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Slip Flow Effects on Unsteady MHD Blood Flow in a Permeable Vessel in the Presence of Heat Source/Sink and Chemical Reaction

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Abstract

The effect of unsteady heat and mass transfer on blood flow in the presence of applied magnetic field has been studied numerically by considering slip effects. The non-linear partial differential equations governing the unsteady blood flow, thermal and concentration fields are first transformed into a set of non-linear ordinary differential equations by using a set of suitable similarity transformations. The resulting system of coupled non-linear differential equations is solved using shooting method by converting into initial value problem. In this method, the system of equations is converted into a set of first order system which is solved by fourth order Runge-Kutta method. Results are compared with the available data and are found to be in excellent agreement. The effects of various controlling parameters on the blood flow, heat and mass transfer are presented graphically and discussed quantitatively. The external magnetic field helps in controlling the flow of blood. Unsteadiness parameter decreases the skin friction coefficient near the wall. Also, heat transfer at the wall reduces approximately by 10% and mass transfer at the wall reduces approximately by 20% as the unsteadiness parameter increases from 0.5 to 0.7. Blood velocity near the wall gets decreased with the increase in the slip parameter.

Keywords: stretching wall, time dependent magnetic field, non-uniform heat source/sink, permeability.

1. INTRODUCTION

Biomagnetic fluids are biological fluids whose flow is influenced by the presence of magnetic field. All biological fluids are considered to be biomagnetic fluids because they contain ions that can interact with an applied magnetic field. The most characteristic biomagnetic fluid is blood. It behaves as a magnetic fluid, due to the complex interaction of the inter-cellular protein, cell membrane and hemoglobin. In the presence of external magnetic field, the erythrocytes orient in such a way that the disk plane is parallel to the magnetic field [1]. Blood behaves as a diamagnetic material when oxygenated and as a paramagnetic material when deoxygenated. Mathematical model for the flow of biomagnetic fluids under the action of an applied magnetic field was first developed by Haik *et al.* [2]. According to biomagnetic fluid dynamics, biofluids are actually treated as ferrofluids, that is, isothermal, electrically non-conducting magnetic fluids. Blood exhibits polarization due to the erythrocytes. Thus, blood can be considered to be a magnetic fluid, with the erythrocytes playing the role of the magnetic dipoles and the plasma playing the role of the liquid carrier. This model is consistent with the principles of ferrohydrodynamics and the dominant force in the flow field is that of magnetization. However, blood also contains ions in the plasma, which interact with an applied magnetic field. Consequently, blood can be considered to be an electrically conducting fluid that simultaneously exhibits magnetization [3], and thus, the principles of magnetohydrodynamics could also be incorporated into the mathematical model. A mathematical model for biomagnetic fluid dynamics that incorporates the principles of both ferrohydrodynamics and magnetohydrodynamics for non-isothermal flows was developed by Tzirtzilakis [4]. Presence of external magnetic field reduces rate of blood flow in human arterial system, which is useful in treatment of certain cardiovascular disorders [5]. The study of behavior of a blood flow when exposed to magnetic field has its applications in the development of magnetic devices for cell separation, targeted transport of drugs using magnetic particles as drug carriers [6, 7] and development of magnetic tracers [8], etc. Extensive research work has been done on the biomagnetic fluid in the presence of magnetic field. Chen [9] analyzed theoretically the effect of magnetic field on blood flow by treating blood as an electrically conducting fluid. Pal *et al.* [10] have studied the effect of uniform transverse magnetic field on the blood flow in the arteries. Misra and Shit [11, 12] developed a mathematical model for the flow of a biomagnetic viscoelastic fluid over a stretching wall under different conditions. The peristaltic transport of magnetohydrodynamic flow of biofluids through a microchannel has been studied mathematically by Shit *et al.* [13]. They have considered the couple stress fluid model to present the effects of rhythmically contracting and expanding walls under the influence of an applied electric field.

Thermal effect in blood flow is an important subject of research, because it has got significant applications in thermal therapeutic procedures. Inclusion of energy equation in the mathematical model is important, because hyperthermia or hypothermia are used for various experimental medical techniques, such as cancer tumor treatment, injury treatment, or open heart surgeries [14]. The components for calculating the total quantity of heat that blood carries when it flows through blood vessels include blood velocity, vessel diameter, thickness of blood, temperature of surrounding tissues and heat transfer coefficient of blood. Transfer of heat near the skin surface takes place by conduction, convection or radiation. A mathematical model for the therapeutic procedure of electromagnetic hyperthermia, where the cancerous tissues are exposed to thermal environment of 42^{o}C, while maintaining the surrounding normal tissues at a suitable temperature has been proposed by Misra *et al.* [15]. Barozzi and Dumas [16] have presented a numerical study on convective heat transfer in blood vessels. Heat is distributed through tissues uniformly to the whole body and so heat cannot accumulate in any part of the tissue. However, for a proper functioning of the human body, the iodel and coordinate system nternal temperature of the body must always must remain consistent. Thus, it is necessary to dissipate the excess heat generated in the body. Nakayama and Kuwahara [17] proposed a mathematical model to study the heat transfer in biological system using volume averaging theory. Problem concerning the cooling of large blood vessel in a heated tissue medium was studied numerically by Colios *et al.* [18]. Metabolic heat generation and blood perfusion was studied by Rai and Rai [19]. A theoretical study on unsteady blood flow over a permeable stretching sheet in the presence of non-uniform heat source and sink was done by Srinivas *et al*.[20] and Sinha *et al.* [21] .

