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On Some Maps Concerning β-Closed Sets and Related Groups

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Abstract

The concept of group of functions, say βch(X,τ) preserving β-closed sets containing homeomorphism group h(X,τ) was studied by Arora,Tahiliani and Maki.In continuation to that,we study some new isomorphisms, mappings, subgroups and their properties.

Keywords: α-open, β-open and β-irresolute mappings.

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1.Introduction and Preliminaries

Throughout this paper we consider spaces on which no separation axiom are assumed unless explicitly stated.The topology of a space(By space we always mean a topological space) is denoted by τ and (X,τ) will be replaced by X if there is no chance of confusion.For $A\subseteq X$,the closure and interior of A in X are denoted by Cl(A) and Int(A) respectively.Let A be a subset of the space (X,τ).Then A is said to be β-open [1] if $A\subseteq Cl(Int(Cl(A)))$.Its complement is β-closed.The family of all β-open sets containing A is denoted by βO(A) and all β-closed sets containing A is denoted by βC(A).A is said to be α-open[6] if $A\subseteq Int(Cl(Int(A)))$ and its complement is α-closed.The union of all β-open sets contained in A is called β-interior of A, denoted by βInt(A)[2].

A map f:(X,τ) →(Y,σ) is called β-irresolute[4] if the inverse image of every β-open set in Y is β-open in X.It is called βc-homeomorphism[5] if f is β-irresolute bijection and f^{-1} is β-irresolute.

2.Subgroups of $\beta ch(X;\tau)$

For a topological space (X,τ) we have $h(X;\tau)=\{f \mid f:(X,\tau) \to (X,\tau)$ is a homeomorphism$\}$[5] and $\beta ch(X;\tau)=\{f \mid f:(X,\tau) \to (X,\tau)$ is a βc-homeomorphism$\}$[5].

In this section, we investigate some structures of $\beta ch(H;\tau|H)$ for a subspace $(H,\tau|H)$ of (X,τ) using two subgroups of $\beta ch(X,\tau)$, say $\beta ch(X, X \setminus H; \tau)$ and $\beta ch_0(X, X \setminus H; \tau)$ below.

Definition2.1. For a topological space (X,τ) and subset H of X,we define the following families of maps:

(i). $\beta ch(X, X \setminus H; \tau)=\{a \mid a\in \beta ch(X;\tau)$ and $a(X \setminus H)= X \setminus H\}$.

(ii). $\beta ch_0(X, X \setminus H; \tau)= \{a \mid a\in \beta ch(X, X \setminus H; \tau)$ and $a(x)=x$ for every $x\in X \setminus H\}$.

Theorem2.2. Let H be a subset of a topological space (X,τ).Then

(i) The family $\beta ch(X, X \setminus H; \tau)$ forms a subgroup of $\beta ch(X,\tau)$.

(ii) The family $\beta ch_0(X, X \setminus H; \tau)$ forms a subgroup of $\beta ch(X, X \setminus H; \tau)$ and hence $\beta ch_0(X, X \setminus H; \tau)$ forms a subgroup of $\beta ch(X,\tau)$.

Proof.(i). It is shown obviously that $\beta ch(X, X \setminus H; \tau)$ is a non empty subset of $\beta ch(X,\tau)$,because $1_X\in \beta ch(X, X \setminus H; \tau)$.Moreover, we have that $\omega_X(a,b^{-1})=b^{-1}o\ a\in \beta ch(X, X \setminus H; \tau)$ for any elements $a,b\in \beta ch(X, X \setminus H; \tau)$,where $\omega_X = \omega|(\beta ch(X, X \setminus H; \tau)\times \beta ch(X, X \setminus H; \tau))$ as ω is the binary operation of the group $\beta ch(X,\tau)$.Evidently, the identity map 1_X is the identity element of $\beta ch(X, X \setminus H; \tau)$.

