed dominating function was introduced by Y.S. Irine Sheela and R. Kala[3]. Frucht And Harary[2] introduced the corona of two graphs in the year 1970. The definition is as follows:

The corona of two graphs G and H is a graph obtained by taking one copy of G and $|V(G)|$ copies of H and then joining the i^{th}- vertex of G to all the vertices of i^{th}- copy of H and this graph is denoted by $G \odot$H.

In this paper corona of a cycle with a complete graph is considered. Let us denote G be a corona graph $C_n \odot K_m$ with vertex set V and let us denote the vertices of a cycle C_n in G by $u_1, u_2, u_3, \dots., u_n$ and the vertices of i^{th} copy of K_m in G by $v_{i1}, v_{i2}, v_{i3}, \dots v_{im}$ where $i = 1$ to n.

2. COMPLEMENTARY INVERSE SIGNED DOMINATING FUNCTIONS

In 2011, the concept of complementary signed domination was introduced by Y.S. Isrine Sheela and R. Kala[3]. The concept of complementary inverse signed domination is discussed and present some results on the complementary inverse signed dominating function of a corona of a cycle with a complete graph and further complementary signed domination number is obtained.

Definition: Let $G(V, E)$ be a graph. A function $f: V \to \{-1, 1\}$ is called a **signed dominating function** (SDF) of G if $f(N[v]) = \sum_{u \in N[v]} f(u) \geq 1$, for each $v \in V$. The weight of a signed dominating function f is defined as $W(f) = \sum_{v \in V} f(v)$. The signed domination number of G is the minimum weight of a SDF of G and it is denoted by $\gamma_s(G)$.

Definition: Let $G(V, E)$ be a graph. A function $f: V \to \{-1,1\}$ is said to be a **complementary signed dominating function** (CSDF) of G if $\sum_{u \notin N[v]} f(u) \geq 1$ for all $v \in V$ with $\deg(v) \neq n-1$. The weight of a complementary signed dominating function f is defined as $W(f) = \sum_{v \in V} f(v)$. The complementary signed domination number of G is the minimum weight of a CSDF of G and it is denoted by $\gamma_{cs}(G)$.

Definition: Let $G(V, E)$ be a graph. A function $f: V \to \{-1,1\}$ is said to be a **complementary inverse signed dominating function** (CISDF) of G if $\sum_{u \notin N[v]} f(u) \leq 0$ for all $v \in V$. The weight of a complementary inverse signed dominating function f is defined as $W(f) = \sum_{v \in V} f(v)$. The complementary inverse signed domination number of G is maximum weight of a CISDF of G and it is denoted by $\gamma_{cs}^0(G)$.

Theorem 2.1: The complementary inverse signed domination number of $C_4 \odot K_m$ is

$$\gamma_{cs}^o(C_4 \odot K_m) = \begin{cases} -4, if\ m\ is\ even \\ 0, if\ m\ is\ odd \end{cases}.$$

Proof: Let G be a corona graph $C_4 \odot K_m$ with vertex set V.

Case I: Suppose m is even.

Let f: V $\rightarrow\{-1, 1\}$ be a function defined as

$$f(u_i) = -1, \ \forall i$$

and $f(v_{ij}) = \begin{cases} 1, for\ \frac{m}{2}\ vertices\ in\ each\ copy\ of\ K_m\ in\ G, \\ -1, otherwise. \end{cases}$

Case 1: Let $u_i \in C_4$ be such that $d(u_i) = m + 2$. Then $N[u_i]$ contains 3 vertices of C_4 and m vertices of K_m in G. The remaining 1 vertex of C_4 and the vertices in 3 copies of K_m are not belongs to $N[u_i]$ in G.

Then $\sum_{w \notin N[u_i]} f(w) = -1 + 3\left(\frac{m}{2}(1) + \frac{m}{2}(-1)\right) = -1$.

