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Fixed Point Theorems in Strong M-Fuzzy Metric Space for Weakly Compatible

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Abstract

In this paper, we examine the common fixed point theorem of weakly compatible mapping and weak** commutative satisfying the implicit relation in strong M- fuzzy metric space.

Keywords: Fuzzy set, M-fuzzy metric space, Strong M-fuzzy metric space, weakly compatible mapping, weak** commutative.

1. INTRODUCTION

Zadeh(1965) [9]introduced the fuzzy set. Kramosil and Michalek(1975) [3]introduced the concept of fuzzy metric space. Popa (1997,1999) [5][6]introduced some fixed point theorems satisfying certain implicit relation. Dhage (1992)[1] introduced the notion of generalized metric or D-metric space and proved several fixed point theorem.

Sedghi and Shoba (2006)[7] gave D* metric space as modification of the definition of D metric introduced by Dhage and also defined M- fuzzy metric space by using the concept of D* metric. Gregori et al (2010) [2] introduced strong fuzzy metric space and proved a fixed point theorem.

In this paper, we obtained fixed point theorem by using implicit relation in strong M-fuzzy metric space.

2. PRELIMINARIES

Definition: 2.1[9]

A fuzzy set A in X is a function with domain X and values in [0,1]

Definition: 2.2[8]

A binary operation $*$: $[0,1] \times [0,1] \rightarrow [0,1]$ is a continuous t-norm if $*$ is satisfying the conditions:

i. **$*$** is commutative and associative,

ii. **$*$** is continuous,

iii. $a*1 = a$ for all $a \in [0,1]$,

iv. $a*b \leq c*d$ whenever $a \leq c$ and $b \leq d$, and $a,b,c,d \in [0,1]$.

Definition:2.3[7]

A 3-tuple (X,M,*) is called a M-fuzzy metric space, if X is an arbitrary (non-empty) set, * is a continuous t-norm and M is a fuzzy set on $X^3 \times (0,\infty)$ satisfying the following condition for each $x,y,z,a \in X$ and $t,s > 0$

i)$M(x,y,z,t) > 0$

(ii)$M(x,y,z,t) = 1$, if and only if $x=y=z$

(iii)$M(x,y,z,t) = M(p\{x,y,z\},t)$ where p is a permutation function

(iv)$M(x,y,a,t) * M(a,z,z,s) \leq M(x,y,z,t+s)$

(v)$M(x,y,z,.) : [0,\infty) \rightarrow [0,1]$ is continuous.

Remark:2.4[7]

Let (X,M,*) be a M-fuzzy metric space. Then for every $t > 0$ and for every $x,y \in X$ we have $M(x,y,y,t) = M(x,x,y,t)$.

Definition:2.5[2]

Let (X,M,*) be a M-fuzzy metric space. The M-fuzzy metric is said to be strong (non-Archimedean) if it satisfies $M(x,y,z,t) \geq M(x,y,a,t)*M(a,z,z,t)$ for each $x,y,z \in X$ and each $t>0$.

Definition:2.6[7]

Let (X,M,*) be a M-fuzzy metric space, for t>0 the open ball BM(x,r,t) with centre x ∈ X and radius 0<r<1 is defined by BM(x,r,t) = {y∈X:M(x,y,y,t)>1-r}. A subset A of X is called open set if for each x ∈ A there exists t>0 and 0<r<1, such that BM(x,r,t) ≤ A.

Definition:2.7[7]

A sequence $\{x_n\}$ in X converges to X if and only if M(x_n,x,t) →1 as n→ ∞ for each t>0. It is called a Cauchy sequence if for each 0<∈<1and t>0 there exists n_0 ∈ N such that M(x_n, x_n, x_m,t) > 1-∈ for each n,m ≥ n_0 the M-fuzzy metric space (X,M,*) is said to be complete if every cauchy sequence is convergent.

Lemma:2.8[7]

Let (X,M,*) be a M-fuzzy metric space. Then M(x,x,y,t) is non – decreasing with respect to t for all x,y,z in X.

Lemma:2.9[7]

Let (X,M,*) be a M-fuzzy metric space. Then M is continuous function on X^3 × (0,∞).

Definition:2.10[7]

Let F and G be two self maps of (X,M,*) then F and G are said to be weakly compatible if there exists v in X with Fv = Gv implies FGv=GFv.