(ii).It is shown that $\beta ch_0(X, X \setminus H; \tau)$ is a non empty subset of $\beta ch(X, X \setminus H; \tau)$ because $1_X\in \beta ch_0(X, X \setminus H; \tau)$.We have that $\omega_{X,0}(a,b^{-1})=b^{-1}o\ a\in \beta ch_0(X, X \setminus H; \tau)$ for any elements $a,b\in \beta ch_0(X, X \setminus H; \tau)$,where $\omega_{X,0}=\omega_X|\ (\beta ch_0(X, X \setminus H; \tau)\times \beta ch_0(X, X \setminus H; \tau)))$(ω_X is the binary operation of the group $\beta ch(X, X \setminus H; \tau)$).Thus $\beta ch_0(X, X \setminus H; \tau)$ is a subgroup of $\beta ch(X, X \setminus H; \tau)$ and the identity map 1_X is the identity element of $\beta ch_0(X, X \setminus H; \tau)$.By using (i), $\beta ch_0(X, X \setminus H; \tau)$ forms a subgroup of $\beta ch(X,\tau)$.

Let H and K be the subsets of X and Y respectively.For a map $f:X\to Y$ satisfying a property $K=f(H)$,we define the following map $r_{H,K}(f):H\to K$ by $r_{H,K}(f)(x)=f(x)$ for every $x\in H$.Then, we have that $j_K o r_{H,K}(f)= f|H:H\to Y$,where $j_K:K\to Y$ be an inclusion defined by $j_K(y)=y$ for every $y\in K$ and $f|H:H\to Y$ is a restriction of f to H defined by $(f|H)(x)=f(x)$ for every $x\in H$.Especially, we consider the following case that $X=Y,H=K\subseteq X$ and $a(H)=H$,$b(H)=H$ for any maps $a,b:X\to X$.Thus $r_{H,H}(boa)= r_{H,H}(b)or_{H,H}(a)$ holds.Moreover ,if a map $a:X\to X$ is a bijection such that $a(H)=H$,then $r_{H,H}:H\to H$ is bijective and $r_{H,H}(a^{-1})= (r_{H,H}(a))^{-1}$.

We recall well known properties on β-open sets of subspace topological spaces:

Theorem 2.3.For a topological space (X,τ) and subsets H and U of X and A⊆H,V⊆H and B⊆H,the following properties hold:

(i).Arbitrary union of β-open sets of (X,τ) is β-open in (X,τ).The intersection of an open set of (X,τ) and a β-open set in (X,τ) is β-open in (X,τ).

(ii).(ii-1).If A is β-open in (X,τ) and A⊆H, then A is β-open in a subspace (H,τ|H).

(ii-2). If H⊆X is open or α-open in (X,τ) and a subset U⊆X is β-open in (X,τ),then H∩U is β-open in a subspace (H,τ|H).

(iii). Let V⊆ H⊆X.

(iii-1). If H is β-open in (X,τ),then $Int_H(V)$ ⊆ βInt(V) holds.

(iii-2). If H is β-open in (X,τ) and V is β-open in a subspace (H,τ|H) then V is β-open in (X,τ).

(iv).Let B⊆ H⊆X.If H is β-closed in (X,τ) and B is β-closed in a subspace (H,τ|H), then B is β-closed in (X,τ).

(v). (v-1).Assume that H is a open subset of (X,τ) .Then,

βO(X,τ)|H⊆ βO(H,τ|H) holds,where βO(X,τ)|H={W∩H| W∈ βO(X,τ)}.

(v-2).Assume that H is a β-open subset of (X,τ) .Then,

βO(H,τ|H) ⊆ βO(X,τ)|H holds.

(v-3). Assume that H is a β-open subset of (X,τ) .Then,

βO(H,τ|H) = βO(X,τ)|H holds.

Proof.(i).Clear from Remark 1.1 of [1] and Theorem 2.7 of [3].

(ii).(ii-1).Clear.(ii-2).Its Lemma 2.5 of [1].

(iii-1).Let x∈Int_H(V).There exists a subset W(x) ∈ τ such that W(x) ∩H⊆V. By (i), W(x) ∩H∈ βO(X,τ).This shows that x∈ βInt(V) and so Int_H(V) ⊆ βInt(V).

(iii-2) and (iv).Its clear from Lemma 2.7 of [1].

