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A Study on Single Server Queuing Model Using DSW Algorithm with Heptagonal and Octagonal Fuzzy Number

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Abstract

In this paper we study (FM/FM/1): (FCFS/∞/∞) queuing model in Heptagonal and Octagonal Fuzzy numbers using DSW algorithm. The performance measures of this model is Heptagonal and Octagonal fuzzy numbers. The numerical example shows the efficiency of this model.

Keywords: Membership function – Heptagonal – Octagonal Fuzzy numbers – α-Cut – DSW algorithm.

1. INTRODUCTION

Queuing theory is the mathematical study of waiting lines. It has great extent applications in service organizations as well as manufacturing firms. Queue is quite common in many fields, for example, in Telephone exchange, in Supermarket, at petrol station, at computer systems etc. In general [7] a queue is formed at a production system when either customers (human beings or physical entities) requiring service wait because number of customers exceeds the number of service facilities, or service facilities do not work efficiently or take more time than prescribed to serve a customer. It can be applied to a variety of situations where it is not possible to accurately predict the arrival rate of customers and service rate of service facility. It helps to determine the balance between cost of offering the service and cost incurred due to delay in offering service.

Queuing theory was introduced by A.K. Erlang in 1909[1]. Queuing models are aids to determine the optimal number of counters so as to satisfy the customers keeping the total cost minimum .It helps the customers should get service with minimum time. It tries to answer the questions like, the mean waiting time in the queue, the mean system response time (waiting time in the queue and service time), expected number of customer in the system and expected number of customer in the queue and such questions.

Fuzzy sets have been introduced by Lofti.A.Zadeh [13] in 1965, professor at university of California at Berkelay. A fuzzy set is a class with no sharp boundary between membership and non membership function. Li.R.J. and Lee.E.S.[14] discussed comparison of fuzzy numbers. Zimmermann.H.J [5] introduced fuzzy linear programming and its application. Shanmugasundaram.S. and Venkadesh.B [7] presented multi server fuzzy queuing model using DSW algorithm..Narayanamoorthy.S and Ramya.L [6] discussed multi server queuing model using DSW algorithm with Hexagonal fuzzy. Durai. K and Karpagam.A [2] defined new membership function and new ranking funct ion on Heptagonal fuzzy number.

The approximate method DSW (Dong, Shah, Wong) algorithm used to define a membership function of the performance measures in single server fuzzy queuing model with Heptagonal and Octagonal fuzzy number.

2. PRELIMINARIES

Kendall's notation:

(a/b/c): (d/e/f), where a = arrivals distribution, b = service time distribution, c = number of servers, d= capacity of the system, e = queue discipline, f = calling source or population.

Fuzzy set:

Let X be a non empty set. A fuzzy set F in X is Characterized by its membership function $\mu_F : X \rightarrow [0,1]$ and $\mu_F(x)$ is interpreted as the degree of membership of element x in fuzzy set F for each $x \in X$. It is clear that F is completely determined by the set of tuples $F = \{(x, \mu_F(x)) / x \in X\}$. Frequently we will write F(x) by $\mu_F(x)$.

α – Cuts:

If a fuzzy set is defined on for any $\alpha \in [0, 1]$ the cuts is represented by the following crisp set.

Strong **α** - cuts

$$A_F^* = \{ x \in X / \mu_A(x) > \alpha : \alpha \in [0,1]\}$$

Weak **α** - cuts

$$A_F = \{x \in X / \mu_A(x) \geq \alpha : \alpha \in [0, 1]\}$$

3. DESCRIPTION OF THE SYSTEM

We consider a traditional queuing system with single server and queue discipline is First come in First serve.

i.e (M/M/1) : (FCFS/∞/∞).

Arrival rate and Service rate are fuzzy numbers denoted by $\tilde{\beta}$ and $\tilde{\gamma}$.

The inter arrival time (A) and Service time (B) are represented by the following fuzzy sets.

$$A= \{(a, \tilde{\gamma}_A (a))/ a \in X\}$$

$$B= \{(b, \tilde{\gamma}_B (b))/ b \in Y\}$$

Where X and Y are crisp universal sets.