Definition:2.11[4]

Two self mappings A and S of fuzzy metric space (X,M, *) is called weak** commuting

if A(X) ⊂ S(X) and for any x ∈ X,

M(A^2S^2x,S^2A^2x,t) ≥M(A^2Sx, S^2Ax,t) ≥
M(AS^2x,SA^2x,t) ≥M(ASx,SAx,t) ≥M(A^2x, S^2x,t)

Remark:2.12[4]

If A and S are idempotent maps i.e. A^2 = A and S^2 = S then weak** commutative reduces to weak commuting pair of (A,S).

3. MAIN RESULTS

Implicit Relation:1

Let Φ be the set of all real continuous functions F: $[0,1]^6$ → R is continuous function such that

(F_1): For u,v > 0 F is non-decreasing in the fifth and sixth variable

(F_{1a}) : F{v(t) , u(t) , v(t) , u(t) , u(t) * v(t) , 1} ≥ 0

then u(t) ≥ u(t) * v(t)

(F_2): F{u(t) , u(t) , u(t) , u(t) , 1 , 1}≥ 0

F{1 , u(t) , 1 , u(t) , u(t) , u(t) }≥ 0

F{u(t) , u(t) , 1 , u(t) , u(t) , 1 }≥ 0

Then u(t) ≥ 1

Example:

Define $f(t_1,t_2,t_3,t_4,t_5,t_6) = 2t_1 + 5t_2 -2t_3 -4t_4 -t_5 -t_6 +1$

Then f ∈ Φ .

Theorem:3.1

Let (X,M,*) be a strong M-fuzzy metric space and (A,S) and (B,T) be self maps with continuous t-norm * defined by a*b = min(a,b) a,b ∈ [0,1]

F{M(Ax,By,Tz,t), M(Ax,Sy,Tz,t), M(By,Sy,Tz,t), M(Ax,Tz,Sy,t), M(By,Ay,Tz,t), M(Bx,Ax,Tz,t)} ≥ 0

- AX ⊆ SX and BX ⊆ TX
- (A,S) and (B,T) are weakly compatible

Then A,B,S,T have common fixed point.

Proof:

Let x_0 ∈ X be any arbitrary point. Since A (X) ⊂ S(X) and B(X) ⊂ T(X) there exists a point x_1 , x_2 ∈ X such that Ax_0 = Sx_1 , Bx_1 = Tx_2 inductively, we get a sequence $\{y_{2n}\}$ as

y_{2n} = Ax_{2n} = Sx_{2n+1} y_{2n+1} = Bx_{2n+1} = Tx_{2n+2} n = 1,2…

Let M_{2n} = M(y_{2n}, y_{2n+1}, y_{2n+2} , t) < 1 for all n,

Put x = x_{2n-1} , y = x_{2n} , z = x_{2n+1}

Substituting in the inequality, we get

F{M(Ax_{2n-1}, Sx_{2n-1}, Tx_{2n+1},t) , M(Ax_{2n-1},Bx_{2n},Sx_{2n+1},t) ,
M(Sx_{2n-1},Ax_{2n-1},Sx_{2n-1},t) , M(Tx_{2n+1}, Ax_{2n-1}, Bx_{2n},t) , M(Ax_{2n+1},Bx_{2n}, Sx_{2n-1},t) ,
M(Tx_{2n+1}, Bx_{2n}, Sx_{2n+1},t)} ≥ 0

F{M(y_{2n-1}, y_{2n-2}, y_{2n},t),M(y_{2n-1},y_{2n}, y_{2n},t),M(y_{2n-2}, y_{2n-1},y_{2n},t),M(y_{2n}, y_{2n-1},y_{2n},t),

M(y_{2n+1},y_{2n}, y_{2n-2},t),M(y_{2n}, y_{2n}, y_{2n},t)} ≥ 0

Here (X,M,*) is strong M-fuzzy metric space ,then

F{M(y_{2n-1}, y_{2n-2}, y_{2n},t),M(y_{2n-1},y_{2n}, y_{2n},t),M(y_{2n-2}, y_{2n-1},y_{2n},t),M(y_{2n}, y_{2n-1},y_{2n},t),

M(y_{2n+1},y_{2n}, y_{2n-1},t)*M(y_{2n-1}, y_{2n-2}, y_{2n-2},t),M(y_{2n}, y_{2n}, y_{2n},t)} ≥ 0

F{M_{2n-2} , M_{2n-1} , M_{2n-2} , M_{2n-1} , M_{2n-1}*M_{2n-2} , 1} ≥ *0*

By using (F_{1a})

Thus, we have $\mathrm{M}_{2n-1} > \mathrm{M}_{2n-1} * \mathrm{M}_{2n-2}$ -------------- (1)

Consider a*b = min{a,b}

Claim:1

$\mathrm{M}_{2n-1} > \mathrm{M}_{2n-1}$

which is not possible.