(v). (v-1).Let V∈ βO(X,τ)|H. For some set W∈ βO(X,τ),V=W∩H and so we have W∩H∈ βO(H,τ|H)(from ii-2). Hence V∈βO(H,τ|H) holds.

(v-2). Let V∈βO(H,τ|H).Since H∈ βO(X,τ),we have V∈ βO(X,τ) by (iii-2).Thus V= V∩H∈ βO(X,τ)|H.

(v-3). It follows from (v-1) and (v-2).

Lemma2.4. (i).If f:(X,τ) →(Y,σ) is β-irresolute and a subset H is α-open in (X,τ),then f|H:(H,τ|H) →(Y,σ) is β-irresolute.

(ii).Let (1) and (2) be properties of two maps $k:(X,\tau) \to (K,\sigma|K)$,where $K \subseteq Y$, and $j_K o k: (X,\tau) \to (Y,\sigma)$ as follows:

(1). $k:(X,\tau) \to (K,\sigma|K)$ is β-irresolute.

(2). $j_K o k: (X,\tau) \to (Y,\sigma)$ is β-irresolute.

Then,the following implications and equivalence hold:

(ii-1). Under the assumption that K is α-open in (Y,σ) ,(1)⇒(2).

(ii-2). Conversely,under the assumption that K is β-open in (Y,σ) ,(2)⇒(1).

(ii-3). Under the assumption that K is β-open in (Y,σ) ,(1)⇔(2).

(iii). If $f:(X,\tau) \to (Y,\sigma)$ is β-irresolute and a subset H is α-open in (X,τ) and f(H) is β-open in (Y,σ),then $r_{H,f(H)}(f): (H,\tau|H) \to (f(H), \sigma|f(H))$ is β-irresolute.

Proof.(i).Let $V \in \beta O(Y,\sigma)$.Then,we have $(f | H)^{-1}(V)=f^{-1}(V) \cap H$ and

$(f| H)^{-1}(V) \in \beta O(H,\tau|H)$.(Theorem2.3 (ii-2)).

(ii).(ii-1) (1)⇒(2).Let $V \in \beta O(Y,\sigma)$.Since $(j_K o k)^{-1}(V)= k^{-1}(V \cap K)$ and

$V \cap K \in \beta O(K,\sigma|K)$ (Theorem 2.3 (ii-2)),we have that $(j_K o k)^{-1}(V) \in \beta O(X,\tau)$ and hence $j_K o k$ is β-irresolute.

(ii-2) (2) ⇒(1).Let $U \in \beta O(K,\sigma|K)$.Since $U \in \beta O(Y,\sigma)$(Theorem2.3 (iii-2)),we have

$k^{-1}(U)=(j_K o k)^{-1}(U) \in \beta O(X,\tau)$.Thus k is β-irresolute.

(ii-3).Obvious in the view of fact that every α-open set is β-open,it is obtained by (ii-1) and(ii-2).

(iii). By (i), $f|H:(H,\tau|H) \to (Y,\sigma)$ is β-irresolute.The map $r_{H,f(H)}(f)$ is β-irresolute, because $f|H=j_{f(H)} o\, r_{H,f(H)}(f)$ holds.

Definition 2.5. For an α-open subset H of (X,τ), the following maps

$(r_H)^*: \beta ch(X, X \setminus H; \tau) \to \beta ch(H;\tau|H)$ and $(r_H)^*{}_{,0}: \beta ch_0(X, X \setminus H; \tau) \to \beta ch(H;\tau|H)$ are well defined as follows(Lemma 2.4 (iii)),respectively:

$(r_H)^*(f)=r_{H,H}(f)$ for every $f \in \beta ch(X, X \setminus H; \tau)$;

$(r_H)^*{}_{,0}(g)= r_{H,H}(g)$ for every $g \in \beta ch_0(X, X \setminus H; \tau)$.Indeed ,in Lemma 2.4 (iii),we assume that $X=Y,\tau=\sigma$ and $H=f(H)$.

Then, under the assumption that H is α-open hence β-open in (X,τ),it is obtained that $r_{H,H}(f) \in \beta ch(H;\tau|H)$ holds for any $f\in\beta ch(X, X \setminus H; \tau)$(resp. $f\in \beta ch_0(X, X \setminus H; \tau)$).