Biochemical reactions that are responsible for secretion of insulin, gastric acid etc., can be accelerated or decelerated by the action of drugs. The strength of a drug is the quantity that is required to be applied in order to have a visible effect. A drug having higher efficiency may be less effective due to its side effects, thus it is essential to pay importance to the presence of chemical reactions during physiological functions. Xu *et al.* [22] developed a theoretical model to study the impact of blood flow on thrombus growth, by considering the interaction between different constituents of blood and chemical reaction. Combined effect of thermal diffusion and chemical reaction on blood flow has a great importance because concentration difference may sometimes produce quantitative and qualitative changes to the rate of heat transfer. In view of the above interests, a series of investigations have been made by different researchers. Misra and Adhikary [23] investigated the MHD oscillatory channel flow, heat and mass transfer in a physiological fluid in presence of chemical reaction.

Viscous fluid normally sticks to a boundary, i.e., there is no slip of the fluid relative to the boundary. The permeability of capillary blood vessel walls, demands the consideration of the slip-velocity at the wall so that the study is closer to the reality. For particulate fluids, although the motion is governed by Navier-Stokes equations, it is desirable that the no-slip condition at the boundary should be replaced by velocity-slip condition. Misra and Shit [24] investigated the role of slip velocity in blood flow through stenosed arteries; also in a different study the thermal slip accompanied by velocity-slip at the capillary wall has been accounted [25]. The aim of the present paper is to study the flow, heat and mass transfer in a stretching capillary blood vessel under the action of an externally applied magnetic field. The problem investigated here involves the effect of non-uniform heat source/sink. The thermal, solutal slip accompanied by velocity slip at the capillary wall has been considered in this study. The similarity solution of the non-linear partial differential equations governing the MHD flow, heat and mass transfer has been studied numerically by using shooting and fourth order Runge-Kutta method. The results for some particular cases are compared with those of Ali [26] and Salem and El-Aziz [27].

2. MATHEMATICAL FORMULATION

Unsteady two-dimensional flow of blood containing chemically reactive substance through a permeable capillary under the action of a time dependent magnetic field $B(t)$ is considered. The $x-$ axis is taken along the lower wall of the capillary and the $y-$ axis is taken normal to it. The coordinate system and the flow model are shown in Fig. 1.

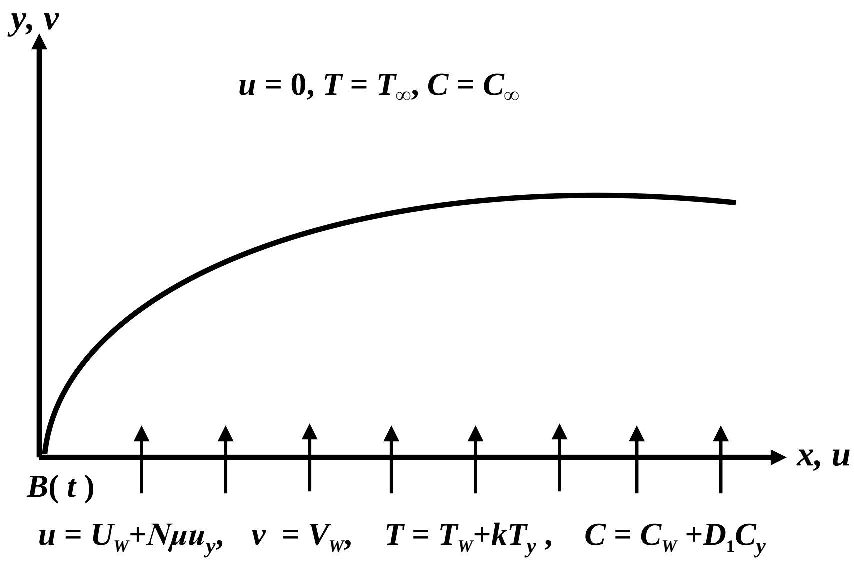

Fig. 1 Physical model and coordinate system

The flow is driven by the stretching motion of the blood capillary given by $U_w\,(x,t) = ax\,/\,(1\text{-}ct)$, where a and c are positive constants and $ct < 1$. The magnetic field is applied in a direction transverse to that of the flow. It is assumed that the magnetic Reynolds number is much less than unity so that induced magnetic field is negligible in comparison with applied magnetic field. Under these assumptions, the governing boundary layer equations can be expressed as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma[B(t)]^2}{\rho}u-\frac{\upsilon}{k_1(t)}u\,, \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{1}{\rho c_p}q'''(t)\,, \tag{3}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-R(t)\left(C-C_\infty\right), \tag{4}$$

where u and v are the velocity components in $x-$ and $y-$ directions respectively, υ is the kinematic viscosity, σ is the electrical conductivity of blood, ρ is density of the blood, $B(t)$ is time dependent magnetic field intensity, $k_1(t)$ is the time dependent permeability parameter of blood, T is the temperature of the blood at any point in the capillary, α is the thermal conductivity of the blood, $q'''(t)$ is the time dependent non-uniform heat generated or absorbed per unit volume, C is the concentration of the solute at any point in the capillary, D is the mass diffusivity, $R(t)$ is the time dependent reaction rate of the solute and C_∞ is the ambient concentration of the solute. The wall being permeable, velocity, thermal and concentration slip is taken in to account and the relevant boundary conditions are

$$u = U_w(x,t) + N\mu\frac{\partial u}{\partial y},\ \ v = V_w(x,t),\ \ T = T_w(x,t) + K\frac{\partial T}{\partial y},$$

$$C = C_w(x,t) + D_1\frac{\partial C}{\partial y}\ \text{ at }\ y = 0, \tag{5}$$

$$u \to 0,\ T \to T_\infty,\ C \to C_\infty\ \text{ as }\ y \to \infty,$$

where $N = N_0(1-ct)^{1/2}$ denotes the velocity slip factor, $K = K_0(1-ct)^{1/2}$ denotes the thermal slip factor and $D_1 = D_0(1-ct)^{1/2}$ denotes the concentration slip factor. In order to obtain similarity solution the following is considered