Claim:2

$\mathrm{M}_{2n-1} > \mathrm{M}_{2n-2}$. Therefore $\mathrm{M}_{2n} > \mathrm{M}_{2n-1}$

Thus {M_{2n},n ≥ 0} is an increasing sequences of positive real numbers in [0,1] and therefore tends to limit L ≤ 1. We claim L =1 for L<1 taking limit in (1), we get L<L which is a contradiction .

Therefore L =1

For any positive integer r

$$\mathrm{M}(y_n, y_n, y_{n+r},t) \geq \mathrm{M}(y_n, y_n, y_{n+1},t/r) * \mathrm{M}(y_{n+1}, y_{n+1}, y_{n+2}, t/r) * \ldots * \mathrm{M}(y_{n+r-1}, y_{n+r-1}, y_{n+r},t/r)$$

$$> (1-\epsilon) \text{ r times} = (1-\epsilon)$$

$M(y_n, y_n, y_{n+r},t) > 1-\in$

For all n,s $\geq n_0$ where $n_0 \in N$. Thus, $\{y_n\}$ is a Cauchy sequence in X. Since X is complete there is a point p ∈ X such that $y_n \to p$. Thus subsequence $\{Ax_{2n}\},\{Sx_{2n-1}\},\{Bx_{2n}\},\{Tx_{2n}\}$ also converges to p. Since $A(X) \subset S(X)$ and $B(X) \subset T(X)$ then there must exist u,v ∈ X such that p= Sv = Tu.

Put x = v ,y = x_{2n} ,z = x_{2n+1}

$F\{M(Av,Sv,Tx_{2n+1},t) , M(Av,Bx_{2n},Sx_{2n+1},t) , M(Sv,Av,Sv,t) , M(Tx_{2n+1},Av,Bx_{2n},t) , M(Ax_{2n+1},Bx_{2n},Sv,t) , M(Tx_{2n+1},Bx_{2n},Sx_{2n+1},t)\} \geq 0$

$F\{M(Av,p,p,t) , M(Av,p,p,t) , M(p,Av,p,t) , M(p,Av,p,t) , M(p,p,p,t) , M(p,p,p,t)\} \geq 0$

$F\{M(Av,p,p,t) , M(Av,p,p,t) , M(p,Av,p,t) , M(p,Av,p,t) , 1 , 1\} \geq 0$

By using F_2 which implies $M(Av,p,p,t) \geq 1$

Av = p

Therefore Av = p = Sv

Put x = v ,y = u , z = x_{2n+1}

$F\{M(Av,Sv,Tx_{2n+1},t) , M(Av,Bu,Sx_{2n+1},t) , M(Sv,Av,Sv,t) , M(Tx_{2n+1},Av,Bu,t) , M(Ax_{2n+1},Bu,Sv,t) , M(Tx_{2n+1},Bu,Sx_{2n+1},t)\} \geq 0$

$F\{M(p,p,p,t) , M(p,Bu,p,t) , M(p,p,p,t) , M(p,p,Bu,t) , M(p,Bu,p,t) , M(p,Bu,p,t)\} \geq 0$

$F\{1 , M(p,Bu,p,t) , 1 , M(p,p,Bu,t) , M(p,Bu,p,t) , M(p,Bu,p,t)\} \geq 0$

By using F_2 which implies $M(Bu,p,p,t) \geq 1$

Bu = p

Therefore Bu = p = Tu

Combine p =Av =Sv=Bu =Tu

Since (A,S) and (B,T) are weak compatible. Therefore ASv =SAv ⇒ Ap =Sp

BTu = TBu ⇒ Bp =Tp

Hence p is a coincidence point of A,B,S and T

Put x = x_{2n-1} , y = p , z = x_{2n+1}

$F\{M(Ax_{2n-1},Sx_{2n-1},Tx_{2n+1},t) , M(Ax_{2n-1},Bp,Sx_{2n+1},t) , M(Sx_{2n-1},Ax_{2n-1},Sx_{2n-1},t) , M(Tx_{2n+1},Ax_{2n-1},Bp,t) , M(Ax_{2n+1},Bp,Sx_{2n-1},t) , M(Tx_{2n+1},Bp,Sx_{2n+1},t) \} \geq 0$