We need the following lemma and then we prove that $(r_H)^*$ and $(r_H)^{*,0}$ are onto homomorphisms under the assumptions that H is α-open and α-closed in (X,τ).

Let $X=U_1\cup U_2$ for some subsets U_1 and U_2 and $f_1:(U_1,\tau|U_1) \to (Y,\sigma)$ and

$f_2:(U_2,\tau|U_2) \to (Y,\sigma)$ be the two maps satisfying a property $f_1(x)=f_2(x)$ for every $x\in U_1\cap U_2$.Then, a map $f_1\nabla f_2$ is well defined as follows:

$(f_1\nabla f_2)(x)= f_1(x)$ for every $x\in U_1$ and $(f_1\nabla f_2)(x)= f_2(x)$ for every $x\in U_2$.

We call this map a combination of $f_1$and f_2.

Lemma 2.6. For a topological space (X,τ),we assume that $X= U_1\cup U_2$,where U_1 and U_2 are subsets of X and $f_1:(U_1,\tau|U_1) \to (Y,\sigma)$ and $f_2:(U_2,\tau|U_2) \to (Y,\sigma)$ be the two maps satisfying a property $f_1(x)=f_2(x)$ for every $x\in U_1\cap U_2$.Then if $U_i\in\beta O(X,\tau)$ for each $i\in\{1,2\}$ and f_1 and f_2 are β-irresolute, then its combination $f_1\nabla f_2 : (X,\tau) \to (Y,\sigma)$ is β-irresolute.

Proof. Clear from Theorem 2.3 (i) and (iii-2).

Theorem 2.7.Let H be a subset of a topological space (X,τ).

(i).(i-1).If H is α-open in (X,τ),then the maps $(r_H)^*: \beta ch(X, X \setminus H;\tau) \to \beta ch(H;\tau|H)$

and $(r_H)^{*,0}: \beta ch_0(X, X \setminus H; \tau) \to \beta ch(H;\tau|H)$ are homomorphism of groups. (Definition 2.5).Morever $(r_H)^* \mid \beta ch_0(X, X \setminus H; \tau)= (r_H)^{*,0}$ holds.

(i-2). If H is α-open and α-closed in (X,τ),then the maps $(r_H)^*: \beta ch(X, X \setminus H; \tau) \to \beta ch(H;\tau|H)$ and $(r_H)^{*}{}_0: \beta ch_0(X, X \setminus H; \tau) \to \beta ch(H;\tau|H)$ are onto homomorphism of groups.

(ii).For an α-open subset H of (X,τ),we have the following isomorphisms of groups:

(ii-1). $\beta ch(X, X \setminus H; \tau) |Ker(r_H)^*$ is isomorphic to $Im(r_H)^*$;

(ii-2). $\beta ch_0(X, X \setminus H; \tau)$ is isomorphic to $Im(r_H)^*$,

where $Ker(r_H)^*=\{a\in \beta ch(X, X\setminus H; \tau) \mid (r_H)^*(a)=1_X\}$ is a normal subgroup of $\beta ch(X, X\setminus H; \tau)$; $Im(r_H)^*=\{(r_H)^*(a) \mid a\in \beta ch(X, X\setminus H; \tau)\}$and $Im(r_H)^{*,0}=\{(r_H)^{*,0} (b) \mid b\in \beta ch_0(X, X\setminus H; \tau)\}$are subgroups of $\beta ch(X,\tau)$.

(iii). For an α-open and α-closed subset H of (X,τ),we have the following isomorphisms of groups:

(iii-1). $\beta ch(H;\tau|H)$ is isomorphic to $\beta ch(X, X\setminus H; \tau) |Ker(r_H)^*$.

(iii-2). $\beta ch(H;\tau|H)$ is isomorphic to $\beta ch_0(X, X \setminus H; \tau)$.

