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Optimal Inventory Policies for a Two-Warehouse Inventory Model under Time Dependent Quadratic Demand Rate

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Abstract

In this paper, we study a two-warehouse inventory model when demand follows a non-linear trend (i.e. Quadratic form) involving deterioration rate under permissible delay in payment. The objective of this study is to obtain minimum total cost. A condition is obtained when to rent a warehouse and the retailer's optimal replenishment policy that minimizes the total cost. The model is tested with suitable numerical examples. Sensitivity analysis is carried out to test the robustness of the model for managerial implications.

1. INTRODUCTION

Many researchers have considered inventory modelling is a mathematical approach to decide on how to order and when to order so as to minimise the total cost. Most of the classical inventory models considered the demand rate as constant or linear function of time (price) or exponential over time (price). Several authors argued that, in realistic terms, the demand need not follow either linear or exponential trend. So, it is reasonable to assume that the demand rate, in certain commodities, due to seasonal variations may follow quadratic function of time [i.e., $D(t) = a + bt + ct^2;\ a \geq 0, b \neq 0, c \neq 0$]. The functional form of time-dependent quadratic demand explains the accelerated growth/decline in the demand patterns which may arise due to seasonal demand rate (Khanra and Chaudhuri)[1]. We may explain different types of realistic demand patterns depending on the signs of *b* and *c*. Bhandari and Sharma[2] have studied a Single Period Inventory Problem with Quadratic Demand Distribution under the Influence of Marketing Policies. Khanra and Chaudhuri[1] have discussed an order-level inventory problem with the demand rate represented by a continuous quadratic function of time. It is well known that the demand for spare parts of new aero planes, computer chips of advanced computer machines, etc. increase very rapidly while the demands for spares of the obsolete aero planes, computers etc. decrease very rapidly with time. This type of phenomena can well be addressed by inventory models with quadratic demand rate. Sana and Chaudhuri [3] have developed a stock-review inventory model for perishable items with uniform replenishment rate and stock-dependent demand. Recently, Ghosh and Chaudhuri [4] have developed an inventory model for a deteriorating item having an instantaneous supply, a quadratic time-varying demand and shortages in inventory. They have used a two-parameter Weibull distribution to represent the time to deterioration. Venkateswarlu and Mohan [5] have developed inventory models for deteriorating items with time dependent quadratic demand and salvage value. Venkateswarlu and Mohan [6] studied inventory model for time varying deterioration and price dependent quadratic demand with salvage value.

Haley and Higgins [7] studied the relationship between inventory policy and credit policy in the context of the classical lot size model. Chapman et al. [8] developed an economic order quantity model which considers possible credit periods allowable by suppliers. This model is shown to be very sensitive to the length of the permissible credit period and to the relationship between the credit period and inventory level. Davis and Gaither [9] developed optimal order quantities for firms that are offered a onetime opportunity to delay payment for an order of a commodity. A mathematical model is developed by Goyal [10] when supplier announces credit period in settling the account, so that no interest charges are payable from the outstanding amount if the account is settled within the allowable delay period. Aggarwal and Jaggi [11] developed mathematical model for deteriorating inventories for which supplier allowed certain fixed period to settle the account. Shah et al. [12] extended the above model by allowing shortages. Mandal and Phaujdar [13], [14] have studied Goyal [10] model by including interest earned from the sales revenue on the stock remaining beyond the settlement period. Carlson and Rousseau [15] examined EOQ under date terms supplier credit by partitioning carrying cost into financial cost and variable holding costs. Chung and Huang [16] extended Goyal [10] model when replenishment rate is finite. Dallenbach [17],[18], Ward and Chapman [19], Chapman and Ward [20] argued that the usual assumptions as to the incidence and the value of the inventory investment opportunity cost made by the traditional inventory theory are correct and also established that if trade credit surplus is taken into account, the optimal ordering quantities decreases rather than increase. Chung [21] established the convexity of the total annual variable cost function for optimal economic order quantity under conditions of permissible delay in payments. Jamal et al. [22] discussed the problem in which the retailer can pay the supplier either at the end of credit period or later incurring interest charges on the unpaid balance for the overdue period. Sarker et al. [23] obtained optimal payment time under permissible delay in payments when units in an inventory are subject to deterioration. Abad and Jaggi [24] considered the seller-buyer channel in which the end demand is price sensitive and the suppler offers trade credit to the buyer. Shinn and Hwang [25] dealt with the problem of determining the retailer's optimal price and order size simultaneously under the condition of order size dependent delay in payments. It is assumed that the length of the credit period is a function of the retailer's order size and also the demand rate is a function of the selling price. Chung et al. [26] determined the economic order quantity under conditions of permissible delay in payments where the delay in payments depends on the quantity ordered when the order quantity is less than the quantity at which the delay in payments is permitted, the payment for the item must be made immediately. Otherwise, the fixed credit period is allowed. Huang [27] examined optimal retailer's replenishment decisions in the EOQ model under two levels of trade credit policy by assuming that the supplier would offer the retailer partially permissible delay in payments when the order quantity is smaller than a predetermined quantity. Teng et. al. [28] derived retailer's optimal ordering policies with trade credit financing. Reddy and Venkateswarlu [29] studied a deterministic inventory model for perishable items with price sensitive quadratic time dependent demand under trade credit policy.