$F\{M(p,p,p,t) , M(p,Tp,p,t) , M(p,p,p,t) , M(p,p,Tp,t) , M(p,Tp,p,t) , M(p,Tp,p,t)\} \geq 0$

F{1, M(p,Tp,p,t) , 1 , M(p,p,Tp,t) , M(p,Tp,p,t) , M(p,Tp,p,t)} ≥ 0

By using F_2 which implies M(Tp,p,p,t) ≥ 1

Tp = p =Bp

Put x =p ,y =x_{2n} , z = x_{2n+1}

F{M(Ap,Sp,Tx_{2n+1},t) , M(Ap,Bx_{2n},Sx_{2n+1},t) , M(Sp,Ap,Sp,t) , M(Tx_{2n+1},Ap,Bx_{2n},t) , M(Ax_{2n+1},Bx_{2n},Sp,t) , M(Tx_{2n+1},Bx_{2n},Sx_{2n+1},t) } ≥ 0

F{M(Sp,Sp,p,t) , M(Sp,p,p,t) , M(Sp,Sp,Sp,t) , M(p,Sp,p,t) , M(p,p,Sp,t) , M(p,p,p,t)} ≥ 0

F{M(Sp,Sp,p,t) , M(Sp,p,p,t) , 1, M(p,Sp,p,t) , M(p,p,Sp,t) , 1}≥ 0

By using F_2 which implies M(Sp,p,p,t) ≥ 1

Sp = p = Ap

Similarly, Ap = Sp = Bp = Tp

Hence p is a common fixed point of A,B,S and T.

Implicit Relation:2

Let Φ be the set of all real continuous functions F: $[0,1]^6$ → R is continuous function such that

(F_1): For u,v >0 F is non-decreasing in the fifth and sixth variable

(F_{1a}) : F{u(t) , v(t) , u(t) , v(t) , 1, u(t) * v(t) } ≥ 0

then u(t) ≥ u(t) * v(t)

(F_2): F{u(t) , 1 , u(t) , u(t) , 1 , 1}≥ 0

F{u(t) , 1 , u(t) , 1 , u(t) , u(t) }≥ 0

F{u(t) , u(t) , u(t) , 1 , 1 , u(t) }≥ 0

F{ u(t) , u(t) , u(t) , 1 , u(t) , u(t)} }≥ 0

Then u(t) ≥ 1

Theorem:3.2

Let (X,M,*) be a complete strong M-fuzzy metric space and (A,S) and (B,T) are weak** commuting pairs of self maps on X satisfying

AX ⊆ SX and BX ⊆ TX

$F\{M(A^2x,B^2y,T^2z,t)$, $M(S^2x, T^2x, S^2z,t)$, $M(A^2x,B^2y, S^2z,t)$, $M(A^2x, S^2x, S^2x,t)$, $M(B^2y, A^2y, A^2y,t)$, $M(A^2z, B^2y, S^2x,t)\} \geq 0$

For all x,y,z ∈ X ,t > 0 then A,B,S and T have a unique common fixed point .

Proof:

Let $x_0 \in X$ be any arbitrary point. Since A (X) ⊂ S(X) and B(X) ⊂ T(X) there exist a point x_1 , $x_2 \in X$ such that $A^2x_0 = S^2x_1$, $B^2x_1 = T^2x_2$ inductively, construct sequence $\{y_{2n}\}$ as

$y_{2n} = A^2x_{2n} = S^2x_{2n+1}$ $y_{2n+1} = B^2x_{2n+1} = T^2x_{2n+2}$ n = 1,2…

Let $M_{2n} = M(y_{2n}, y_{2n+1}, y_{2n+2}, t) < 1$ for all n,

Put $x = x_{2n-1}$, $y = x_{2n}$, $z = x_{2n+1}$

$F\{M(A^2x_{2n-1}, B^2x_{2n}, T^2x_{2n+1},t)$, $M(T^2x_{2n-1}, S^2x_{2n-1}, S^2x_{2n+1},t)$, $M(A^2x_{2n-1}, B^2x_{2n}, S^2x_{2n+1},t)$, $M(A^2x_{2n-1}, S^2x_{2n-1}, S^2x_{2n-1},t)$, $M(B^2x_{2n}, A^2x_{2n}, A^2x_{2n},t)$, $M(A^2x_{2n+1}, B^2x_{2n}, S^2x_{2n-1},t)\} \geq 0$

