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A Note on Fuzzy B* Sets

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Abstract

In this paper, the conditions for fuzzy simply* open sets to become fuzzy B* sets in fuzzy topological spaces are obtained. It is established that fuzzy pre -open sets with fuzzy Baire property, fuzzy β - open sets with fuzzy Baire property in fuzzy topological spaces and fuzzy residual sets with fuzzy Baire property in fuzzy P - spaces, are fuzzy B* sets. The conditions for fuzzy hyperconnected spaces to become fuzzy Baire spaces, fuzzy Volterra spaces are also obtained.

Keywords: Fuzzy G_δ - set, fuzzy first category set, fuzzy simply open set, fuzzy residual set, fuzzy simply* open set, fuzzy Baire property, fuzzy Baire space, fuzzy hyperconnected space.

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1. INTRODUCTION

In order to deal with uncertainties, the idea of fuzzy sets, fuzzy set operations was introduced by **L.A.ZADEH [19]** in 1965. By applying the fuzzy set notions to general topology **C.L.CHANG [5]** introduced the theory of fuzzy topological spaces. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts.

D. K,GANGULY AND CHANDARANI MITRA [7**]** introduced and studied the concept of B* sets in classical topology. This notion in fuzzy setting was introduced and studied by the authors in [**17**]. The purpose of this paper is to study several characterizations of fuzzy B* sets in fuzzy topological spaces. In section 3, the conditions for fuzzy simply* open sets to become fuzzy B* sets in fuzzy topological spaces, are obtained. It is established that fuzzy pre -open sets with fuzzy Baire property, fuzzy β - open sets with fuzzy Baire property in fuzzy topological spaces and fuzzy residual sets with fuzzy Baire property in fuzzy P- spaces, are fuzzy B* sets. It is also established that in fuzzy topological spaces where fuzzy first category sets are not fuzzy dense sets, fuzzy residual sets with fuzzy Baire property, are fuzzy B* sets and fuzzy dense and fuzzy G_δ - sets with fuzzy Baire property in fuzzy GID spaces, fuzzy strongly irresolvable spaces are fuzzy B* sets. In section 4, the conditions under which hyper connected spaces become fuzzy Baire spaces, fuzzy Volterra spaces, are obtained.

2. PRELIMINARIES

In order to make the exposition self-contained, some basic notions and results used in the sequel are given. In this work (X,T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non- empty set and I, the unit interval [0, 1]. A fuzzy set λ in X is a function from X into I . The null set 0 is the function from X into I which assumes only the value 0 and the whole fuzzy set 1 is the function from X into I takes the value 1 only.

Definition 2.1 [5]: Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). The interior and the closure of λ defined as follows

(i) $\text{Int}(\lambda) = \vee \{ \mu / \mu \leq \lambda, \mu \in T \}$.

(ii) $\text{Cl}(\lambda) = \wedge \{ \mu / \lambda \leq \mu, 1 - \mu \in T \}$.

Lemma 2.1 [1]: For a fuzzy topological space X,

(i) $1 - \text{int}(\lambda) = \text{cl}(1 - \lambda)$.

(ii) $1 - \text{cl}(\lambda) = \text{int}(1 - \lambda)$.

Definition 2.2 : A fuzzy set λ in a fuzzy topological space (X, T) is called

(i). ***fuzzy dense*** if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$ **[13].**

(ii). ***fuzzy nowhere dense*** if there exists no non - zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$. That is, int cl (λ) = 0, in (X, T) **[13].**

(iii). ***fuzzy somewhere dense*** if int cl (λ) ≠ 0, in (X,T) **[10].**

(iv). ***fuzzy first category set*** if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy second category **[13].**

(v). ***fuzzy simply open set*** if Bd (λ) is a fuzzy nowhere dense set in (X,T).