Proof.(i).(i-1).Let a,b∈ βch(X, X\H; τ).Since H is α-open in (X,τ),the maps $(r_H)^*$ and $(r_H)^{*,0}$ are well defined(Definition2.5).Then we have that$(r_H)^*(\omega_X(a,b))= (r_H)^*(b o a)= r_{H,H}(b o a)= r_{H,H}(b)$ o $r_{H,H}(a)= \omega_X((r_H)^*(a), (r_H)^*(b))$ hold,where ω_H is a binary operation of βch(H;τ|H)([5] Theorem 4.4 (iv)).Thus $(r_H)^*$ is a homomorphism of groups.For the map $(r_H)^{*}{}_0$: $\beta ch_0(X, X \setminus H; \tau) \to \beta ch(H;\tau|H)$,we have that $(r_H)^{*,0}(\omega_{X,0}(a,b))= (r_H)^{*,0}(b o a)= r_{H,H}(b o a)= r_{H,H}(b)$ o $r_{H,H}(a)= \omega_X((r_H)^*(a), (r_H)^*(b))$ hold, where ω_X is a binary operation of βch(H;τ|H)(Theorem 2.3 (ii)).Thus $(r_H)^{*,0}$ is also a homomorphism of groups.It is obviously shown that $(r_H)^* \mid \beta ch_0(X, X \setminus H; \tau)=(r_H)^{*,0}$ holds.(Definitions 2.1 and 2.5).

(i-2).In order to prove that $(r_H)^*$ and $(r_H)^*{}_0$ are onto ,let h∈ βch(H;τ|H).Let j_H: (H;τ|H) →(X,τ) and $J_{X\setminus H}$: (X \ H,τ| X \ H) →(X,τ) be the inclusions defined $j_H(x)=x$ for every x∈H and $J_{X\setminus H}(x)=x$ for every x∈X \ H.We consider the combination $h_1=(j_H o h) \nabla(j_{X\setminus H}$ o $1_{X\setminus H})$: (X,τ) →(X,τ).By Lemma 2.4 (ii-1),under the assumption of α-openness on H, it is shown that two maps j_H oh : (H;τ|H) →(X,τ) and j_H oh^{-1} : (H;τ|H) →(X,τ) are β-irresolute; moreover under the assumption of α-openness on X \ H, $J_{X\setminus H}$ o $1_{X\setminus H}$: (X \ H,τ| X \ H) →(X,τ) is β-irresolute.Using lemma 2.6, for a β-open cover {H, X \ H} of X,the combination above h_1: (X,τ) →(X,τ) is β-irresolute. Since h_1 is bijective, its inverse map $h_1^{-1}=(j_H o h^{-1}) \nabla(j_{X\setminus H}$ o $1_{X\setminus H})$ is also β-irresolute.Thus under the assumption that both H and X \ H are β-open in (X,τ),we have h_1∈ βch(X,τ).Since $h_1(x)=x$ for every point x∈X \ H,we conclude that $h_1 \in \beta ch_0(X, X \setminus H; \tau)$ and so $h_1 \in \beta ch(X, X \setminus H; \tau)$.Moreover, $(r_H)^{*,0}(h_1)= (r_H)^*(h_1)= r_{H,H}((h_1)=h$, hence $(r_H)^*$ and $(r_H)^{*,0}$ are onto,under the assumption that H is α-open and α-closed subset of (X,τ).

(ii).By (i-1) above and the first isomorphism theorem of group theory, it is shown that there are group isomorphism below, under the assumption that H is α-open in (X,τ):

(*). βch(X, X\H; τ) |Ker$(r_H)^*$ is isomorphic to Im$(r_H)^*$; and (**). βch_0(X, X \ H; τ)|Ker$(r_H)^*{}_0$ is isomorphic to Im$(r_H)^{*,0}$

where Ker$(r_H)^*{}_0$ ={a∈βch_0(X, X\H; τ) |$(r_H)^*{}_0(a)=1_X$}.Moreover, under the assumption of α-openness on H,it is shown that Ker$(r_H)^*{}_0$ ={1_H}.Therefore,using (**) above, we have the isomorphism (ii-2).

(iii).By (i-2) above, it is shown that $(r_H)^*$ and $(r_H)^{*,0}$ are onto homomorphism of groups,under the assumption that H is α-open and α-closed in (X,τ).Therefore ,by (ii) above, the isomorphisms (iii-1) and (iii-2) are obtained.