Case 2: Let $v_{ij} \in K_m$ be such that $d(v_{ij}) = m$, where $i = 1$ to 4, $j = 1$ to m. Then $N[v_{ij}]$ contains '1' vertex of C_4 and m vertices of K_m in G. The remaining 3 vertices in C_4 and the vertices in 3 copies of K_m are not belongs to $N[v_{ij}]$ of G.

Then $\sum_{w \notin N[v_{ij}]} f(w) = 4(-1) - (-1) + 3\left(\frac{m}{2}(1) + \frac{m}{2}(-1)\right) = -3$.

Therefore for all possibilities, we get $\sum_{w \notin N[v]} f(w) \leq 0$ for all $v \in V$.

Hence f is a complementary inverse signed dominating function of G.

Since $\sum_{w \notin N[u_i]} f(w) = -1$, the labelling is maximum with respect to the vertices of C_4. If $f(u_2) = 1$, then $\sum_{w \notin N[u_4]} f(w) = 1$.

Therefore $W(f) = \sum_{v \in V} f(v) = 4(-1) + (4)\left(\frac{m}{2}(1) + \frac{m}{2}(-1)\right) = -4$.

Hence $\gamma_{cs}^0(C_4 \odot K_m) = -4$, if m is even.

Case II: Suppose m is odd.

Let f: V $\rightarrow\{-1, 1\}$ be a function defined as

$$f(u_i) = 1 \ \forall i$$

and $f(v_{ij}) = \begin{cases} 1, for\ \dfrac{m-1}{2}\ vertices\ in\ each\ copy\ of\ K_m\ in\ G, \\ -1, otherwise. \end{cases}$

Case 3: Let $u_i \in C_n$ in G be such that $d(u_i) = m + 2$.

As we have seen in case 1 of case I,

$$\sum_{w \notin N[u_i]} f(w) = 1 + 3\left(\frac{m-1}{2}(1) + \frac{m+1}{2}(-1)\right) = -2.$$

Case 4: Let $v_{ij} \in K_m$ in G be such that $d(v_{ij}) = m$, where $i = 1$ to 4, $j = 1$ to m.

As we have seen in Case 2 of case I,

$$\sum_{w \notin N[v_{ij}]} f(w) = 4(1) - 1 + 3\left(\frac{m-1}{2}(1) + \frac{m+1}{2}(-1)\right) = 0.$$

Therefore for all possibilities, we get $\sum_{w \notin N[v]} f(w) \leq 0$ for all $v \in V$.

Hence f is a complementary inverse signed dominating function of G.

Since $\sum_{w \notin N[v_{ij}]} f(w) = 0$, the labelling is maximum with respect to the vertices of K_m in G. If at least one v_{ij} such that $f(v_{ij}) = 1$, then $\sum_{w \notin N[v_{ij}]} f(w) = 2$.

Therefore $W(f) = \sum_{v \in V} f(v) = n(1) + (n)\left(\frac{m-1}{2}(1) + \frac{m+1}{2}(-1)\right) = 0$.

Hence $\gamma_{cs}^{0}(C_4 \odot K_m) = 0$, if m is odd.

Theorem 2.2: The complementary inverse signed domination number of $C_n \odot K_m$ for $n \neq 4$ is $\gamma_{cs}^{0}(C_n \odot K_m) = \begin{cases} -1, & if n\ is\ odd\ and\ m\ is\ even \\ 0, & if\ n\ is\ odd\ and\ m\ is\ odd \\ -2, & if\ n\ is\ even\ and\ m\ is\ even \\ 0, & if\ n\ is\ even\ and\ m\ is\ odd \end{cases}$.

Proof: Let G be a corona graph $C_n \odot K_m$ with vertex set V.

Case I: Suppose n is odd and m is even.