That is, λ is a fuzzy simply open set in (X,T) if [cl (λ) ∧ cl (1− λ)], is a fuzzy nowhere dense set in (X,T) **[6].**

(vi). ***fuzzy simply* open set*** if $\lambda = \mu \vee \delta$, where μ is a fuzzy open set and δ is a fuzzy nowhere dense set in (X,T) and 1 – λ is called a fuzzy simply* closed set in (X,T) **[6].**

(vii). ***fuzzy*** **G_δ - set** in (X,T) if $\lambda = \bigwedge_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i \in T$ for $i \in I$ **[2].**

(viii). ***fuzzy*** **F_σ - set** in (X,T) if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$ where $1 - \lambda_i \in T$ for $i \in I$ **[2].**

(ix). ***fuzzy*** **β -open** in (X,T) if λ ≤ cl int cl(λ) and fuzzy closed if int cl int(λ) ≤ λ **[3].**

(x). ***fuzzy strongly first category set*** if $\lambda = \bigwedge_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy strongly nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be a fuzzy strongly second category set in (X,T) **[9].**

(xi). ***fuzzy pre- open*** if λ ≤ int cl (λ) and ***fuzzy pre closed*** if cl int (λ) ≤ λ **[4].**

Definition 2.3 [13] : Let λ be a fuzzy first category set in a fuzzy topological space (X,T). Then, 1 – λ is called a fuzzy residual set in (X,T).

Definition 2.4 [9]: Let (X,T) be a fuzzy topological space. A fuzzy set λ defined on X is said to have the fuzzy Baire property, if $\lambda = (\mu \vee \delta) \wedge \eta$, where μ is a fuzzy open set, δ is a fuzzy residual set and η is a fuzzy first category set in (X,T).

Definition 2.5 : A fuzzy topological space (X,T) is called a

(i). ***fuzzy Baire space*** if int $(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where $(\lambda_i)'$s are fuzzy nowhere dense sets in (X,T) **[11].**

(ii). ***fuzzy sub maximal space*** if for each fuzzy set λ in (X,T) such that cl(λ) = 1,then λ ϵ T in (X,T) **[2].**

(iii). ***fuzzy strongly Baire space*** if $\text{cl}(\wedge_{i=1}^{N}(\lambda_i)) = 1$, where (λ_i)'s are fuzzy nowhere dense sets in (X,T) **[9].**

(iv). ***fuzzy GID space*** if for each fuzzy dense and fuzzy G_δ set λ in (X,T), clint(λ) = 1in (X,T) **[16].**

(v). ***fuzzy P- space*** if each fuzzy G_δ – set in (X,T) is a fuzzy open set in (X,T) **[12].**

(vi). ***fuzzy hyper-connected*** if each non-null fuzzy open subset of (X,T) is fuzzy dense set in (X,T). That is, a fuzzy topological space (X,T) is fuzzy hyper-connected if $\text{cl}(\mu_i) = 1$, for all $\mu_i \in T$ **[8].**

(vii). ***fuzzy Volterra space*** if $\text{cl}(\wedge_{i=1}^{N}(\lambda_i)) = 1$, where (λ_i)'s are fuzzy dense and G_δ - sets in (X,T) **[15].**

(viii). ***fuzzy first category space*** if the fuzzy set 1_X is a fuzzy first category set in (X,T). That is, $1_X = (\vee_{i=1}^{\infty}(\lambda_i))$, where $(\lambda_i)'$s are fuzzy nowhere dense sets in (X,T) **[1].** Otherwise (X,T) is said to be of fuzzy second space **[13].**

Theorem 2.1 [6] : If λ is a fuzzy simply* open set in a fuzzy topological space (X,T), then int (λ) ≠ 0, in (X,T).

Theorem 2.2 [6]: If λ is a fuzzy simply* open set in a hyper connected space (X,T), then λ is a fuzzy simply open set in (X,T).

Theorem 2.3 [17]: If λ is a fuzzy B* set in a fuzzy hyper connected space (X,T), then $1 - \lambda$ is a fuzzy nowhere dense set in (X,T).

Theorem 2.4 [18]: If λ is a fuzzy residual set in a fuzzy P-space (X,T), then λ is a fuzzy somewhere dense set in (X,T).

Theorem 2.5 [18]: If λ is a fuzzy residual set in a fuzzy topological space (X,T) in which fuzzy first category sets are not fuzzy dense sets, then λ is a fuzzy somewhere dense set in (X,T).

Theorem 2.6 [9]: If (X,T) is a fuzzy hyperconnected space (X,T), then (X,T) is a fuzzy strongly Baire space.