$$B(t) = B_0(1-ct)^{-1/2},\ k_1(t) = k_2(1-ct),\ R(t) = R_0(1-ct)^{-1},$$

$$T_w(x,t) = T_\infty + \frac{bx}{1-ct},\ C_w(x,t) = C_\infty + \frac{ex}{1-ct},\ V_w(x,t) = \left(\frac{\upsilon U_w(x,t)}{x}\right)^{1/2} S, \tag{6}$$

where T_∞ denote the ambient temperature, B_0, k_2, R_0, S, b and e are constants. The non-uniform heat generated or absorbed per unit volume is taken as

$$q''' = kU_w(x,t)\left(A^*(T_w - T_\infty)e^{-\eta} + B^*(T - T_\infty)\right)/x\upsilon, \tag{7}$$

where A^* and B^* denote the parameters of space and temperature dependent heat generation/ absorption. A^* and B^* are both positive in the case of internal heat source and negate ve in the case of internal heat sink. Thus $q''' > 0$ in the case of heat generation and $q''' < 0$ in the case of heat absorption. The following similarity transformations

$$\eta = y\left(\frac{U_w}{\upsilon x}\right)^{1/2},\ \psi = (\upsilon x U_w)^{\frac{1}{2}} f(\eta),\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \tag{8}$$

are used in Eqs. (1) – (4), where ψ is the stream function, which is defined in the usual form as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. The Eq. (1) is identically satisfied and the velocity components u and v are obtained as

$$u = U_w\, f'(\eta),\ v = -\sqrt{\frac{\upsilon\, a}{1-ct}}\ f(\eta), \tag{9}$$

where primes denote differentiation with respect to η. On substituting the similarity variables (8), Eqs. (2) - (4) are reduced to the following set of ordinary differential equations

$$f''' + ff'' - f'^2 - A\left(f' + \frac{\eta}{2} f'' \right) - M^2 f' - \frac{1}{k_3} f' = 0, \tag{10}$$

$$\theta'' - \Pr\left[A\left(\theta + \frac{\eta}{2}\theta' \right) - f\theta' + \theta f' \right] + A^* e^{-\eta} + B^*\theta = 0, \tag{11}$$

$$\phi'' - Sc\left[A\left(\phi + \frac{\eta}{2}\phi' \right) - f\phi' + \phi f' - \gamma\phi \right] = 0, \tag{12}$$

subject to the transformed boundary conditions

$$\begin{gathered} f(0) = S, \quad f'(0) = 1 + S_f f''(0), \quad \theta(0) = 1 + S_T\theta'(0), \quad \phi(0) = 1 + S_C\,\phi'(0), \\ f'(\eta_\infty) = 0,\ \theta(\eta_\infty) = 0,\ \ \phi(\eta_\infty) = 0, \end{gathered} \tag{13}$$

where η_∞ is the edge of the boundary layer. Further, $A = c/a$ is the unsteadiness parameter, $M^2 = \sigma B_0^2/\rho a$ is the magnetic parameter, $k_3 = a k_2/\upsilon$ is the permeability parameter, $\Pr = \upsilon/\alpha$ is the Prandtl number, $Sc = \upsilon/D$ is the Schmidt number, $\gamma = R_0/a$ is the reaction rate parameter, $S > 0$ denotes suction, $S < 0$ denotes injection, $S_f = N_0\rho\sqrt{a\upsilon}$ is the non dimensional velocity slip factor, $S_T = K_0\sqrt{a/\upsilon}$ is the non dimensional thermal slip factor and $S_C = D_0\sqrt{a/\upsilon}$ is the non dimensional concentration slip factor. The physical quantities of interest are the local skin friction coefficient C_{fx}, local Nusselt number Nu_x and the local Sherwood number Sh_x, which are defined as

$$C_{fx} = -2\,\mathrm{Re}_x^{-1/2}\ f''(0),\ \ Nu_x = -\mathrm{Re}_x^{1/2}\,\theta'(0),\ \ Sh_x = -\mathrm{Re}_x^{1/2}\,\phi'(0) \tag{14}$$

where $\mathrm{Re}_x = U_w / \nu x$ is the Reynolds number.

3. METHOD OF SOLUTION

The non-linear ordinary differential equations (10) – (12) along with the boundary conditions (13) form a two point boundary value problem and are solved by shooting method, by converting into an initial value problem. In this procedure, the system of Eqs. (10) – (12) is converted into the set of following first order system.

$$f' = p, \quad p' = q, \quad q' = -fq + p^2 + A\left(p + \frac{\eta}{2} q \right) + M^2 p + \frac{p}{k_3}, \tag{15}$$

$$\theta' = r, \quad r' = \Pr\left(A\left(\theta + \frac{\eta}{2} r\right) - f\,r + \theta\,p \right) - A^* e^{-\eta} + B^*\theta, \tag{16}$$

$$\phi' = s, \quad s' = Sc\left(A\left(\phi + \frac{\eta}{2} s\right) - f\,s + \phi\,p - \gamma\,\phi \right), \tag{17}$$

with the initial conditions

$$f(0) = f_0, \quad p(0) = 1 + S_f\, q(0), \quad \theta(0) = 1 + S_T\, r(0), \quad \phi(0) = 1 + S_C\, s(0). \tag{18}$$

To solve the system of Eqs. (15) – (17) as an initial value problem, the values of $q(0), r(0)$ and $s(0)$ i.e., $f''(0)$, $\theta'(0)$ and $\phi'(0)$ are required but those values are not given in the problem. The initial guess value of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ are chosen and the fourth order Runge-Kutta method is applied to obtain the solution. Finally, the computed values $f'(\eta), \theta(\eta)$ and $\phi(\eta)$ at a suitably chosen finitely large values of η, say $\eta = \eta_\infty$ are compared with the given boundary conditions $f'(\eta_\infty) = 0$, $\theta(\eta_\infty) = 0$, $\phi(\eta_\infty) = 0$. The initial guess values of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ are improved iteratively using secant method to get better approximation for the solution. The step size is taken as 0.01. A convergence criteria based on the relative difference between the current and previous iteration is employed. When the difference reaches 10^{-4}, the solution is assumed to be converged and the iterative process is terminated.