*Remark 2.8.*Under the assumption that H is α-open and α-closed in (X,τ),Theorem 2.7 (iii) is proved. Let (X,τ) be a topological space where X={a,b,c} and τ={φ, X,{a},{b,c}}, and (H;τ|H) is a subspace of (X,τ),where H={a}.Then βO(X,τ)=P(X)(the power set of X) and H is α -open and α -closed in (X,τ).We apply Theorem 2.7 (iii) to the present case,we have the group isomorphisms.Directly, we obtain the following date on groups: βch(X,τ) is isomorphic to S_3,the symmetric group of degree 3, βch(X, X \ H; τ)={1_X, h_a},Ker(r_H)*={1_X, h_a}, βch(H; τ \ H)={1_H} and so $βch_0$(X, X \ H; τ)= {1_X},where h_a:(X,τ) →(X,τ) is a map defined by h_a(a)=a, h_a(b)=c and h_a(c)=b.Therefore in this example, we have βch(H;τ|H) is isomorphic to βch(X, X \ H; τ) |Ker(r_H)* and βch(H;τ|H) is isomorphic to $βch_0$(X, X \ H; τ).Moreover we have h(X,τ)={1_X, h_a}.

(iii). Even if a subset H of a topological space (X,τ) is not α-closed and it is α-open, we have the possibilities to investigate isomorphisms of groups corresponding to a subspace (H, τ|H) and (r_H)* using Theorem 5.7(ii).For example, Let (X,τ) be a topological space where X={a,b,c} and τ={φ, X, {a,b}},and (H;τ|H) is a subspace of (X,τ),where H={a,b}.Then βO(X,τ)=P(X)(the power set of X) except {c} $τ^α = τ$ .The subset H is α-open but not α-closed in (X,τ). By theorem 2.7(i)(i-1), the maps (r_H)*: βch(X, X \ H;τ) → βch(H;τ|H) and (r_H)*$_{,0}$: $βch_0$(X, X \ H; τ) → βch(H;τ|H) are homomorphism of groups and by theorem 5.7(ii) two isomorphisms of groups are obtained:

(*-1). βch(X, X \ H; τ)/Ker(r_H)* is isomorphic to Im(r_H)*. (*-2). $βch_0$(X, X \ H; τ)/Ker(r_H)* is isomorphic to Im(r_H)*$_{,0}$.

We need notation on maps as follows: let h_c: (X,τ) →(X,τ) and $t_{a,b}$: (H,τ|H) → (H,τ|H) are the maps defined by h_c(a)=b, h_c(b)=a, h_c(c)=c and $t_{a,b}$(a)=b, $t_{a,b}$ (b)=a,respectively.Then it is directly shown that βch(X, X \ H; τ)={1_X, h_c}which is isomorphic to Z_2, $(h_c)^2$= 1_X, and Ker(r_H)*={a∈ βch(X, X \ H; τ)| (r_H)*(a)= 1_H }={a∈{1_X, h_c}|(r_H)*(a)= 1_H }={1_X },because (r_H)*(1_X)= 1_H and (r_H)*(h_c)= $t_{a,b}$ not equal to 1_H .By using (*-1) above, Im(r_H)* is isomorphic to βch(X, X \ H; τ) ={1_X, h_c}and so Im(r_H)*={ 1_H, $r_{H,H}$(h_c)}={1_H, $t_{a,b}$}.Since Im(r_H)*⊆ βch(H;τ|H) ⊆{1_H, $t_{a,b}$},we have that Im(r_H)* = βch(H,τ|H)= {1_H, $t_{a,b}$} and hence (r_H)*

is onto. Namely, we have an isomorphism (r_H)*: βch(X, X \ H; τ) is isomorphic to βch(H ;τ|H) which is isomorphic to Z_2.Morever it is shown that $βch_0$(X, X \ H; τ)={a∈ βch(X, X \ H; τ)|a(x)=x for any x∈{c}}={1_X, h_c}= βch(X, X \ H; τ) hold and so (r_H)*= (r_H)*$_{,0}$ holds.

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