The membership function of A and B are

$$A (\alpha) = \{a \in X, \tilde{\gamma}_A (a) \geq \alpha\}$$

$$B (\alpha) = \{b \in Y, \tilde{\gamma}_B(b) \geq \alpha\}$$

Where A (α) and B(α) are crisp sets using α cuts.

Using different levels of confidence intervals, the interval time and service time are represented.

4. INTERVAL ANALYSIS

Let I_1 and I_2 be two interval numbers defined by ordered pairs of real numbers with lower and upper bounds.

$$I_1 = [a_1 ,a_2,] , a_1 \leq a_2 , \ I_2 = [a_3,a_4] , a_3 \leq a_4$$

To define a general arithmetic property with the symbol * , where * = [+,-,×,÷]

(i.e) $I_1 * I_2 = [a_1,a_2] * [a_3,a_4]$

Where $I_1 * I_2$ represent another interval. The interval calculation depends on the magnitudes and signs of the element a_1, a_2, a_3, a_4.

$$[a_1 , a_2]+ [a_3, a_4] = [a_1+a_3, a_2 +a_4]$$

$$[a_1, a_2] - [a_3, a_4] = [a_1-a_3, a_2 -a_4]$$

$$[a_1, a_2] \times [a_3, a_4] = [\min(a_1a_3, a_1a_4, a_2a_3, a_2a_4), \max(a_1a_3, a_1a_4, a_2a_3, a_2a_4)]$$

$$[a_1, a_2] \div [a_3, a_4] = [a_1, a_2] \cdot \left[\frac{1}{a3}, \frac{1}{a4}\right]$$

Provided that $0 \notin [a_3, a_4]$

$$\alpha\,[a_1, a_2] = \begin{cases} [\alpha\, a_1, \alpha\, a_2] & \text{for } \alpha > 0 \\ [\alpha\, a_2, \alpha\, a1] & \text{for } \alpha < 0 \end{cases}$$

5. DSW ALGORITHM

DSW (Dong, Shah, Wong) is one of the approximate methods make use of intervals at various α cut levels in defining membership functions. It was the full α cut intervals in a standard interval analysis.

The DSW algorithm greatly simplifies manipulation of the extension principle for continuous valued fuzzy variables, such as fuzzy numbers defines on the real line. It prevent abnormality in the output membership function due to application of the discrimination reaching on the fuzzy variables domain,it can prevent the widening of the resulting functional expression by conventional interval analysis methods.

Any continuous membership function can be represented by a continuous sweep α cut interm from $\alpha = 0$ to $\alpha = 1$. Suppose we have single input mapping given by $Y = f(X)$ that is to be extended for fuzzy sets $\tilde{B} = f(\tilde{A})$ and we want to decompose $\tilde{A}$ in to the series of α cut intervals say I_α . It uses the full α cut intervals in a standard interval analysis. The DSW algorithm [6] consists of the following steps:

1. Select a α-cut value where $0 \le \alpha \le 1$.
2. Find the intervals in the input membership functions that correspond to this α.
3. Using standars binary interval operations compute theinterval for the output membership function for the selected α-cut level.
4. Repeat Steps 1to 3 for different values of α to complete α-cut representation of the solution.

Heptagonal membership function is constructed by

$$\gamma_{hep}(x) = \begin{cases} 0, & \text{for } x < c_1 \\ \frac{1}{2}\left(\frac{x-c1}{c2-c1}\right), & \text{for } c_1 \leq x \leq c_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-c2}{c3-c2}\right), & \text{for } c_2 \leq x \leq c_3 \\ 1\,, & \text{for } c_3 \leq x \leq c_4 \\ 1 - \frac{1}{4}\left(\frac{x-c4}{c5-c4}\right), & \text{for } c_4 \leq x \leq c_5 \\ \frac{3}{4} - \frac{1}{2}\left(\frac{x-c5}{c6-c5}\right), & \text{for } c_5 \leq x \leq c_6 \\ \frac{1}{4}\left(\frac{x-c7}{c6-c7}\right), & \text{for } c_6 \leq x \leq c_7 \\ 0, & \text{for } x > c_7 \end{cases}$$