4. RESULTS AND DISCUSSION

Computations have been carried out for various values of Sc ($1.0 \le Sc \le 5.0$), Pr ($1.0 \le \Pr \le 4.0$), A ($0 \le A \le 1.0$), S ($-1.0 \le S \le 1.0$), M ($1.0 \le M \le 3.0$), S_f ($1.0 \le S_f \le 3.0$), S_T ($1.0 \le S_T \le 3.0$), S_C ($0.2 \le S_C \le 2.0$), γ ($0.1 \le \gamma \le 0.5$), A^* ($-0.2 \le A^* \le 0.2$), B^* ($-0.2 \le B^* \le 0.2$), k_3 ($1.0 \le k_3 \le 3.0$). In order to verify the correctness of the present numerical approach, the computed results are compared with those of Ali [26] and Salem and El-Aziz [27]. The results are found to be in excellent agreement and some of the comparisons are shown in Table 1.

Table 1 Comparison of values of $\theta'(0)$ for various values of Pr when $A = A^* = B^* = M = S = 0$ with those of Ali [26] and Salem and El-Aziz [27]

Pr	Ali [26]	Salem and El-Aziz [27]	Present results
0.7	- 0.45255	- 0.45605	- 0.45194
1.0	- 0.59988	- 0.58223	- 0.59966
10	- 2.29589	- 2.30798	- 2.29462

The velocity distribution for different values of magnetic parameter (*M*) is presented in Fig. 2. As then value of magnetic parameter increases, the velocity distribution and the boundary layer thickness gets decreased. For example, as the magnetic parameter increases from $M = 1$ to $M = 3$ a significant decrease in boundary layer thickness η_∞ is observed that is from $\eta_\infty \approx 3.78$ to $\eta_\infty \approx 1.5$. This happens due to the Lorentz force arising from the interaction of magnetic and electric fields during the motion of an electrically conducting fluid. The generated Lorentz force opposes the motion of the blood in boundary layer region, thereby reducing the momentum boundary layer thickness. Figure 3 illustrates the effect of Prandtl number (Pr) on temperature profile ($\theta(\eta)$). It is observed that the thermal boundary layer thickness reduces with the increase in Prandtl number. Quantitatively, for Pr = 1.0, the thermal boundary layer thickness is approximately 15.6, whereas the thermal boundary layer thickness $\eta_\infty \approx 9.0$ for Pr = 4.0 (See Fig. 3). This figure further indicates that the temperature gradient at the surface increases with increase in Prandtl number. This implies that an increase in Prandtl number is accompanied by an enhancement of the heat transfer rate at the wall of the blood vessel. When blood attains a higher Prandtl number, its thermal conductivity is lowered down and so its heat conduction capacity diminishes. Thereby boundary layer thickness gets reduced. The impact of Schmidt number (Sc) over the concentration distribution elucidated through Fig. 4. It is noted that the effect of *Sc* is to reduce the thickness of the concentration boundary layer largely. For example, for $k_3 = 1.0$, $A = 0.1$, $A^* = 0.1$, $B^* = -1.0$, $M = 0.5$, Pr = 2.0, $\gamma = 0.2$, $S = 0.1$, $S_T = 2.0$, $S_f = 2.5$ and $S_C = 2.5$, the concentration boundary layer thickness reduces from $\eta_\infty \approx 12.0$ to $\eta_\infty \approx 7.0$ when *Sc* increases from 2.0 to 5.0, respectively. Physically, the increase of *Sc* means decrease of molecular diffusivity (*D*) and the concentration profile shows significant variation for different values of *Sc*.

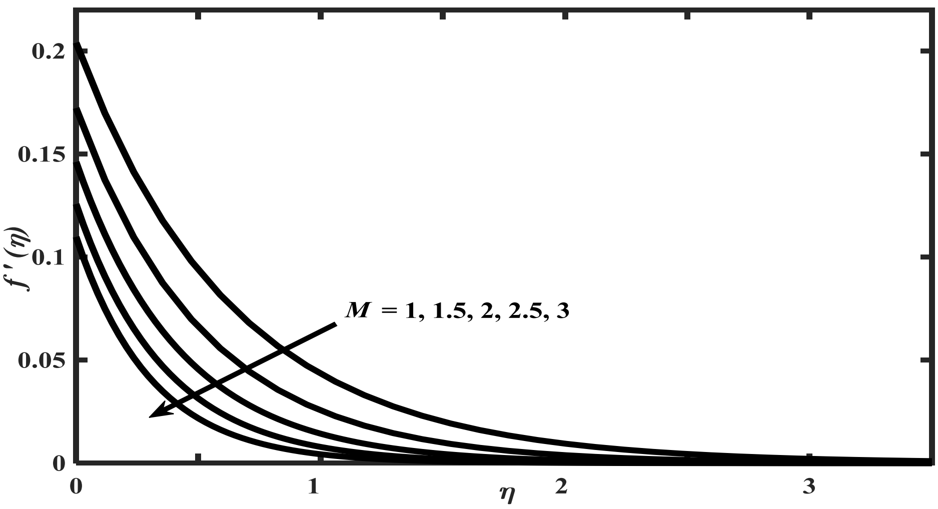

Fig. 2 Effect of *M* on velocity profile for $k_3 = 1.0$, $A = 0.1$, $A^* = 0.1$, $B^* = -1.0$, $Sc = 1.0$, Pr = 1.0, $\gamma = 0.3$, $S = 0.1$, $S_T = 2.0$, $S_f = 2.5$ and $S_C = 0.25$