3. FUZZY B* SETS

Definition 3.1 [17]: Let (X,T) be a fuzzy topological space. A fuzzy set λ defined on X is called a fuzzy B* set, if λ is a fuzzy set with fuzzy Baire property in (X,T) such that int cl(λ) $\neq 0$, in (X,T). That is, if λ is a fuzzy somewhere dense set having fuzzy Baire property in (X,T), then λ is a fuzzy B* set in (X,T).

Proposition 3.1: If λ is a fuzzy simply* open set with fuzzy Baire property in a fuzzy topological space (X,T), then λ is a fuzzy B* set in (X,T).

Proof: Let λ be a fuzzy simply* open set with fuzzy Baire property in (X,T). Since λ is a fuzzy simply* open set in (X,T), by theorem 2.1, int (λ) $\neq 0$ in (X,T). Now int (λ) $\leq$ int cl(λ) implies that intcl(λ) $\neq 0$ in (X,T). Thus λ is a fuzzy somewhere dense set in (X,T) with the fuzzy Baire property. Hence λ is a fuzzy B* set in (X,T).

Proposition 3.2 : If λ is a fuzzy simply* open set with fuzzy Baire property in a fuzzy hyperconnected space (X,T), then λ is a fuzzy B* set in (X,T) such that intcl [bd (λ)] = 0.

Proof: Let λ be a fuzzy simply* open set with fuzzy Baire property in (X,T). Then, by proposition 3.1, λ is a fuzzy B* set in (X,T). Since (X,T) is a fuzzy hyperconnected space by theorem 2.2, the fuzzy simply* open set λ is a fuzzy simply open set in (X,T). Then, intcl [bd (λ)] = 0, in (X,T). Thus, λ is a fuzzy B* set in (X,T) such that int cl [bd (λ)] = 0, in (X,T).

Proposition 3.3: If λ is a fuzzy simply* open set with fuzzy Baire property in a fuzzy hyperconnected space (X,T), then λ is a fuzzy B* set such that int cl(λ) $\leq$ cl int (λ), in (X,T).

Proof : Let λ be a fuzz simply* open set with the fuzzy Baire property in (X,T). Since (X,T) is a fuzzy hyperconnected space, by proposition 3.2, λ is a fuzzy B* set in (X,T) such that int cl [bd (λ)] = 0, in (X,T). Now int cl [bd (λ)] = intcl [cl (λ) $\wedge$ cl (1 $-$ λ)] implies that int cl [cl (λ) $\wedge$ cl (1 $-$ λ)] = 0 in (X,T). But, int [cl (λ) $\wedge$ cl (1 $-\lambda$)] $\leq$ int cl [cl (λ) $\wedge$ cl (1 $-\lambda$)] implies that int [cl (λ) $\wedge$ cl (1 $-$ λ)] = 0, in (X,T). Then, [int cl (λ)] $\wedge$ [int cl (1$-$ λ)] = 0 and thus int cl (λ) $\leq$ (1 – [int cl (1 – λ)]) and thus int cl (λ) $\leq$ cl int (λ) in (X,T). Hence λ is a fuzzy B* set in (X,T) such that int cl (λ) $\leq$ cl int (λ), in (X,T).

Proposition 3.4 : If λ is a fuzzy simply* open set with fuzzy Baire property in a fuzzy hyperconnected space (X,T), then λ is a fuzzy B* set such that int cl $(1 - \lambda) = 0$, in (X,T).

Proof : Let λ be a fuzzy simply* open set with fuzzy Baire property in a fuzzy hyperconnected space (X,T). Then by proposition 3.1, λ is a fuzzy B* set in (X,T). Since (X,T) is a fuzzy hyper-connected space, by theorem 2.3, 1 – λ is a fuzzy nowhere dense set in (X,T) and thus λ is a fuzzy B* set such that int cl $(1- \lambda) = 0$, in (X,T).

Proposition 3.5 : If λ is a non- zero fuzzy pre -open set with fuzzy Baire property in a fuzzy topological space (X,T), then λ is a fuzzy B* set in (X,T).

Proof : Let λ be a non-zero fuzzy pre-open set in (X,T). Then $\lambda \leq$ int cl (λ), in (X,T). Then, int cl (λ) $\neq 0$, in (X,T). Thus, λ is a fuzzy somewhere dense set with fuzzy Baire property in (X,T) and hence λ is a fuzzy B* set in (X,T).