Let $f: V \rightarrow \{-1, 1\}$ be a function defined as

$$f(u_i) = \begin{cases} -1\ , & if\ i\ is\ odd, \\ 1\ , & if\ i\ is\ even. \end{cases}$$

and $f(v_{ij}) = \begin{cases} 1, for\ \frac{m}{2}\ vertices\ in\ each\ copy\ of\ K_m\ in\ G, \\ -1, otherwise. \end{cases}$

Case 1: Let $u_i \in C_n$ be such that $d(u_i) = m + 2$. Then the neighbourhood $N[u_i]$ contains 3 vertices of C_n and m vertices K_m in G. The remaining $n - 3$ vertices of C_n

and the vertices in $(n-1)$ copies of K_m does not belongs to $N[u_i]$.

If $i=1$, then

$$\sum_{w\notin N[u_1]} f(w) = \sum_{i=1}^{n} f(u_i) - \big(f(u_n)+f(u_1)+f(u_2)\big) + \Big(\sum_{i=2}^{n}\Big(\sum_{j=1}^{m} f\big(v_{ij}\big)\Big)\Big)$$

$$= \left(\tfrac{n-1}{2}\right)(1) + \left(\tfrac{n+1}{2}\right)(-1) - (-1-1+1) + (n-1)\left(\tfrac{m}{2}(1) + \tfrac{m}{2}(-1)\right) = 0.$$

Similarly for $i=n,$ we get $\sum_{w\notin N[u_n]} f(w) = 0$.

Also we have $\sum_{w\notin N[u_n]} f(w) = 0$ if i is even.

If $i=3$, then

$$\sum_{w\notin N[u_3]} f(w) = \sum_{i=1}^{n} f(u_i) - \big(f(u_2)+f(u_3)+f(u_4)\big) + \sum_{\substack{i=1\\ i\neq 3}}^{n}\Big(\sum_{j=1}^{m} f\big(v_{ij}\big)\Big)$$

$$= \left(\tfrac{n-1}{2}\right)(1) + \left(\tfrac{n+1}{2}\right)(-1) - (1-1+1) + (n-1)\left(\tfrac{m}{2}(1) + \tfrac{m}{2}(-1)\right) = -2.$$

Similarly for $2<i<n-1$ and i is odd, we get $\sum_{w\notin N[u_i]} f(w) = -2$.

Case 2: Let $v_{ij}\in K_m$ be such that $d\big(v_{ij}\big)=m$, where $i=1$ to $n,$ $j=1$ to $m.$

Then $N[v_{ij}]$ contains 1 vertex of C_n and m vertices o f K_m in G. The remaining $n-1$ vertices of C_n and the vertices in $(n-1)$ copies of K_m does not belongs to $N[v_{ij}]$.

If i is odd and $j=1$ to m, then

$$\sum_{w\notin N[v_{ij}]} f(w) = \sum_{k=1}^{n} f(u_k) - f(u_i) + \sum_{\substack{k=1\\ k\neq i}}^{n}\Big(\sum_{j=1}^{m} f\big(v_{kj}\big)\Big)$$

$$= \left(\tfrac{n-1}{2}\right)(1) + \left(\tfrac{n+1}{2}\right)(-1) + 1 + (n-1)\left(\tfrac{m}{2}(1) + \tfrac{m}{2}(-1)\right) = 0.$$

If i is even and $j=1$ to m, then

$$\sum_{w\notin N[v_{ij}]} f(w) = \left(\tfrac{n-1}{2}\right)(1) + \left(\tfrac{n+1}{2}\right)(-1) - 1 + (n-1)\left(\tfrac{m}{2}(1) + \tfrac{m}{2}(-1)\right) = -2.$$

Therefore for all possibilities, we get $\sum_{w\notin N[v]} f(w) \leq 0$ for all $v\in V.$

Hence f is a complementary inverse signed dominating function of G.

Since $\sum_{w\notin N[u_i]} f(w) = 0$, the labelling is maximum with respect to the vertices $u_1, u_2, u_4, \dots u_{n-3}, u_{n-1}, u_n$ of G. If $f(u_n)=1,$ then $\sum_{w\notin N[u_2]} f(w) = 2.$

Therefore $W(f) = \sum_{v\in V} f(v) = = \left(\frac{n-1}{2}\right)(1) + \left(\frac{n+1}{2}\right)(-1) + (n)\left(\frac{m}{2}(1) + \frac{m}{2}(-1)\right) = -1.$

Hence $\gamma_{cs}^{0}(C_n \odot K_m) = -1$, if n is odd and m is even.