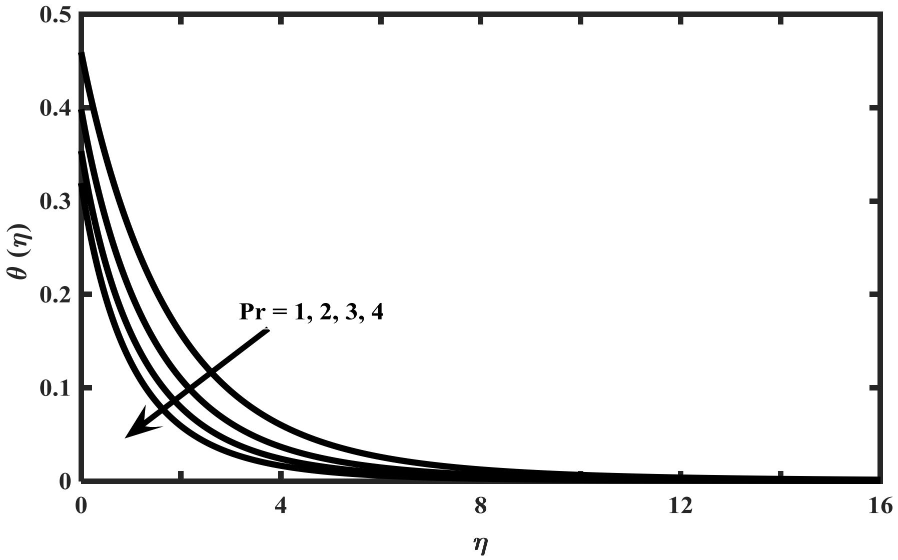

Fig. 3 Effect of Pr on temperature profile for k_3 = 1.0, A = 0.1, A^* = 0.1, B^* = -1.0, Sc = 1.0, M = 2.0, γ = 0.3, S = 0.1, S_T = 2.0, S_f = 2.5 and S_C = 0.25

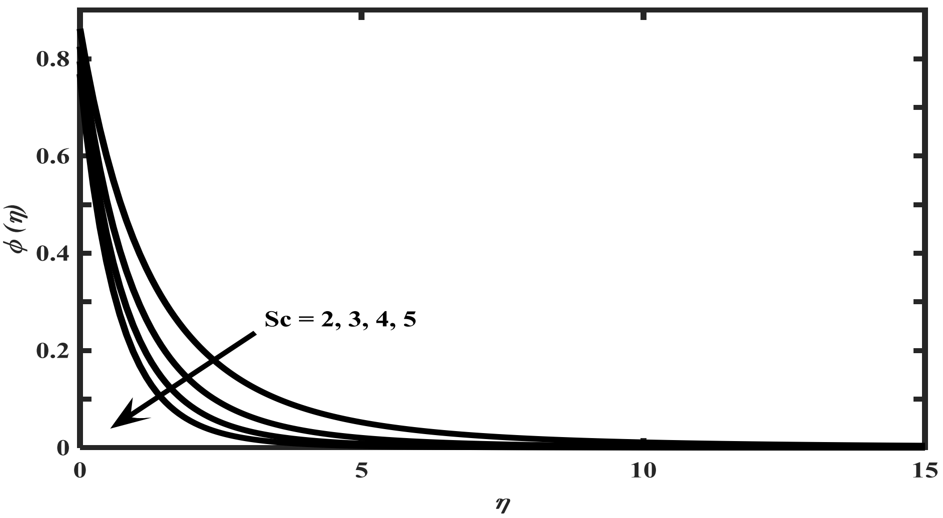

Fig. 4 Effect of Sc on concentration profile for k_3 = 1.0, A = 0.1, A^* = 0.1, B^* = -1.0, M = 0.5, Pr = 2.0, γ = 0.2, S = 0.1, S_T = 2.0, S_f = 2.5 and S_C = 2.5

The characteristics of temperature profiles ($\theta(\eta)$) for different values of space (A^*) and temperature (B^*) dependent heat source/sink parameters is depicted in Fig. 5. In case of heat generation (when B^* > 0), temperature of blood increases with increasing B^*, while reverse trend is observed in case of heat absorption. For example, $\eta_\infty \approx 15$ for B^* = 0.2 (heat generation) whereas, for B^* = -0.2 (heat absorption), the boundary layer thickness reduces to 6.7 (approximately). Similar effect is also observed for space dependent parameter A^*. Approximately 95% decrease in thermal boundary layer thickness is noticed as the value of A^* decreases from 0.1 (heat generation) to -0.1 (heat absorption). The velocity distribution for different values of velocity slip parameter (S_f) is depicted in Fig. 6. It is inferred from the figure that the presence of slip velocity within the boundary layer causes the velocity level along the vessel wall to decrease. This is due to the fact that the quantity $1-f'(0)$ increases monotonically with S_f. So for large values of S_f the frictional resistance between the blood velocity and the vessel wall is eliminated. Initially, for the increasing values of the slip parameter the blood velocity distribution gets decreased near the surface of the wall since not all the pulling force of the vessel wall can be transmitted to the blood but it gets increased away from the wall. Figure 7 highlights the impact of thermal slip factor (S_T) over the dimensionless temperature distribution. The blood temperature near the vessel wall gets decreased for increasing values of S_T. Since the increase in thermal slip parameter, increases the thermal accommodation coefficient, the thermal diffusion towards the blood flow reduces. Similarly, it is observed from Fig. 8 that increases in solutal slip parameter (S_C) causes decrease in the concentration of the solute near the vessel wall. For example, approximately 47% decrease in the concentration is noticed as S_C increases from 0.25 to 1.75.