Proposition 3.6 : If λ is a non- zero fuzzy β-open set with fuzzy Baire property in a fuzzy topological space (X,T), then λ is a fuzzy B* set in (X,T).

Proof : Let λ be a non zero fuzzy β-open set with fuzzy Baire property in (X,T). Since λ is a fuzzy β –open set in (X,T), $\lambda \leq$ cl int cl(λ), in (X,T). Then, int cl (λ) $\neq 0$, in (X,T) [otherwise, int cl (λ) $= 0$, will implies that $\lambda \leq$ cl(0) and in turn it will be that $\lambda = 0$, a contradiction]. Thus, λ is a fuzzy somewhere dense set with the fuzzy Baire property and hence λ is a fuzzy B* set in (X,T).

Proposition 3.7: If λ is a non-zero fuzzy β-open set with cl (λ) having the fuzzy Baire property in a fuzzy topological space (X,T), then cl (λ) is a fuzzy B* set in (X,T).

Proof : Let λ be a non zero fuzzy β-open set in (X,T). Then, as in the proof of proposition 3.6, int cl (λ) $\neq 0$ in (X,T) and int cl(λ) $\leq$ int cl [cl (λ)] implies that int cl [cl (λ)] $\neq 0$ and thus cl (λ) is a fuzzy somewhere dense set in (X,T). By hypothesis cl (λ) is a fuzzy set with fuzzy Baire property in (X,T). Thus, the fuzzy somewhere dense set cl (λ) with the fuzzy Baire property, is a fuzzy B* set in (X,T).

Proposition 3.8 : If λ is a fuzzy residual set with fuzzy Baire property in a fuzzy P -space (X,T), then λ is a fuzzy B* set in (X,T).

Proof : Let λ be a fuzzy residual set with fuzzy Baire property in (X,T). Since (X,T) is a fuzzy P-space, by theorem 2.5, the fuzzy residual set λ is a fuzzy somewhere dense set in (X,T). Hence λ is a fuzzy B* set in (X,T).

Proposition 3.9: If λ is a fuzzy residual set with fuzzy Baire property in a fuzzy topological space (X,T) in which fuzzy first category sets are not fuzzy dense sets, then λ is a fuzzy B* set in (X,T).

Proof: Let λ be a fuzzy residual set with fuzzy Baire property in (X,T). By hypothesis the fuzzy first category sets are not fuzzy dense sets in (X,T). Then, by theorem 2.6, λ is a fuzzy somewhere dense set in (X,T). Hence the fuzzy residual set λ is a fuzzy somewhere dense set with fuzzy Baire property and thus λ is a fuzzy B* set in (X,T).

Proposition 3.10: If $\lambda \leq \mu$ and λ is a fuzzy somewhere dense set and μ is a fuzzy set with fuzzy Baire property in a fuzzy topological space (X,T), then μ is a fuzzy B* set in (X,T).

Proof: Suppose that $\lambda \leq \mu$ in (X,T). Then, int cl $(\lambda) \leq$ int cl(μ) in (X,T). Since λ is a fuzzy somewhere dense set in (X,T) int cl$(\lambda) \neq 0$ in (X,T) and then int cl$(\mu) \neq 0$ Thus μ is a fuzzy somewhere dense set with fuzzy Baire property in (X,T). Hence μ is a fuzzy B* set in (X,T).

Proposition 3.11: If $(\lambda \vee \mu)$ is a fuzzy set with fuzzy Baire property, where λ is a fuzzy set defined on X and μ is a fuzzy somewhere dense set in (X,T), then $(\lambda \vee \mu)$ is a fuzzy B* set in (X,T).

Proof: Now int cl $(\lambda \vee \mu) =$ int[cl$(\lambda) \vee$ cl(μ)] $\geq$ int cl$(\lambda) \vee$ intcl$(\mu) \geq$ intcl(μ), where λ and μ are fuzzy sets defined on X. Since μ is a fuzzy somewhere dense set in (X,T), int cl$(\mu) \neq 0$ in (X,T) and thus int cl $(\lambda \vee \mu) \geq 0$. This implies that $(\lambda \vee \mu)$ is a fuzzy somewhere dense set in (X,T). By hypothesis $(\lambda \vee \mu)$ is a fuzzy set with fuzzy Baire property in (X,T) and thus $(\lambda \vee \mu)$ is a fuzzy somewhere dense set with fuzzy Baire property in (X,T). Hence $(\lambda \vee \mu)$ is a fuzzy B* set in (X,T).