Generally inventory modeling includes the deterioration of most of the items. Every firm in general has its own warehouse (OW) with a limited capacity. When retailers purchase more goods than the capacity of OW, the excess quantity can be stored in a rented warehouse (RW). Even in this situation, the deterioration is unavoidable. So to attract more number of customers, the retailers can give permissible delay in payments to settle the accounts. Hartley [30] developed the first two warehouse inventory model. Sarma [31] developed the inventory model which included two levels of storage and the optimum release rule. Sarma [32] extended his previous model to the case of infinite refilling rate with shortages. Ghosh and Chakrabarty [33] developed an order level inventory model with two levels of storage for deteriorating items. An EOQ model with two levels of storage was studied by Dave [34], considering distinct stage production schemes. Several researchers developed inventory models for deteriorating goods. The deterioration of goods is defined as damage, spoilage, and dryness of items like groceries, pictographic film, electronic equipment, etc. Pakkala and Acharya [35] developed a two warehouse inventory model for deteriorating items with finite replenishment rate and shortages. Benkherouf [36] developed a two warehouse model with deterioration and continuous release pattern. Lee and Ma [37] studied an optimal inventory policy for deteriorating items with two warehouse and time dependent demand. Zhou [38] developed two warehouse inventory models with time varying demand. Yanlai Liang and Fangming Zhou [39] developed a two warehouse inventory model for deteriorating items under conditionally permissible delay in payment. Sumdara Rajan and Uthayakumar [40] studied a two-warehouse inventory model for deteriorating items with permissible delay under exponentially increasing demand. Naresh Kumar et al [41] developed a two warehouse inventory model for deteriorating item with exponential demand rate and permissible delay in payment.

2. ASSUMPTIONS AND NOTATIONS

In developing the mathematical model of the inventory system for this study, the following assumptions are used.

2.1 Assumptions

1. The replenishment rate is infinite.
2. Lead time is zero.
3. The inventory model deals with single item
4. Deterioration occurs as soon as items are received into inventory
5. There is no replacement or repair of deteriorating items during the period under consideration
6. The Demand rate $D(t)$ at time'ť' is assumed to be $D(t) = a + bt + ct^2$ Where $a \geq 0,\ b \neq 0,\ c \neq 0$ Here 'a' is the initial rate of demand, 'b' is the initial rate of change of the demand and 'c' is the acceleration of demand rate.
7. Shortages are not allowed to occur.
8. The OW has a fixed capacity of W units and the RW has unlimited or infinite capability.
9. The RW is utilized only after OW is full, but stocks in RW are dispatched first.
10. The holding cost is h per unit of time (excluding interest charges), when $h = h_o$ for items in OW and $h = h_r$ for items in RW and $h_r > h_o$.
11. The items deteriorate at a constant rate α in OW and at θ in RW.

2.2 Notations

In developing the mathematical model of the inventory system for this study, the following assumptions are used.