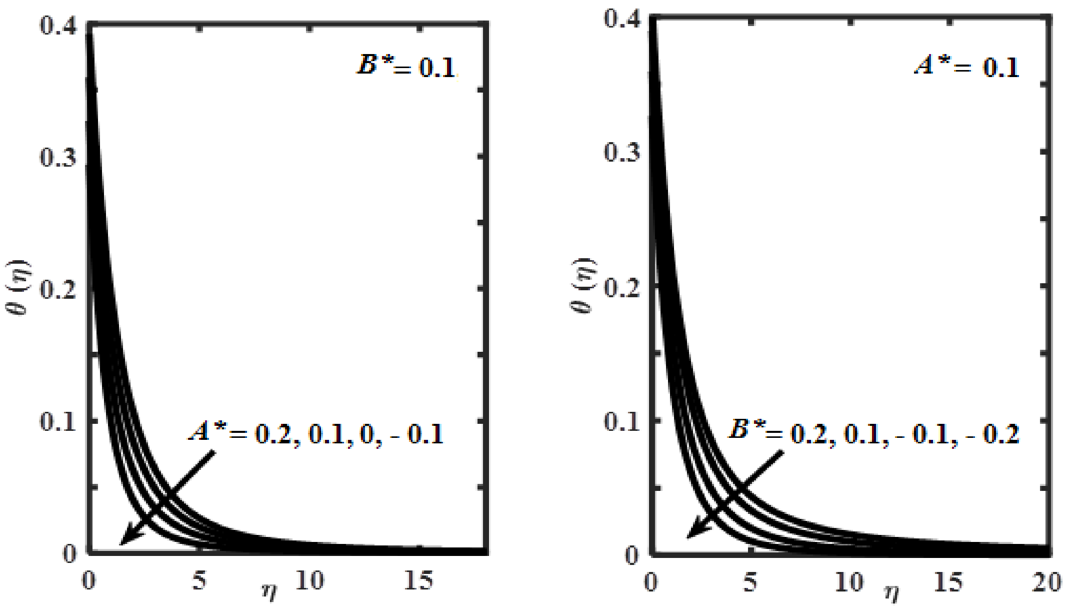

Fig. 5 Effect of A^* and B^* on temperature profile for $k_3 = 1.0$, $A = 0.1$, Pr = 1.0, $Sc = 1.0$, $M = 2.0$, $\gamma = 0.3$, $S = 0.1$, $S_T = 2.0$, $S_f = 2.5$ and $S_C = 0.25$

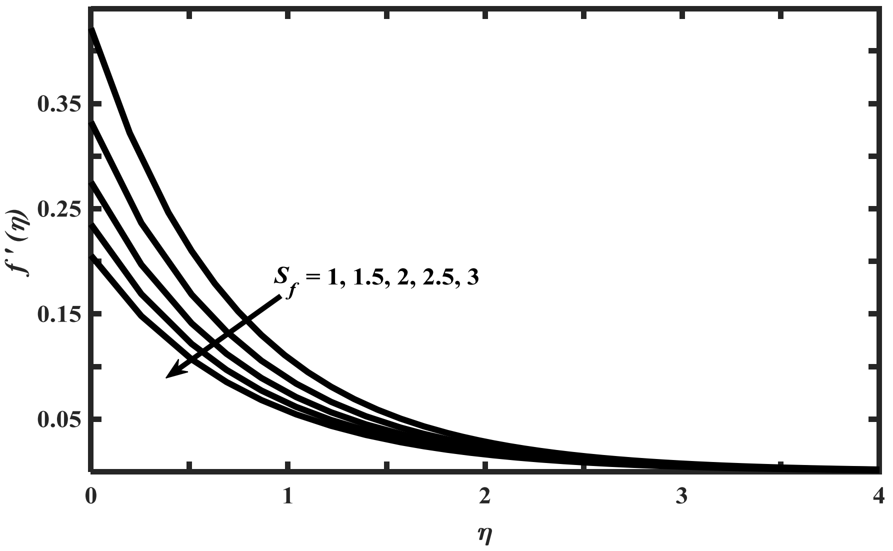

Fig. 6 Effect of S_f on velocity profile for k_3 = 1.0, A = 0.1, Pr = 1.0, A^* = 0.1, B^* = -1.0, Sc = 1.0, M = 2.0, γ = 0.3, S = 0.1, S_T = 2.0 and S_C = 0.25

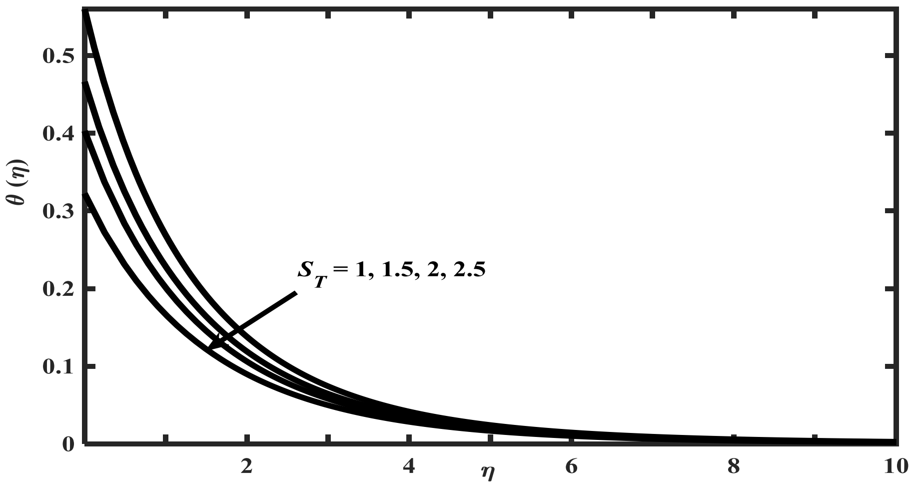

Fig. 7 Effect of S_T on temperature profile for k_3 = 1.0, A = 0.1, Pr = 1.0, A^* = 0.1, B^* = -1.0, Sc = 1.0, M = 2.0, γ = 0.3, S = 0.1, S_C = 0.25 and S_f = 2.5