$F\{M(y_{2n-1}, y_{2n}, y_{2n},t)$, $M(y_{2n-2}, y_{2n-2}, y_{2n},t)$, $M(y_{2n-1}, y_{2n}, y_{2n},t)$, $M(y_{2n-1}, y_{2n-2}, y_{2n-2},t)$, $M(y_{2n}, y_{2n}, y_{2n},t)$, $M(y_{2n+1}, y_{2n}, y_{2n-2},t)\} \geq 0$

Using strong M-fuzzy metric space

$F\{M(y_{2n-1}, y_{2n}, y_{2n},t)$, $M(y_{2n-2}, y_{2n-2}, y_{2n},t)$, $M(y_{2n-1}, y_{2n}, y_{2n},t)$, $M(y_{2n-1}, y_{2n-2}, y_{2n-2},t)$, 1 , $M(y_{2n+1}, y_{2n}, y_{2n-1},t)*M(y_{2n-1}, y_{2n-2}, y_{2n-2},t)\} \geq 0$

$F\{M_{2n-1}, M_{2n-2}, M_{2n-1}, M_{2n-2},1, M_{2n-1} * M_{2n-2}\} \geq 0$

By using F_{1a}

Thus, we have $M_{2n-1} > M_{2n-1} * M_{2n-2}$ -------------- (2)

Consider a*b = min{a,b}

Claim:1

$M_{2n-1} > M_{2n-1}$

which is not possible.

Claim:2

$M_{2n-1} > M_{2n-2}$.

Therefore $M_{2n} > M_{2n-1}$

Thus$\{M_{2n}, n \geq 0\}$ is an increasing sequence of positive real numbers in [0,1] and therefore tends to limit L ≤ 1. We claim L =1 for L<1 taking limit in (2), we get L<L which is a contradiction .

Therefore L =1

For any positive integer r

$M(y_n, y_n, y_{n+r},t) \geq M(y_n, y_n, y_{n+1},t/r) * M(y_{n+1}, y_{n+1}, y_{n+2},t/r)$
$*\ldots\ldots*M(y_{n+r-1}, y_{n+r-1}, y_{n+r},t/r)$

$> (1-\epsilon)$ r times = $(1-\epsilon)$

$M(y_n, y_n, y_{n+r},t) > 1-\epsilon$

For all n,s $\geq n_0$ where $n_0 \in N$. Thus, $\{y_n\}$ is a Cauchy sequence in X. Since X is complete there is a point $p \in X$ such that $y_n \to p$. Thus subsequence $\{Ax_{2n}\},\{Sx_{2n-1}\},\{Bx_{2n}\},\{Tx_{2n}\}$ also converges to p.

Case:I

S(X) is complete

Take $p \in S(X)$ there exist $v \in X$ such that $p = S^2v$

Put $x = v$, $y = x_{2n}$, $z = x_{2n+1}$

$F\{M(A^2v,B^2x_{2n},T^2x_{2n+1},t) , M(S^2v, T^2x_{2n}, S^2x_{2n+1},t) , M(A^2v, B^2x_{2n}, S^2x_{2n+1},t) , M(A^2v, S^2v, S^2v,t) , M(B^2x_{2n}, A^2x_{2n}, A^2x_{2n},t) , M(A^2x_{2n+1}, B^2x_{2n}, S^2v,t)\} \geq 0$

$F\{M(A^2v,p,p,t) , M(p,p,p,t) , M(A^2v,p,p,t) , M(A^2v,p,p,t) , 1 , 1\} \geq 0$

By using (F_2)