1. **A** is the Ordering cost per order.
2. ***p*** is the unit purchase cost
3. ***s*** is the unit selling price $(s > p)$.
4. $\boldsymbol{h_r}$ is unit stock holding cost per unit of time in rented warehouse (excluding interest charges).
5. $\boldsymbol{h_o}$ is unit stock holding cost per unit of time in owned warehouse (excluding interest charges)
6. ***Q(t)*** is the Ordering quantity at time t=0
7. ***Ie*** is the interest earned per year per unit of time by retailer
8. ***Ic*** is the interest charged per stocks per year per unit of time by supplier.
9. ***w*** is the capacity of the owned warehouse (OW)
10. $\boldsymbol{w_1}$ is The maximum inventory level.
11. $\boldsymbol{t_w}$ is the time that inventory level reduce to W (decision variable)
12. ***M*** is the retailer's trade credit period offered by supplier per year, $0<M<1$.
13. ***T*** is the interval between two successive orders.
14. $\boldsymbol{I_r(t)}$ is the inventory level at time $t \in [0, tw]$ in rented warehouse (RW)
15. $\boldsymbol{I_o(t)}$ is the inventory level at time $t \in [0, T]$ in owned warehouse (OW)
16. $\boldsymbol{TC_1\ (t_w, T)}$, $\boldsymbol{TC_2\ (t_w, T)}$ and $\boldsymbol{TC_3\ (t_w, T)}$ are the total cost per unit time in a two-warehouse model.

3. FORMULATION AND SOLUTION OF THE MODEL

A lot size of particular units enters into the inventory system at time $t = 0$. In OW, *w* units are kept and the remaining units are stored in RW. The items stored in OW are consumed only when the items in RW are consumed first. The stock in RW decreases owing to combined effects of demand and deterioration during the interval [0, t_w], and it vanishes at $t = t_w$. However, the stock in OW depletes due to deterioration only during [0, t_w]. But during [t_w, T], the stock decreases due to combined effects of demand and deterioration. At time T, both the warehouses are empty. The entire process is repeated for every replenishment cycle

The inventory level in RW and OW at time $t \in [0, t_w]$ is described by following differential equations:

$$\frac{dI_r(t)}{dt} + \theta I_r(t) = -(a + bt + ct^2),\ \ 0 \le t \le t_w \tag{1}$$

With the boundary condition $I_r(t) = 0$

and

$$\frac{dI_0(t)}{dt} = -\alpha I_0(t),\ \ 0 \le t \le t_w \tag{2}$$

With the boundary condition $I_0(0) = w$

The inventory depletes due to demand and deterioration during $[t_w, T]$. At time T, the inventory level becomes zero and both warehouses are empty. The inventory level in OW *i.e.*, $I_0(t)$ is described by the following differential equation

$$\frac{dI_0(t)}{dt} + \alpha I_0(t) = -(a + bt + ct^2),\ \ t_w < t \le T \tag{3}$$

With the boundary condition $I_0(T) = 0$

The solutions of the above differential equations from (1) to (3) are

$$I_r(t) = a(t_w - t) + (a\theta + b)\left(\frac{t_w^{\ 2}}{2} - \frac{t^2}{2}\right) + (b\theta + c)\left(\frac{t_w^{\ 3}}{3} - \frac{t^3}{3}\right) + c\theta\left(\frac{t_w^{\ 4}}{4} - \frac{t^4}{4}\right) - (\theta t)\left[a(t_w - t) + b\left(\frac{t_w^{\ 2}}{2} - \frac{t^2}{2}\right) + c\left(\frac{t_w^{\ 3}}{3} - \frac{t^3}{3}\right)\right] \tag{4}$$

$$I_0(t) = we^{-\alpha t} \tag{5}$$

$$I_0(t)=a(T-t)+(a\alpha+b)\left(\frac{T^2}{2}-\frac{t^2}{2}\right)+(b\alpha+c)\left(\frac{T^3}{3}-\frac{t^3}{3}\right)+ c\alpha\left(\frac{T^4}{4}-\frac{t^4}{4}\right)-(\alpha t)\left[a(T-t)+b\left(\frac{T^2}{2}-\frac{t^2}{2}\right)+c\left(\frac{T^3}{3}-\frac{t^3}{3}\right)\right] \quad (6)$$

The maximum inventory level w_1 is given by $w_1 = I_r(0)+I_0(0)$

$$w_1 = at_w+(a\theta+b)\left(\frac{t_w{}^2}{2}\right)+(b\theta+c)\left(\frac{t_w{}^3}{3}\right)+c\theta\left(\frac{t_w{}^4}{4}\right)+w \quad (7)$$

The total relevant costs, TC, comprise following elements:

1. The ordering cost = A

2. Stock holding cost per year:

The increasing the inventory in RW during the interval $[0, t_w]$ and in OW during the interval [0,T] is

$$HC = h_r\int_0^{t_w} I_r(t)dt + h_0\int_0^{T} I_0(t)dt = h_r\int_0^{t_w} I_r(t)dt + h_0\int_0^{t_w} I_0(t)dt + h_0\int_{t_w}^{T} I_0(t)dt$$

$$\therefore HC = h_r\left(\frac{at_w^2}{2}+\frac{bt_w^3}{3}+\frac{ct_w^4}{4}+\frac{at_w^3\theta}{6}+\frac{bt_w^4\theta}{8}+\frac{ct_w^5\theta}{10}\right)-\frac{h_0w\left(e^{-\alpha t_w}-1\right)}{\alpha}+ \frac{h_0(T-t_w)^2}{120}\begin{pmatrix}60a+30T^2c+10ct_w^2+40Tb+20bt_w+20Ta\alpha\\ -20Tct_w-20at_w\alpha+15T^2b\alpha+12T^3c\alpha-5bt_w^2\alpha\\ -2ct_w^3\alpha-10Tbt_w\alpha-4Tct_w^2\alpha-6T^2ct_w\alpha\end{pmatrix} \quad (8)$$

3. The Deteriorating cost per year:

Cost of the deteriorating item per year in RW and OW during the interval [0, T] is

$$DC = \theta\int_0^{t_w} I_r(t)dt + \alpha\int_0^{T} I_0(t)dt = \theta\int_0^{t_w} I_r(t)dt + \alpha\int_0^{t_w} I_0(t)dt + \alpha\int_{t_w}^{T} I_0(t)dt$$

$$\therefore DC = \theta\left(\frac{at_w^2}{2}+\frac{bt_w^3}{3}+\frac{ct_w^4}{4}+\frac{at_w^3\theta}{6}+\frac{bt_w^4\theta}{8}+\frac{ct_w^5\theta}{10}\right)-\frac{\alpha w\left(e^{-\alpha t_w}-1\right)}{\alpha}+ \frac{\alpha(T-t_w)^2}{120}\begin{pmatrix}60a+30T^2c+10ct_w^2+40Tb+20bt_w+20Ta\alpha\\ -20Tct_w-20at_w\alpha+15T^2b\alpha+12T^3c\alpha-5bt_w^2\alpha\\ -2ct_w^3\alpha-10Tbt_w\alpha-4Tct_w^2\alpha-6T^2ct_w\alpha\end{pmatrix} \quad (9)$$

4. Case-1: The interest is payable.

Based on the parameters t_w, T and M there are three cases to be considered.

Case (a): $M \le t_w < T$

In this case, the interest payable is

$$IP_1 = pI_c \int_M^{t_w} I_r(t)dt + pI_c \int_M^{t_w} I_0(t)dt + pI_c \int_{t_w}^{T} I_0(t)dt$$

$$\therefore IP_1 = pI_c \begin{pmatrix} \frac{M^2a}{2} + \frac{M^3b}{6} + \frac{M^4c}{12} - \frac{at_w^2}{2} - \frac{bt_w^3}{6} - \frac{ct_w^4}{12} + Tat_w - \\ \frac{MT^2b}{2} - \frac{MT^3c}{3} - \frac{M^3a\theta}{6} - \frac{M^4b\theta}{24} - \frac{M^5c\theta}{60} + \\ \frac{T^2bt_w}{2} + \frac{T^3ct_w}{3} + \frac{a\theta t_w^3}{6} + \frac{b\theta t_w^4}{24} + \frac{c\theta t_w^5}{60} - MTa + \\ \frac{M^2T^2b\theta}{4} + \frac{M^2T^3c\theta}{6} - \frac{T^2b\theta t_w^2}{4} - \frac{T^3c\theta t_w^2}{6} - \\ \frac{MT^2a\theta}{2} + \frac{M^2Ta\theta}{2} - \frac{MT^3b\theta}{3} - \frac{MT^4c\theta}{4} - \frac{Ta\theta t_w^2}{2} + \\ \frac{T^2a\theta t_w}{2} + \frac{T^3b\theta t_w}{3} + \frac{T^4c\theta t_w}{4} \end{pmatrix}$$
$$- \frac{pI_c w\left(e^{-\alpha t_w} - e^{-\alpha M}\right)}{\alpha} + \qquad (10)$$
$$\frac{pI_c\left(T - t_w\right)^2}{120} \begin{pmatrix} 60a + 30T^2c + 10ct_w^2 + 40Tb + 20bt_w + 20Ta\alpha \\ - 20Tct_w - 20at_w\alpha + 15T^2b\alpha + 12T^3c\alpha - 5bt_w^2\alpha \\ - 2ct_w^3\alpha - 10Tbt_w\alpha - 4Tct_w^2\alpha - 6T^2ct_w\alpha \end{pmatrix}$$