Octagonal membership function is constructed by

$$\gamma_{oct}(x) = \begin{cases} 0, & \text{for } x < c_1 \\ k\left(\frac{x-c1}{c2-c1}\right), & \text{for } c_1 \leq x \leq c_2 \\ k, & \text{for } c_2 \leq x \leq c_3 \\ k + (1-k)\left(\frac{x-c3}{c4-c3}\right), & \text{for } c_3 \leq x \leq c_4 \\ 1\,, & \text{for } c_4 \leq x \leq c_5 \\ k + (1-k)\left(\frac{c6-x}{c6-c5}\right), & \text{for } c_5 \leq x \leq c_6 \\ k\,, & \text{for } c_6 \leq x \leq c_7 \\ k\left(\frac{c8-x7}{c8-c7}\right), & \text{for } c_7 \leq x \leq c_8 \\ 0, & \text{for } x > c_8 \end{cases}$$

(FM/FM/1): (FCFS/∞/∞) Model

In this model we consider an infinite source population with first come first served where both inter arrival time and service time follow exponential distribution with ratio λ and μ respectively.

The expected number of customer in the system

$$L_s = \frac{\beta}{\gamma-\beta}$$

The expected number of customer in the queue

$$L_q = \frac{\beta^2}{\gamma(\gamma-\beta)}$$

The average waiting time in the system

$$W_s = \frac{1}{\gamma-\beta}$$

The average waiting time in the queue

$$W_q = \frac{\beta}{\gamma(\gamma-\beta)}$$

6. NUMERICAL EXAMPLE

(i) Heptagonal fuzzy number

Consider a FM/FM/1 queue, where both the arrival rate and service rate are Heptagonal fuzzy number represented by $\tilde{\beta}$ = [1, 2, 3, 4, 5, 6, 7] and $\tilde{\gamma}$ = [11, 12, 13, 14, 15, 16, 17]. The interval of confidence at possibility level α as [1 + α , 7 – α] and [11 + α , 17 –α]. Where,

x = [1 + α , 7 – α] and y = [11 + α , 17 –α].

$$L_s = \frac{x}{y-x}, \quad L_q = \frac{x^2}{y(y-x)}, \quad W_s = \frac{1}{y-x}, \quad W_q = \frac{x}{y(y-x)}$$

Table 1: The α- cuts of Ls, Lq, Ws, Wq at α values

α	L_s	L_q	W_s	W_q
0	[0.0625,1.7500]	[0.0037,1.1136]	[0.0625,0.2500]	[0.0037,0.1591]
0.1	[0.0696,1.6429]	[0.0045,1.0212]	[0.0633,0.2381]	[0.0041,0.1480]
0.2	[0.0769,1.5455]	[0.0054,0.9383]	[0.0641,0.2273]	[0.0038,0.1380]
0.3	[0.0844,1.4565]	[0.0066,0.8636]	[0.0649,0.2174]	[0.0051,0.1289]
0.4	[0.0921,1.3750]	[0.0078,0.7961]	[0.0658,0.2083]	[0.0055,0.1206]
0.5	[0.1000,1.3000]	[0.0091,0.7348]	[0.0667,0.2000]	[0.0061,0.1130]
0.6	[0.1081,1.2308]	[0.0105,0.6790]	[0.0676,0.1923]	[0.0066,0.1061]
0.7	[0.1164,1.1667]	[0.0121,0.6282]	[0.0685,0.1852]	[0.0071,0.0997]
0.8	[0.1250,1.1071]	[0.0139,0.5817]	[0.0694,0.1786]	[0.0077,0.0938]
0.9	[0.1338,1.0517]	[0.0158,0.5391]	[0.0704,0.1724]	[0.0083,0.0884]
1	[0.1429,1.0000]	[0.0179,0.5000]	[0.0714,0.1667]	[0.0089,0.0833]