Case II : Suppose n is odd and m is odd.

Let f: V $\rightarrow\{-1, 1\}$ be a function defined as

$f(u_i) = 1\ \forall$i

and $f(v_{ij}) = \begin{cases} 1, for\ \dfrac{m-1}{2}\ vertices\ in\ each\ copy\ of\ K_m\ in\ G, \\ -1, otherwise. \end{cases}$

As we have seen in case 1 of case I,

$\sum_{w\notin N[u_i]} f(w) = n(1) - (1+1+1) + (n-1)\left(\frac{m-1}{2}(1) + \frac{m+1}{2}(-1)\right) = -2$, $\forall u_i \epsilon\ C_n$of G.

We have seen in case 2 of case I,

$\sum_{w\notin N[v_{ij}]} f(w) =\ n(1) - 1 + (n-1)\left(\frac{m-1}{2}(1) + \frac{m+1}{2}(-1)\right) = 0$, $\forall v_{ij}\epsilon\ K_m$ of G.

Therefore for all possibilities, we get $\sum_{w\notin N[v]} f(w) \leq 0$ for all $v \in V$.

Hence f is a complementary inverse signed dominating function of G.

Since $\sum_{w\notin N[v_{ij}]} f(w) = 0$, the labelling is maximum with respect to the vertices of K_m in G. If at least one v_{ij} such that $f(v_{ij}) = 1$, then $\sum_{w\notin N[v_{ij}]} f(w) = 2$.

Therefore $W(f) = \sum_{v\in V} f(v) = = n(1) + (n)\left(\frac{m-1}{2}(1) + \frac{m+1}{2}(-1)\right) = 0$.

Hence $\gamma_{cs}^{0}(C_n \odot K_m) = 0$, if n is odd and m is odd.

Case III : Suppose n is even and m is even.

Let f: V $\rightarrow\{-1, 1\}$ be a function defined as

$f(u_1) = 1, f(u_2) = f(u_n) = -1,$

for $3 \leq i \leq n-1, f(u_i) = \begin{cases} 1\ , \text{if}\ i\ is\ even, \\ -1\ , \text{if}\ i\ is\ odd. \end{cases}$

and $f(v_{ij}) = \begin{cases} 1, for\ \frac{m}{2}\ vertices\ in\ each\ copy\ of\ K_m\ in\ G, \\ -1, otherwise. \end{cases}$

Case 3: Let $u_i \in C_n$ of G.

If $i = 1$, then

$\sum_{w \notin N[u_1]} f(w) = \sum_{i=1}^{n} f(u_i) - \big(f(u_n) + f(u_1) + f(u_n)\big) + \sum_{i=2}^{n}\big(\sum_{j=1}^{m} f(v_{ij})\big)$

$= \frac{n+2}{2}(-1) + \frac{n-2}{2}(1) — (1 - 1 + 1) + (n-1)\left(\frac{m}{2}(1) + \frac{m}{2}(-1)\right) = -1.$

Similarly for $i = 3$ and $n - 1$, we get $\sum_{w \notin N[u_i]} f(w) = -1$.

Also we have $\sum_{w \notin N[u_i]} f(w) = -1$, if i is even.

If $i = 5$, then

$\sum_{w \notin N[u_5]} f(w) = \sum_{i=1}^{n} f(u_i) - \big(f(u_4) + f(u_5) + f(u_6)\big) + \sum_{i=5}^{n}\big(\sum_{j=1}^{m} f(v_{ij})\big)$

$= \frac{n+2}{2}(-1) + \frac{n-2}{2}(1) — (1 - 1 + 1) + (n-1)\left(\frac{m}{2}(1) + \frac{m}{2}(-1)\right) = -3.$

Also we have $\sum_{w \notin N[u_i]} f(w) = -3$, if i is odd and $5 \leq i \leq n - 2$.