Case (b): $t_w \le M < T$

In this case, the interest payable is

$$IP_2 = pI_c \int_M^{T} I_0(t)dt$$

$$\therefore IP_2 = \frac{pI_c\left(M - T\right)^2}{120} \begin{pmatrix} 60a + 30T^2c + 10cM^2 + 40Tb + 20bM + 20Ta\alpha \\ - 20TcM - 20aM\alpha + 15T^2b\alpha + 12T^3c\alpha - 5bM^2\alpha \\ - 2cM^3\alpha - 10TbM\alpha - 4TcM^2\alpha - 6T^2cM\alpha \end{pmatrix} \qquad (11)$$

Case (c): $M > T$

In this case, the interest payable is zero

5. Case-2: The interest earned.

There are two cases to be considered.

Case (a): $M < T$

In this case the interest earned is

$$IE_1 = sI_e \int_0^M D(t)tdt = sI_e \int_0^M \left(a + bt + ct^2\right)(t)dt$$

$$\therefore IE_1 = sI_e \left(\frac{M^2(3cM^2 + 4bM + 6a)}{12} \right) \tag{12}$$

Case (b): $M > T$

In this case the interest earned is

$$IE_2 = sI_e \int_0^T D(t)tdt + D(T)T(M - T)$$

$$\therefore IE_2 = sI_e \left(\frac{T^2(3cT^2 + 4bT + 6a)}{12} + T(M - T)(cT^2 + bT + a) \right) \tag{13}$$

Thus, the total relevant cost per year for the retailer is given by

$$TC(t_w, T) = \left(\frac{1}{T}\right) \begin{pmatrix} \text{Ordering cost} + \text{Stock holding cost in RW} \\ + \text{Stock holding cost in OW} + \text{Deterioration cost} \\ + \text{Opportunity cost with interest - Interest earned} \end{pmatrix}$$

The total relevant costs for the retailer are given as:

$$TC(t_w, T) = \begin{cases} TC_1 & M \le t_w < T \\ TC_2 & t_w \le M < T \\ TC_3 & M > T \end{cases}$$

where,

$$TC_1 = \frac{1}{T}\left[\begin{array}{l} A + (h_r + \theta)\left(\frac{at_w^2}{2} + \frac{bt_w^3}{3} + \frac{ct_w^4}{4} + \frac{at_w^3\theta}{6} + \frac{bt_w^4\theta}{8} + \frac{ct_w^5\theta}{10}\right) + \\ pI_c\left(\begin{array}{l} \frac{M^2a}{2} + \frac{M^3b}{6} + \frac{M^4c}{12} - \frac{at_w^2}{2} - \frac{bt_w^3}{6} - \frac{ct_w^4}{12} + Tat_w - \frac{MT^2b}{2} - \frac{MT^3c}{3} - \\ \frac{M^3a\theta}{6} - \frac{M^4b\theta}{24} - \frac{M^5c\theta}{60} + \frac{T^2bt_w}{2} + \frac{T^3ct_w}{3} + \frac{a\theta t_w^3}{6} + \frac{b\theta t_w^4}{24} + \frac{c\theta t_w^5}{60} - \\ MTa + \frac{M^2T^2b\theta}{4} + \frac{M^2T^3c\theta}{6} - \frac{T^2b\theta t_w^2}{4} - \frac{T^3c\theta t_w^2}{6} - \frac{MT^2a\theta}{2} + \frac{M^2Ta\theta}{2} - \\ \frac{MT^3b\theta}{3} - \frac{MT^4c\theta}{4} - \frac{Ta\theta t_w^2}{2} + \frac{T^2a\theta t_w}{2} + \frac{T^3b\theta t_w}{3} + \frac{T^4c\theta t_w}{4} \end{array}\right) \\ - \frac{(h_0 + \alpha)w\left(e^{-\alpha t_w} - 1\right)}{\alpha} - \frac{pI_c w\left(e^{-\alpha t_w} - e^{-\alpha M}\right)}{\alpha} - sI_e\left(\frac{M^2(3cM^2 + 4bM + 6a)}{12}\right) + \\ \frac{(h_0 + \alpha + pI_c)(T - t_w)^2}{120}\left(\begin{array}{l} 60a + 30T^2c + 10ct_w^2 + 40Tb + 20bt_w + 20Ta\alpha - \\ 20Tct_w - 20at_w\alpha + 15T^2b\alpha + 12T^3c\alpha - 5bt_w^2\alpha - \\ 2ct_w^3\alpha - 10Tbt_w\alpha - 4Tct_w^2\alpha - 6T^2ct_w\alpha \end{array}\right) \end{array}\right] \quad (14)$$