Proposition 3.12: If λ is a fuzzy B* set in a fuzzy topological space (X,T), then there exists a fuzzy closed set μ in (X,T) such that int $(1 - \lambda) \leq \mu$

Proof: Let λ be a fuzzy B* set in (X,T). Then λ is a fuzzy somewhere dense set with fuzzy Baire property in (X,T). Since λ is a fuzzy somewhere dense set in (X,T), int cl $(\lambda) \neq 0$ in (X,T) and then $1-$ int cl$(\lambda) \neq 1$ and hence cl int $(1 - \lambda) \neq 1$, in (X,T). Thus int $(1- \lambda)$ is not a fuzzy dense set in (X,T). Then there exist a fuzzy closed set μ in (X,T) such that int $(1 - \lambda) \leq \mu$.

Proposition 3.13: If λ is a fuzzy G_δ -set with fuzzy Baire property such that cl$(\lambda) = 1$ in a fuzzy GID space (X,T), then λ is a fuzzy B* set in (X,T).

Proof: Let λ be a fuzzy G_δ-set with fuzzy Baire property in (X,T). Now cl (λ) = 1, in (X,T) implies that λ is a fuzzy dense and fuzzy G_δ -set in (X,T). Since (X,T) is a fuzzy GID space, cl int(λ) = 1 in (X,T). Then, int (λ) ≠ 0 and this implies that int cl (λ) ≠ 0 in (X,T). Hence λ is a fuzzy somewhere dense set in (X,T) with fuzzy Baire property. Hence λ is a fuzzy B* set in (X,T).

Proposition 3.14: If λ is a fuzzy G_δ-set with fuzzy Baire property such that cl (λ) = 1 in a fuzzy strongly irresolvable space, then λ is a fuzzy B* set in (X,T).

Proof: Let λ be a fuzzy G_δ set with fuzzy Baire property in (X,T). By hypothesis cl(λ) = 1 in (X,T). Since (X,T) is a fuzzy strongly irresolvable space, for the fuzzy dense set λ in (X,T), cl int (λ) = 1 in (X,T). Then, int (λ) ≠ 0, in (X,T). Since int(λ) ≤ int cl(λ), int c(λ) ≠ 0 in (X,T) and thus λ is a fuzzy somewhere dense set in (X,T). Hence λ is a fuzzy somewhere dense set fuzzy Baire property in (X,T) and therefore λ is a fuzzy B* set in (X,T).

Proposition 3.15 : If λ is a fuzzy simply* open set with fuzzy Baire property in a fuzzy hyperconnected space (X,T), then λ is a fuzzy dense set in (X,T) .

Proof: Let λ be a fuzzy simply* open set with fuzzy Baire property in (X,T). Since (X,T) is a fuzzy hyperconnected spaces, by proposition 3.4, int cl(1 − λ) = 0, in (X,T). Then, 1 – cl int (λ) = 0, and thus clint(λ) = 1, in (X,T). But clint (λ) ≤ cl(λ) implies that 1 = cl(λ) in (X,T). Hence λ is a fuzzy dense set in (X,T).

4. FUZZY B* SETS, FUZZY STRONGLY BAIRE SPACES, FUZZY BAIRE SPACES

The following propositions give conditions for fuzzy hyperconnected spaces to become fuzzy Baire spaces

Theorem 4.1 [9]: If cl (μ) = 1, for a fuzzy strongly first category set μ in a fuzzy topological space (X,T), then (X,T) is a fuzzy strongly Baire space.

Proposition 4.1: If cl ($\wedge_{i=1}^{\infty}(\lambda_i)$) = 1, where (λ_i)'s are fuzzy simply* open sets with fuzzy Baire property in a fuzzy hyperconnected space (X,T), then (X,T) is a fuzzy Baire space.