Case 4: Let $v_{ij} \in K_m$ of G.

Let $i = 1$ and $j = 1$ to m, we have

$\sum_{w \notin N[v_{1j}]} f(w) = \sum_{k=1}^{n} f(u_k) - f(u_1) + \sum_{k=2}^{n}\big(\sum_{j=1}^{m} f(v_{kj})\big)$

$= \frac{n+2}{2}(-1) + \frac{n-2}{2}(1) - 1 + (n-1)\left(\frac{m}{2}(1) + \frac{m}{2}(-1)\right) = -3.$

Also we have $\sum_{w \notin N[v_{ij}]} f(w) = -3$, if i is even and $3 \leq i \leq n - 1$.

Let $i = 2$ and $j = 1$ to m, we have

$\sum_{w \notin N[v_{ij}]} f(w) = \sum_{k=1}^{n} f(u_k) - f(u_i) + \sum_{\substack{k=1 \\ k \neq i}}^{n}\big(\sum_{j=1}^{m} f(v_{kj})\big)$

$= \frac{n+2}{2}(-1) + \frac{n-2}{2}(1) + 1 + (n-1)\left(\frac{m}{2}(1) + \frac{m}{2}(-1)\right) = -1.$

Also we have $\sum_{w \notin N[v_{ij}]} f(w) = -1$, if i is odd and $3 \leq i \leq n - 1$.

Therefore for all possibilities, we get $\sum_{w \notin N[v]} f(w) \leq 0$ for all $v \in V$.

Hence f is a complementary inverse signed dominating function of G.

Since $\sum_{w \notin N[u_2]} f(w) = -1$, the labelling is maximum with respect to the vertices $u_1, u_2, u_3, u_4, u_6, \ldots, u_{n-1}, u_n$. If at least one u_i such that $f(u_i) = 1$, then $\sum_{w \notin N[u_i]} f(w) = 1$.

Therefore $W(f) = \sum_{v \in V} f(v) = \frac{n+2}{2}(-1) + \frac{n-2}{2}(1) + (n)\left(\frac{m}{2}(1) + \frac{m}{2}(-1)\right) = -2.$

Hence $\gamma_{cs}^{0}(C_n \odot K_m) = -2$, if n is even and m is even.

Case IV: Suppose n is even and m is odd

Let f: V $\to\{-1, 1\}$ be a function defined as

$f(u_i) = 1\ \forall$i

and $f(v_{ij}) = \begin{cases} 1, & for\ \dfrac{m-1}{2}\ vertices\ in\ each\ copy\ of\ K_m\ in\ G, \\ -1, & otherwise. \end{cases}$

As we have seen in case II,

$\sum_{w\notin N[u_i]} f(w) = n(1) - (1+1+1) + (n-1)\left(\frac{m-1}{2}(1) + \frac{m+1}{2}(-1)\right) = -2$, $\forall u_i \epsilon\ C_n$ of G.

Also we have $\sum_{w\notin N[v_{ij}]} f(w) = n(1) - 1 + (n-1)\left(\frac{m-1}{2}(1) + \frac{m+1}{2}(-1)\right) = 0$, $\forall v_{ij} \epsilon\ K_m$ of G.

Therefore for all possibilities, we get $\sum_{w\notin N[v]} f(w) \leq 0$ for all $v \in V$.

Hence f is a complementary inverse signed dominating function of G.

Since $\sum_{w\notin N[v_{ij}]} f(w) = 0$, the labelling is maximum with respect to the vertices of K_m in G. If at least one v_{ij} such that $f(v_{ij}) = 1$, then $\sum_{w\notin N[v_{ij}]} f(w) = 2$.

Therefore $W(f) = \sum_{v\in V} f(v) = n(1) + (n)\left(\frac{m-1}{2}(1) + \frac{m+1}{2}(-1)\right) = 0$.

Hence $\gamma_{cs}^0(C_n \odot K_m) = 0$, if n is even and m is odd.

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