$M(A^2v,p,p,t) \geq 1$ which implies $A^2v = p$

$P = A^2v = S^2v$

Using (A,S) is weak ** commuting

$M(S^2A^2v, A^2S^2v,t) \geq M(S^2Av, A^2Sv,t) \geq M(SA^2v,AS^2v,t) \geq M(SAv,ASv,t) \geq M(S^2v, A^2v,t)$

which implies $S^2A^2v = A^2S^2v$

$S^2p = A^2p$

Put $x = p$, $y = x_{2n}$, $z = x_{2n+1}$

$F\{M(A^2p,B^2x_{2n},T^2x_{2n+1},t) , M(S^2p, T^2x_{2n}, S^2x_{2n+1},t) , M(A^2p, B^2x_{2n}, S^2x_{2n+1},t) , M(A^2p, S^2p, S^2p,t) , M(B^2x_{2n}, A^2x_{2n}, A^2x_{2n},t) , M(A^2x_{2n+1}, B^2x_{2n}, S^2p,t)\} \geq 0$

$F\{M(A^2p,p,p,t) , M(A^2p,p,p,t) , M(A^2p,p,p,t) , 1, 1 , M(p,p, A^2p)\} \geq 0$

By using (F_2)

$M(A^2p,p,p,t) \geq 1$ which implies $A^2p = p$

Hence p = $A^2p = S^2p$

Case:II

T(X) is complete

Take p ∈ T(X) there exist v ∈ X such that p = T^2v

Put x = x_{2n-1} , y = v , z = x_{2n+1}

$F\{M(A^2x_{2n-1},B^2v,T^2x_{2n+1},t)$, $M(S^2x_{2n-1}, T^2v, S^2x_{2n+1},t)$, $M(A^2x_{2n-1}, B^2v, S^2x_{2n+1},t)$, $M(A^2x_{2n-1}, S^2x_{2n-1}, S^2x_{2n-1},t)$, $M(B^2v, A^2v, A^2v,t)$, $M(A^2x_{2n+1}, B^2v, S^2x_{2n-1},t)\} \geq 0$

$F\{M(p, B^2v ,p,t)$, $M(p,p,p,t)$, $M(p, B^2v ,p,t)$, $M(p,p,p,t)$, $M(B^2v,p,p,t)$, $M(p, B^2v ,p,t)\} \geq 0$

$F\{M(p, B^2v ,p,t)$, 1, $M(p, B^2v ,p,t)$, 1, $M(B^2v,p,p,t)$, $M(p, B^2v ,p,t)\} \geq 0$

By using (F_2)

$M(B^2v,p,p,t) \geq 1$ which implies $B^2v = p$

$P = B^2v = T^2v$

Using (B,T) is weak ** commuting

$M(T^2B^2v, B^2T^2v,t) \geq M(T^2Bv, B^2Tv,t) \geq M(TB^2v,BT^2v,t) \geq M(TBv,BTv,t) \geq M(T^2v, B^2v,t)$

which implies $T^2B^2v = B^2T^2v$

$T^2p = B^2p$

Put x = x_{2n-1} , y = p , z = x_{2n+1}

$F\{M(A^2x_{2n-1},B^2p,T^2x_{2n+1},t)$, $M(S^2x_{2n-1} , T^2p, S^2x_{2n+1},t)$, $M(A^2x_{2n-1} , B^2p, S^2x_{2n+1},t)$, $M(A^2x_{2n-1} , S^2x_{2n-1} , S^2x_{2n-1} ,t)$, $M(B^2p, A^2p, A^2p,t)$, $M(A^2x_{2n+1}, B^2p, S^2x_{2n-1} ,t)\} \geq 0$

$F\{M(p, B^2p ,p,t)$, $M(p, B^2p\ p,t)$, $M(p, B^2p\ p,t)$, 1, $M(B^2p,p,p,t)$, $M(p, B^2p,p,t)\} \geq 0$

By using (F_2)

$M(B^2p,p,p,t) \geq 1$ which implies $B^2p = p$

Hence p = $B^2p = T^2p$

Put x = Ap , y = p , z = p

$F\{M(A^2(Ap), B^2p, T^2p,t)$, $M(S^2(Ap), T^2p, S^2p,t)$, $M(A^2(Ap), B^2p, S^2p,t)$, $M(A^2(Ap), S^2(Ap), S^2(Ap),t)$, $M(B^2p, A^2p, A^2p,t)$, $M(A^2p, B^2p, S^2(Ap),t)\} \geq 0$

F{M(Ap,p,p,t) , M(Ap,p,p,t) , M(Ap,p,p,t) , 1,1,M(Ap,p,Ap,t)} $\geq$0

Ap = p

Similarly ,show that Bp = p ,Sp = p and Tp = p

P = Ap = Bp = Sp = Tp

p is common fixed point of A,B,S and T.