$$TC_2 = \frac{1}{T}\left[\begin{array}{l} A + (h_r + \theta)\left(\frac{at_w^2}{2} + \frac{bt_w^3}{3} + \frac{ct_w^4}{4} + \frac{at_w^3\theta}{6} + \frac{bt_w^4\theta}{8} + \frac{ct_w^5\theta}{10}\right) - \frac{(h_0 + \alpha)w\left(e^{-\alpha t_w} - 1\right)}{\alpha} \\ + \frac{(h_0 + \alpha)(T - t_w)^2}{120}\left(\begin{array}{l} 60a + 30T^2c + 10ct_w^2 + 40Tb + 20bt_w + 20Ta\alpha \\ - 20Tct_w - 20at_w\alpha + 15T^2b\alpha + 12T^3c\alpha - 5bt_w^2\alpha \\ - 2ct_w^3\alpha - 10Tbt_w\alpha - 4Tct_w^2\alpha - 6T^2ct_w\alpha \end{array}\right) \\ + \frac{pI_c(M - T)^2}{120}\left(\begin{array}{l} 60a + 30T^2c + 10cM^2 + 40Tb + 20bM + 20Ta\alpha \\ - 20TcM - 20aM\alpha + 15T^2b\alpha + 12T^3c\alpha - 5bM^2\alpha \\ - 2cM^3\alpha - 10TbM\alpha - 4TcM^2\alpha - 6T^2cM\alpha \end{array}\right) \\ - sI_e\left(\frac{M^2(3cM^2 + 4bM + 6a)}{12}\right) \end{array}\right] \quad (15)$$

$$TC_3 = \frac{1}{T}\left[\begin{array}{l} A+\left(h_r+\theta\right)\left(\frac{at_w^2}{2}+\frac{bt_w^3}{3}+\frac{ct_w^4}{4}+\frac{at_w^3\theta}{6}+\frac{bt_w^4\theta}{8}+\frac{ct_w^5\theta}{10}\right) \\ -\frac{\left(h_0+\alpha\right)w\left(e^{-\alpha t_w}-1\right)}{\alpha} \\ +\frac{\left(h_0+\alpha\right)\left(T-t_w\right)^2}{120}\left(\begin{array}{l} 60a+30T^2c+10ct_w^2+40Tb+20bt_w+20Ta\alpha \\ -20Tct_w-20at_w\alpha+15T^2b\alpha+12T^3c\alpha-5bt_w^2\alpha \\ -2ct_w^3\alpha-10Tbt_w\alpha-4Tct_w^2\alpha-6T^2ct_w\alpha \end{array}\right) \\ -sI_e\left(\frac{T^2(3cT^2+4bT+6a)}{12}+T(M-T)(cT^2+bT+a)\right) \end{array}\right] \tag{16}$$

The optimal values of t_w and T are obtained by solving

$$\frac{\delta TC_i(t_w,T)}{\delta t_w}=0 \text{ and } \frac{\delta TC_i(t_w,T)}{\delta T}=0 \text{ for i =1, 2, 3}$$

The necessary and sufficient conditions to minimise total relevant cost per unit time is

$$\frac{\delta^2 TC_i(t_w,T)}{\delta t_w^2}>0, \quad \frac{\delta^2 TC_i(t_w,T)}{\delta T^2}>0 \tag{17}$$

and

$$\left(\frac{\delta^2 TC_i(t_w,T)}{\delta t_w^2}\right)\left(\frac{\delta^2 TC_i(t_w,T)}{\delta T^2}\right)-\left(\frac{\delta^2 TC_i(t_w,T)}{\delta t_w \delta T}\right)^2>0 \tag{18}$$

Using these optimal values of t_w and T the optimal value of w_1 can be obtained from the equation (7)

4. NUMERICAL EXAMPLE

The following hypothetical data is taken to validate the effectiveness of the models developed:

T =1,	a = 500,	b = 25,	c = 0.5,	A = 200,	α = 0.1,
θ =0.06,	h_r = 3,	h_0 = 1,	W = 100,	M = 0.25,	I_c = 0.12,
I_e = 0.09,	s = 12,	p = 8,	t_w = 0.05		

The optimality conditions given by (17) and (18) are satisfied all types of Total costs with the choice of the parameters given above. For these values the optimum values of t_w, cycle time T, total cost TC_1 and the maximum inventory level w_1 of the system are 0.066, 0.593, 581.534 and 125.069 respectively. Table-1 shows the results of various models. It is observed that the behaviour maximum inventory level w_1 of the system is similar and the values of t_w, cycle time (T) and total cost TC_1 slightly changes in these models.

Table-1

Case	t_w	T	$TC(t_w,T)$	w_1
1	0.066	0.593	581.534	125.069
2	0.124	0.647	597.587	125.069
3	0.119	0.627	618.883	125.069

5. SENSITIVE ANALYSIS

We now study sensitivity of the models developed to examine the implications of underestimating and overestimating the parameters individually on optimal value of total cost. The Sensitive analysis is performed by changing each of the parameter by -15%, -5%, +5% and +15% taking one parameter at a time and keeping the remaining parameters are unchanged. Since all models show slightly variation in results, we will present the sensitivity for total cost for the first case. The results are shown in Table-2. The following observations are made from this table:

(i) The Total cost function TC_1 is highly sensitive to the changes in the parameter 'a', 'A', 'h_0', 'α', 'M', 'I_c', and 'p'.

(ii) The Total cost function TC_1 is moderately sensitive to the changes in the parameter 'b', 'w', 'Ie', 's',

(iii) The Total cost function TC_1 is less sensitive to all other parameters namely *'c'*, 'θ', and 'h_r'.

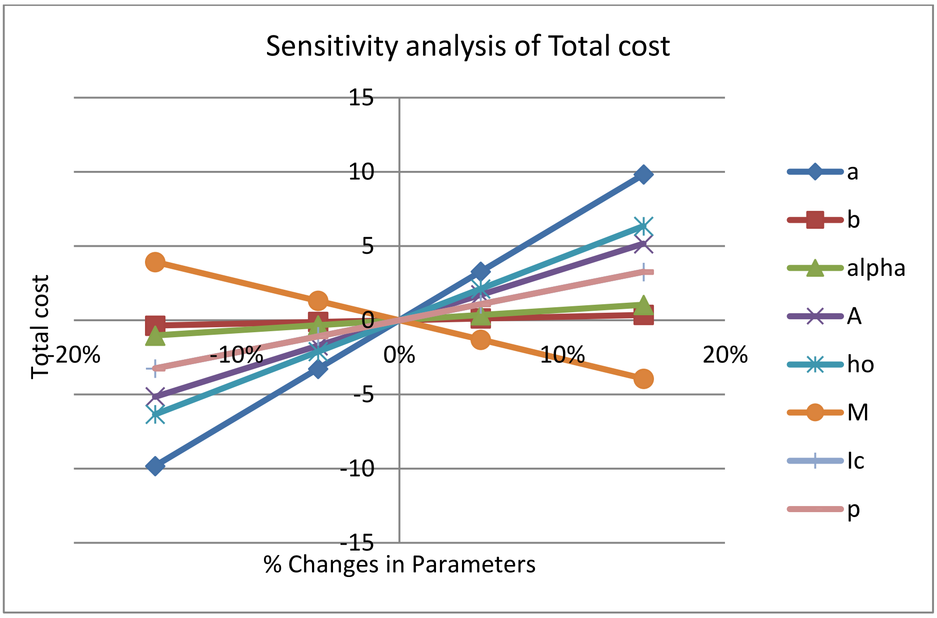
(iv) The maximum inventory level w_1 is highly sensitive to the changes in the parameter 'w', 'a' and there is no sensitive to all other parameters 'A', 'h_0', 'α', 'M', 'I_c', 'b', 'w', 'Ie', 's', 'p', *'c'*, 'θ', and 'h_r'.