Proof: Suppose that cl($\wedge_{i=1}^{\infty}(\lambda_i)$) = 1, where (λ_i)'s are fuzzy simply* open sets with fuzzy Baire property in (X,T). Since (X,T) is a fuzzy hyperconnected space, by proposition 3.4, (1− λ_i)'s are fuzzy nowhere dense sets in (X,T). Now cl ($\wedge_{i=1}^{\infty}(\lambda_i)$) = 1, implies that $1 - cl(\wedge_{i=1}^{\infty}(\lambda_i)) = 0$ and then $int(1 - \wedge_{i=1}^{\infty}(\lambda_i)) = 0$. Then $int(\vee_{i=1}^{\infty}(1 - \lambda_i)) = 0$ in (X,T). Hence, $int(\vee_{i=1}^{\infty}(1 - \lambda_i)) = 0$, where (1− λi)'s are fuzzy nowhere dense sets in (X,T), implies that (X,T) is a fuzzy Baire space.

Theorem 4.2 [9] : If 1 – λ is a fuzzy nowhere dense set in a fuzzy topological space (X,T), then λ is a fuzzy strongly nowhere dense set in (X,T).

Proposition 4.2: If $cl(\vee_{i=1}^{\infty}(\lambda_i)) = 1$, where $(\lambda_i)'s$ are fuzzy B* sets in a fuzzy hyperconnected space (X,T), then (X,T) is a fuzzy strongly Baire space.

Proof: Let $(\lambda_i)'s$ ($i = 1$ to ∞) be fuzzy B* sets in (X,T). Since (X,T) is a fuzzy hyperconnected space, by theorem 2.4, $(1- \lambda_i)$'s are fuzzy nowhere dense sets in (X,T). Then, by theorem 4.2, $(\lambda_i)'s$ are fuzzy strongly nowhere dense sets in (X,T). Then $\vee_{i=1}^{\infty}(\lambda_i)$ is a fuzzy strongly first category set in (X,T). Let $\mu = \vee_{i=1}^{\infty}(\lambda_i)$. Then μ is a fuzzy strongly first category set in (X,T). The hypothesis $cl(\vee_{i=1}^{\infty}(\lambda_i)) = 1$, implies that cl (μ) = 1, in (X,T). Then by theorem 4.1, (X,T) is a fuzzy strongly Baire space.

Theorem 4.3 [9] : If int (μ) = 0 for a fuzzy strongly first category set μ in a fuzzy topological space (X,T), then (X,T) is a fuzzy Baire space.

Proposition 4.3 : If $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where $(\lambda_i)'s$ are fuzzy B* sets in a fuzzy hyperconnected space (X,T), then (X,T) is a fuzzy Baire space.

Proof: Let $(\lambda_i)'s$ ($i = 1$ to ∞) be fuzzy B* sets in (X,T). Since (X,T) is a fuzzy hyperconnected space, by theorem 2.4, $(1- \lambda_i)$'s are fuzzy nowhere dense sets in (X,T) and then, by theorem 4.2, $(\lambda_i)'s$ are fuzzy strongly nowhere dense sets in (X,T). Then $\vee_{i=1}^{\infty}(\lambda_i)$ is a fuzzy strongly first category set in (X,T). Let $\mu = \vee_{i=1}^{\infty}(\lambda_i)$. Then μ is a fuzzy strongly first category set in (X,T). The hypothesis $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, implies that int (μ) = 0, in (X,T). Then, by theorem 4.3, (X,T) is a fuzzy Baire space.

The following proposition gives a condition for fuzzy hyperconnected and fuzzy GID space to become a fuzzy Volterra space.

Theorem 4.4 [14]: If the fuzzy topological space (X,T) is a fuzzy Baire and fuzzy GID space, then (X,T) is a fuzzy Volterra space.

Proposition 4.4: If $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where$(\lambda_i)'s$ are fuzzy B* sets in a fuzzy hyperconnected and fuzzy GID space (X,T), then (X,T) is a fuzzy Volterra space.

Proof: Let $(\lambda_i)'s$ $(i = 1 \text{ to } \infty)$ be fuzzy B* sets in (X,T) such that int $(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$ in (X,T). Since (X,T) is a fuzzy hyperconnected space, by proposition 4.3, (X,T) is a fuzzy Baire space. Also since (X,T) is a fuzzy GID space, by theorem 4.4, (X,T) is a fuzzy Volterra space.

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