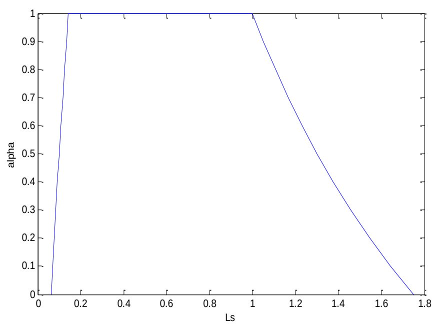


Fig 1: Ls

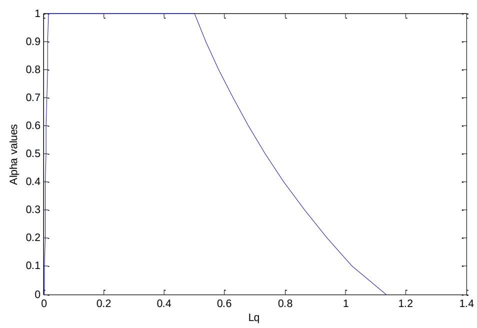


Fig 2: Lq

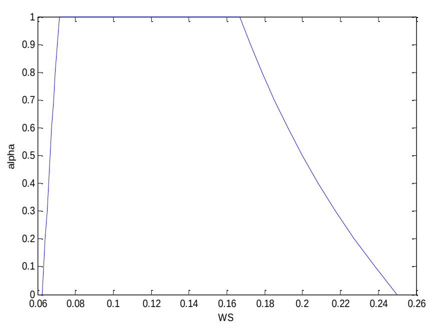


Fig 3 : Ws

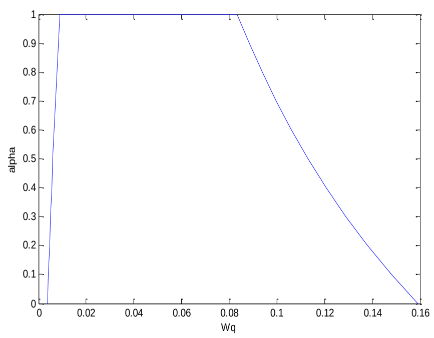


Fig 4 : Wq

(ii) Octagonal fuzzy number

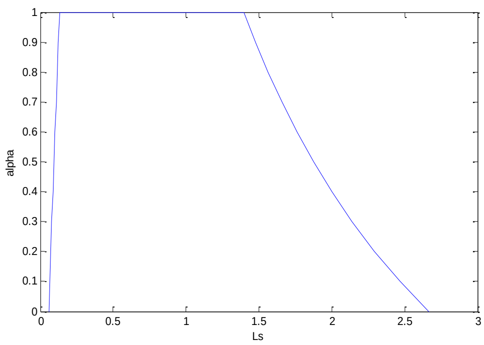
Take both the arrival rate and service rate are Octagonal fuzzy number represented by $\tilde{\beta}$ = [1, 2, 3, 4, 5, 6, 7,8] and $\tilde{\gamma}$ = [11, 12, 13, 14, 15, 16, 17,18].

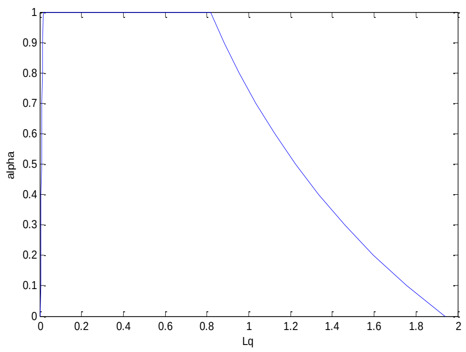
The interval of confidence at possibility level α as [1 + α , 8 – α] and [11 + α , 18 –α]. Where, x = [1 + α , 8 – α] and y = [11 + α , 18 –α].