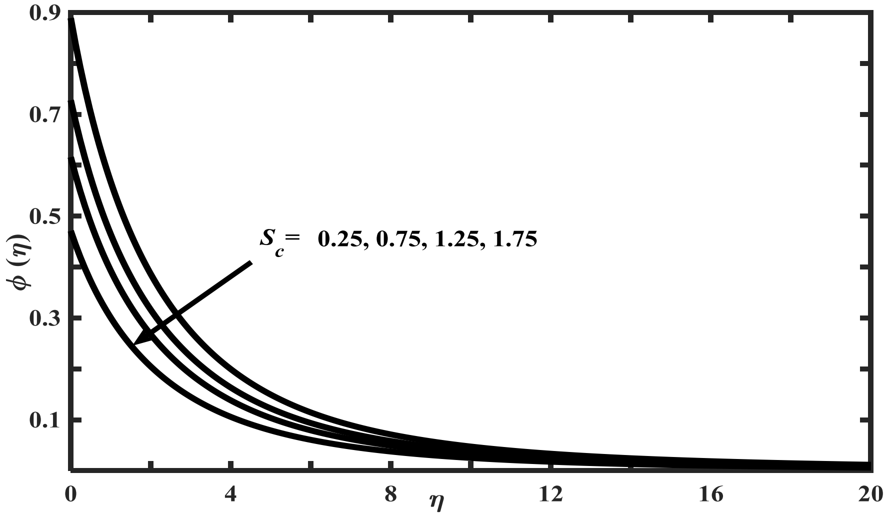

Fig. 8 Effect of S_C on concentration profile for k_3 = 1.0, A = 0.1, Pr = 1.0, A^* = 0.1, B^* = -1.0, Sc = 1.0, M = 2.0, γ = 0.3, S = 0.1, S_T = 2.0 and S_f = 2.5

The variation of skin friction coefficient $f''(0)$, heat transfer $\theta'(0)$ and mass transfer $\phi'(0)$ at the surface with S for different values of unsteadiness parameter (A) and permeability parameter (k_3) are displayed in Figs. 9 – 11. The result presented in Fig. 9 indicates that skin friction increases with suction and decreases for injection. The physical meaning for the above behavior is that the injection carries blood away from the surface of the wall giving rise to a thicker momentum boundary layer thereby decreasing velocity gradient at the surface. It is also observed that skin friction increases with increase in permeability parameter (k_3). In particular, approximately 2.75% increase in $f''(0)$ is noticed as k_3 increases from 1 to 3 for A = 0.5, S = -0.3. Also, unsteadiness parameter decreases the skin friction coefficient near the wall. Similar effect is observed for heat transfer near the surface ($\theta'(0)$) in Fig. 10. It can also be noticed that as the effect of permeability (k_3) reduces with increase in the value of unsteadiness parameter. In particular, for A = 0.1, about 3% decrease in heat transfer at the wall is noticed as k_3 increases from 1 to 3 whereas only 1% decrease is noticed for A = 0.5. The effect of reaction parameter (γ) on the concentration gradient ($\phi'(0)$) for different values of unsteadiness parameter and suction/ injection parameter is presented in Fig. 11. The curves representing the concentration gradient confirm that the thickness of concentration boundary layer decreases with increasing reaction rate parameter (γ). In the case of distribution of reactive solute, the reaction rate parameter is a decelerating agent and it thins the solute boundary layer formed in the neighborhood of the surface. It is noticed that in case of injection ($S < 0$), the fluid is carried away from the surface causing reduction in concentration gradient as it tries to maintain the same concentration over a small region near the surface, and this effect is reversed in case of suction ($S > 0$). Injection causes decrease in the steepness of the concentration profile at the wall but the steepness of the concentration profile increases with suction.

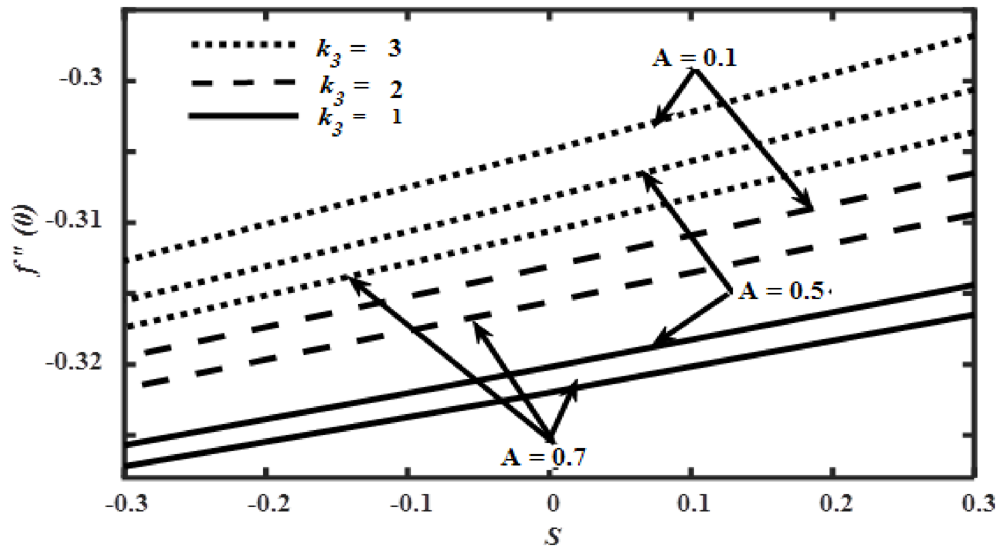

Fig. 9 Variation of $f''(0)$ with S for A and k_3 when Pr = 1.0, $A^* = 0.1$, $B^* = -1.0$, $Sc = 1.0$, $M = 2.0$, $\gamma = 0.3$, $S_T = 2.0$, $S_C = 0.25$ and $S_f = 2.5$