Uniqueness:

Put x = p ,y = p , z = q

F{M(p,p,q,t) , M(p,p,q,t) , M(p,p,q,t) , M(p,p,p,t) , M(p,p,p,t) , M(q,p,p,t)} $\geq$0

F{M(p,p,q,t) , M(p,p,q,t) , M(p,p,q,t) , 1,1 , M(q,p,p,t)} $\geq$0

M(p,p,q,t) $\geq$ 1

Therefore p = q

Hence p is a unique fixed point of A,B,S and T.

Theorem:3.3

Let (X,M,*) be complete strong M-fuzzy metric space. Let A,B,S and T be self maps satisfying the following condition

A(X) $\subseteq$S(X) and B(X) $\subseteq$ T(X)

(A,S) and (B,T) are weak compatible pair

M(Ax,By,Sz,t) $\geq$ min{M(Ax,Ty,Tz,t) , M(Ax,Sx,Sz,t) , M(Bz,Tz,Sx,t) , M(By,Sz,Tz,t)}

Then A,B,S and T have a unique fixed point .

Proof:

Let $x_0 \in$ X be any arbitrary point. Since A (X) $\subset$ S(X) and B(X) $\subset$ T(X) there exist a point x_1 , $x_2 \in$ X such that $Ax_0 = Sx_1$, $Bx_1 = Tx_2$ inductively, we get a sequence $\{y_{2n}\}$ as

$$y_{2n} = Ax_{2n} = Sx_{2n+1} \qquad y_{2n+1} = Bx_{2n+1} = Tx_{2n+2} \quad n = 1,2\ldots$$

Let $M_{2n} = M(y_{2n}, y_{2n+1}, y_{2n+2}, t) < 1$ for all n,

Put $x = x_{2n-1}$, $y = x_{2n}$, $z = x_{2n+1}$

Substituting in the inequality, we get

$M(Ax_{2n-1},Bx_{2n}\ ,Sx_{2n+1},t) \geq \min\{M(Ax_{2n-1},Tx_{2n}\ ,Tx_{2n+1},t)$,
$M(Ax_{2n-1},Sx_{2n-1},Sx_{2n+1},t)$, $M(Bx_{2n+1},Tx_{2n+1},Sx_{2n-1},t)$,
$M(Bx_{2n},Sx_{2n+1},Tx_{2n+1},t)\}$

$M(y_{2n+1}, y_{2n}, y_{2n}, t) \geq \min\ \{M(y_{2n-1}, y_{2n-1}, y_{2n},t)$, $M(y_{2n-1}, y_{2n-2}, y_{2n},t)$,
$M(y_{2n+1}, y_{2n}, y_{2n-2},t)$, $M(y_{2n}, y_{2n}, y_{2n},t)$

Using strong M- Fuzzy metric space

$M(y_{2n-1}, y_{2n}, y_{2n}, t) \geq \min\ \{M(y_{2n-1}, y_{2n-1}, y_{2n},t)$, $M(y_{2n-1}, y_{2n-2}, y_{2n},t)$,
$M(y_{2n+1}, y_{2n}, y_{2n-1},t)*M(y_{2n-1}, y_{2n-2}, y_{2n-2},t)$, $M(y_{2n}, y_{2n}, y_{2n},t)$

$M_{2n-1} \geq \min\{M_{2n-1}, M_{2n-2}, M_{2n-1}*M_{2n-2},1\}$

Thus, we have $M_{2n-1} > M_{2n-1} * M_{2n-2}$ -------------- (3)

Consider a*b = min{a,b}

Claim:1

$M_{2n-1} > M_{2n-1}$

which is not possible.

Claim:2

$M_{2n-1} > M_{2n-2}$.

Therefore $M_{2n} > M_{2n-1}$

Thus$\{M_{2n},n \geq 0\}$ is an increasing sequence of positive real numbers in [0,1] and therefore tends to limit L ≤ 1. We claim L =1 for L<1 taking limit in (3), we get L<L which is a contradiction .