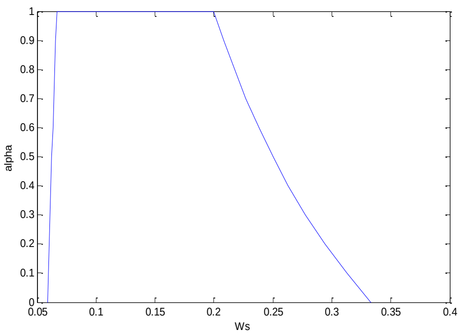
$$L_s = \frac{x}{y-x}, \quad L_q = \frac{x^2}{y(y-x)}, \quad W_s = \frac{1}{y-x}, \quad W_q = \frac{x}{y(y-x)}$$

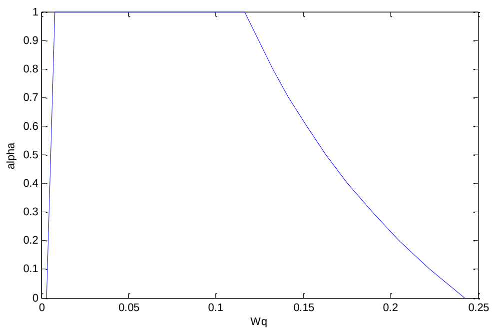
Table 2: The α- cuts of Ls, Lq, Ws, Wq at α values

α	L_s	L_q	W_s	W_q
0	[0.0588,2.6667]	[0.0033,1.9394]	[0.0588,0.3333]	[0.0033,0.2424]
0.1	[0.0655,2.4688]	[0.0040,1.7570]	[0.0595,0.3125]	[0.0037,0.2224]
0.2	[0.0723,2.2941]	[0.0049,1.5977]	[0.0602,0.2941]	[0.0041,0.2048]
0.3	[0.0793,2.1389]	[0.0058,1.4575]	[0.0610,0.2778]	[0.0045,0.1893]
0.4	[0.0864,2.0000]	[0.0069,1.3333]	[0.0617,0.2632]	[0.0049,0.1754]
0.5	[0.0938,1.8750]	[0.0080,1.2228]	[0.0625,0.2500]	[0.0054,0.1630]
0.6	[0.1013,1.7619]	[0.0093,1.1240]	[0.0633,0.2381]	[0.0058,0.1419]
0.7	[0.1090,1.6591]	[0.0107,1.0352]	[0.0641,0.2273]	[0.0063,0.1418]
0.8	[0.1169,1.5652]	[0.0122,0.9550]	[0.0649,0.2174]	[0.0068,0.1326],
0.9	[0.1250,1.4792]	[0.0139,0.8825]	[0.0658,0.2083]	[0.0073,0.1243]
1	[0.1333,1.4000]	[0.0157,0.8167]	[0.0667,0.2000]	[0.0078,0.1167]









We performed by using MATLAB α cuts of arrival rate and service rate and fuzzy expected number of jobs in queue at eleven distinct α levels: 0, 0.1, 0.2, 0.3,0.4,0.5, 0.6, 0.7, 0.8, 0.9,1. Crisp intervals for fuzzy expected number of jobs in queue at different possibility α levels are presented in table. The performance measures such as expected number of customers in the system (L_s), expected length of queue (L_q), the average waiting time in the system (W_s), and the average waiting time of a customer in the queue (W_q) also derived in table.

From table:1

1. Expected number of customers in the system

   when α = 1 is [0.1429 , 1.0000] and α = 0 is [0.0625 ,1.7500]
2. Expected number of customers in the queue

   when α = 1 is [0.0179 , 0.5000] and α = 0 is [0.0037 ,1.1136]
3. Average waiting time of a customer in the system

   when α = 1 is [0.0714 ,0.1667] and α = 0 is [0.0625, 0.2500]
4. Average waiting time of a customer in the queue

   when α = 1 is [0.0089 ,0.0833] and α = 0 is [0.0037, 0.1591].

From table:2

1. Expected number of customers in the system

   when α = 1 is [0.1333 , 1.4000] and α = 0 is [0.0588 ,2.6667]
2. Expected number of customers in the queue

   when α = 1 is [0.0157 , 0.8167] and α = 0 is [0.0037 ,1.9394]
3. Average waiting time of a customer in the system

   when α = 1 is [0.0667 ,0.2000] and α = 0 is [0.0588, 0.3333]
4. Average waiting time of a customer in the queue

   when α = 1 is [0.0078 ,0.1167] and α = 0 is [0.0033, 0.2424].

7. CONCLUSION

In the above discussion we study the performance measures of Heptagonal and Octagonal fuzzy number using α-cut the arrival and service times are fuzzy in nature. The numerical example shows its coincidence.

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