Parameters	%change	Change in t_w (%)	Change in T(%)	Change in TC_1 (%)	Change in W_1 (%)
a	-15%	-7.5758	7.4199	-9.8237	-3.0031
	-5%	-3.0303	2.3609	-3.2746	-1.0010
	5%	0.0000	-2.1922	3.2746	1.0010
	15%	3.0303	-6.0708	9.8237	3.0023
b	-15%	0.0000	0.1686	-0.3584	-0.0040
	-5%	0.0000	0.1686	-0.1195	-0.0016
	5%	0.0000	0.0000	0.1193	0.0008
	15%	-1.5152	-0.1686	0.3582	0.0040
c	-15%	0.0000	0.0000	-0.0057	0.0000
	-5%	0.0000	0.0000	-0.0019	0.0000

Parameters	**%change**	**Change in t_w (%)**	**Change in T(%)**	**Change in TC_1 (%)**	**Change in W_1 (%)**
	5%	0.0000	0.0000	0.0019	0.0000
	15%	0.0000	0.0000	0.0055	0.0000
θ	-15%	0.0000	0.0000	0.0549	-0.0048
	-5%	0.0000	0.0000	0.0182	-0.0016
	5%	-1.5152	0.0000	-0.0184	0.0016
	15%	-1.5152	0.0000	-0.0549	0.0040
α	-15%	-3.0303	0.3373	-1.0326	0.0000
	-5%	-1.5152	0.1686	-0.3449	0.0000
	5%	0.0000	-0.1686	0.3455	0.0000
	15%	1.5152	-0.3373	1.0383	0.0000
A	-15%	-24.2424	-9.2749	-5.1588	0.0000
	-5%	-9.0909	-3.0354	-1.7196	0.0000
	5%	7.5758	2.8668	1.7196	0.0000
	15%	21.2121	8.4317	5.1588	0.0000
h_r	-15%	13.6364	0.6745	-0.0485	0.0000
	-5%	3.0303	0.1686	-0.0162	0.0000
	5%	-4.5455	-0.1686	0.0162	0.0000
	15%	-12.1212	-0.5059	0.0485	0.0000
h_0	-15%	-13.6364	2.6981	-6.3417	0.0000
	-5%	-4.5455	0.8432	-2.1139	0.0000
	5%	3.0303	-0.8432	2.1139	0.0000
	15%	10.6061	-2.3609	6.3417	0.0000
w	-15%	25.7576	1.8550	0.3463	-11.9934
	-5%	9.0909	0.6745	0.1154	-3.9978
	5%	-9.0909	-0.6745	-0.1156	3.9978
	15%	-28.7879	-2.0236	-0.3463	11.9934
M	-15%	1.5152	1.1804	3.9301	0.0000
	-5%	0.0000	0.3373	1.3110	0.0000
	5%	-1.5152	-0.3373	-1.3120	0.0000
	15%	-3.0303	-1.1804	-3.9389	0.0000
I_c	-15%	22.7273	5.0590	-3.2548	0.0000
	-5%	6.0606	1.6863	-1.0851	0.0000
	5%	-7.5758	-1.5177	1.0849	0.0000
	15%	-22.7273	-4.5531	3.2548	0.0000

Parameters	**%change**	**Change in t_w (%)**	**Change in T(%)**	**Change in TC_1 (%)**	**Change in W_1 (%)**
I_e	-15%	1.5152	0.8432	0.4388	0.0000
	-5%	0.0000	0.3373	0.1463	0.0000
	5%	-1.5152	-0.1686	-0.1463	0.0000
	15%	-3.0303	-0.6745	-0.4390	0.0000
s	-15%	1.5152	0.8432	0.4388	0.0000
	-5%	0.0000	0.3373	0.1463	0.0000
	5%	-1.5152	-0.1686	-0.1463	0.0000
	15%	-3.0303	-0.6745	-0.4390	0.0000
p	-15%	1127.2727	5.0590	-3.2548	0.0000
	-5%	6.0606	1.6863	-1.0851	0.0000
	5%	-7.5758	-1.5177	1.0849	0.0000
	15%	-22.7273	-4.5531	3.2548	0.0000

Fig-1 also shows the variations of the system Total cost for the change made in some parameters.



Figer-1: Variations of total cost w.r.t the values of some important parameters

6. CONCLUSIONS

In this paper we have developed an inventory model for two-warehouse having quadratic time dependent demand rate with constant rates of deterioration with trade credit policy. It is observed that the model is highly sensitive to initial demand, ordering cost and holding cost of own-warehouse. The total cost of the inventory system increases (decreases) with the increase in initial demand, ordering cost and holding cost of own-warehouse. Also it is noted that the changes in total cost of the system are insignificant with respect to the changes in holding cost of rented-warehouse, selling price and the capacity of owned-warehouse.

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