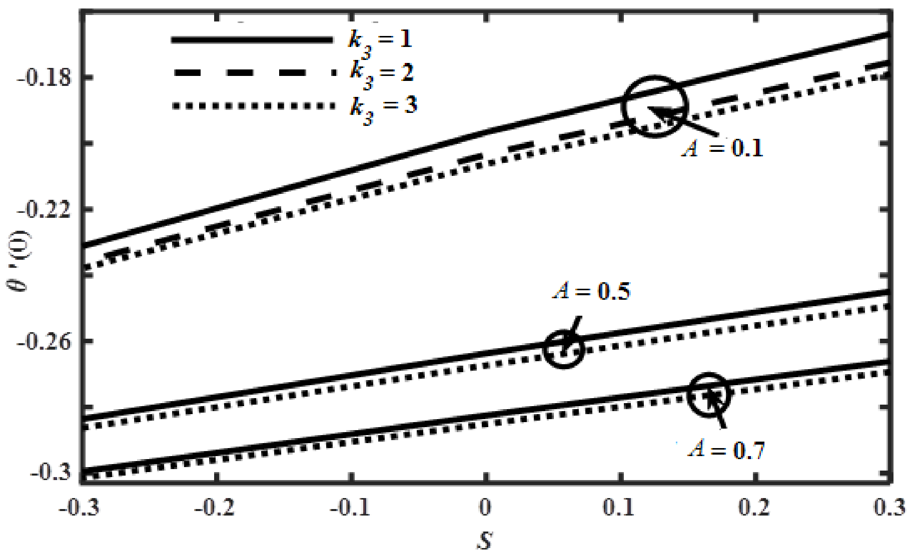

Fig. 10 Variation of $\theta'(0)$ with S for A and k_3 when Pr = 1.0, $A^* = 0.1$, $B^* = -1.0$, $Sc = 1.0$, $M = 2.0$, $\gamma = 0.3$, $S_T = 2.0$, $S_C = 0.25$ and $S_f = 2.5$

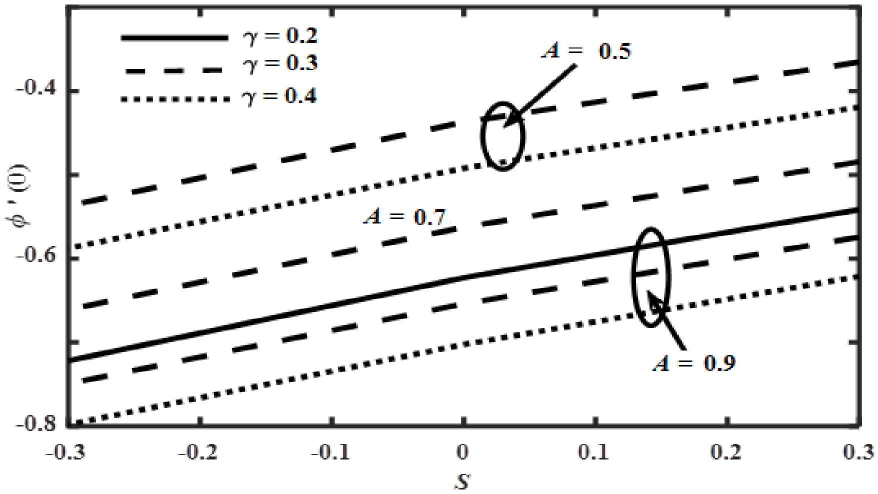

Fig. 11 Variation of $\phi'(0)$ with S for A and γ when Pr = 1.0, $A^* = 0.1$, $B^* = -1.0$, $Sc = 1.0$, $M = 2.0$, $k_3 = 1.0$, $S_T = 2.0$, $S_C = 0.25$ and $S_f = 2.5$

5. CONCLUSION

Unsteady two dimensional magnetohydrodynamic flow of blood containing chemically reactive substance through a permeable blood vessel having a heat source/sink is studied numerically. The erythrocyte slip at the vessel wall, the thermal and the solutal slip has been duly accounted. Conclusions of the study are as follows

- The velocity of the blood can be controlled by suitably regulating the intensity of the external magnetic field. The increase in the value of magnetic parameter from $M = 1$ to $M = 3$ reduces the velocity boundary layer thickness by 60%.
- The temperature of blood inside the boundary layer reduces with the increase in Prandtl number. Similarly, the concentration of the solute in the boundary layer reduces with the increase in Schmidt number.
- Increase in erythrocyte slip leads to the decrease in the blood flow near the wall. Approximately 15% decrease in temperature of the blood near the wall is observed with increase of thermal slip and 14% decrease in concentration of the solute in the blood is observed with the increase of solutal slip.
- Thermal boundary layer thickness increases with heat generation whereas, heat absorption reduces the thickness. Approximately 33% increase in thermal boundary layer thickness is noticed for increase in heat generation and 16% decrease in thermal boundary layer thickness is observed for increase in heat absorption.
- Skin friction of the blood increases with the increase in permeability of the blood vessel. Unsteadiness parameter reduces the skin friction, heat transfer and mass transfer near the wall.

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