Therefore L =1

For any positive integer r

$M(y_n, y_n, y_{n+r},t) \geq M(y_n, y_n, y_{n+1},t/r) * M(y_{n+1}, y_{n+1}, y_{n+2},t/r)$
$*\ldots\ldots*M(y_{n+r-1}, y_{n+r-1}, y_{n+r},t/r)$

$>$ (1-∈) r times = (1-∈)

$M(y_n, y_n, y_{n+r},t) > 1-\in$

For all n,s $\geq n_0$ where $n_0 \in$ N. Thus, $\{y_n\}$ is a Cauchy sequence in X. Since X is complete there is a point p ∈ X such that $y_n \to p$. Thus subsequence

$\{Ax_{2n}\},\{Sx_{2n-1}\},\{Bx_{2n}\},\{Tx_{2n}\}$ also converges to p. since A(X) ⊂ S(X) and B(X) ⊂ T(X) then there must exist u,v ∈ X such that p= Su = Tv.

Put x = u ,y = x_{2n} ,z = x_{2n+1}

$M(Au,Bx_{2n},Sx_{2n+1},t) \geq \min\{M(Au,Tx_{2n},Tx_{2n+1},t) , M(Au,Su,Sx_{2n+1},t) , M(Bx_{2n+1},Tx_{2n+1},Su,t) , M(Bx_{2n},Sx_{2n+1},Tx_{2n+1},t)\}$

$M(Au,p,p,t) \geq \min\{M(Au,p,p,t) , M(Au,p,p,t) , M(p,p,p,t) , M(p,p,p,t)\}$

$M(Au,p,p,t) \geq \min\{M(Au,p,p,t) , M(Au,p,p,t) ,1,1\}$

$M(Au,p,p,t) \geq M(Au,p,p,t)$ which is a contradiction

Au =p

Hence Au = p =Su

Put x = u , y = v, z = x_{2n+1}

$M(Au,Bv,Sx_{2n+1},t) \geq \min\{M(Au,Tv,Tx_{2n+1},t) , M(Au,Su,Sx_{2n+1},t) , M(Bx_{2n+1},Tx_{2n+1},Su,t) , M(Bv,Sx_{2n+1},Tx_{2n+1},t)\}$

$M(p,Bv,p,t) \geq \min \{M(p,p,p,t) , M(p,p,p,t) , M(p,p,p,t) , M(Bv,p,p,t)\}$

$M(p,Bv,p,t) \geq \min \{1,1,,1, M(Bv,p,p,t)\}$

which is a contradiction

Bv = p = Tv

Since (A,S) and (B,T) are weak compatible, ASu =SAu ⇒ Ap =Sp

BTv = TBv ⇒ Bp =Tp

Hence p is a coincidence point of A,B,S and T

Put x = u, y = p, z = x_{2n+1}

$M(Au,Bp,Sx_{2n+1},t) \geq \min\{M(Au,Tp,Tx_{2n+1},t) , M(Au,Su,Sx_{2n+1},t) , M(Bx_{2n+1},Tx_{2n+1},Su,t) , M(Bp,Sx_{2n+1},Tx_{2n+1},t)\}$

$M(p,Bp,p,t) \geq \min \{M(p,Bp,p,t) , M(p,p,p,t) , M(p,p,p,t) , M(Bp,p,p,t)\}$

$M(p,Bp,p,t) \geq \min \{M(p,Bp,p,t) , 1,1, M(Bp,p,p,t)\}$

$M(p,Bp,p,t) \geq \{M(p,Bp,p,t)$ which is a contradiction

Bp = p =Tp

Similarly, Ap = p =Sp

p is the common fixed point.

Uniqueness:

Put x = p, y = p, z = w

$M(Ap,Bp,Sw,t) \geq \min\{M(Ap,Tp,Tw,t) , M(Ap,Sp,Sw,t) , M(Bw,Tw,Sp,t) , M(Bp,Sw,Tw,t)\}$

$M(p,p,w,t) \geq \min\{M(p,p,w,t) , M(p,p,w,t) , M(w,w,p,t) , M(p,w,w,t)\}$

$M(p,p,w,t) \geq M(p,p,w,t)$ which is contradiction

p = w

p is the unique fixed point of A,B,S and T.

Corollary:3.4

Let (X,M,*) be complete strong M-fuzzy metric space. Let A,B,S and T be self maps satisfying the following condition

$A(X) \subseteq S(X)$ and $A(X) \subseteq T(X)$

(A,S) and (A,T) are weakly compatible

$M(Ax,Ay,Sz,t) \geq \min\{M(Ax,Sx,Sz,t) , M(Az,Tz,Sz,t) , M(Ay,Sz,Sx,t)\}$

Then A,S and T have a unique fixed point .

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