

# Investigation of Effects of Asset Price Fluctuations on Option Value

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## Abstract

In this paper, the numerical effects of asset price fluctuation on the value of an option using a two-dimensional Black-Scholes-Merton partial differential equation have been investigated. The equation governing the value of an option was solved numerically using the Crank-Nicolson finite difference scheme and simulated in MATLAB software to obtain the profiles of the option values. The numerical results obtained from the present study have been presented graphically and also discussed. The effects of varying risk-free interest rate, the volatility of the two assets prices, correlation coefficient between the two asset prices, and dividend payout on the value of an option have been determined. It was observed that an increase in volatility of the two asset prices results in an increase in the values of both the call and put options. It was also noted that an increase in risk-free interest rate results in an increase in the value of a call option but a decrease in the value of a put option. Furthermore, the results revealed that an increase in the dividend payout and correlation coefficient between the two asset prices results in a decrease in the value of a call option but an increase in the value of a put option. The results obtained from the present study are useful for investors wishing to maximize the profits from their investments.

**Keywords:** Black-Scholes-Merton, Crank-Nicolson, options, stochastic-differential-equation.

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## 1. INTRODUCTION

The trading of options has gained a lot of interest from various researchers due to imperfect market liquidity, which results in fluctuations in prices of the underlying asset. In the nineteenth century, a contemporary of Einstein applied random phenomenon (i.e., a stochastic process) on the theory of speculation and the concept of Brownian motion to price the value of options. Black, Scholes, and Merton later came up with the celebrated modern partial differential equation for option pricing. Thus, Black-Scholes-Merton model became the most powerful and significant tool for the valuation of an option. Due to the fluctuations in market prices of the underlying asset, rigorous mathematical and probabilistic concepts through the theory of stochastic process (also known as Wiener process) resolving old challenges and giving mathematical ideas to new problems in engineering and finance and other different fields were formulated.

There was a further rigorous treatment to stochastic process and stochastic differential equations which ended up with the laws that govern stochastic integration and solutions to stochastic differential equations. Stochastic differential equations are fundamental in describing and understanding random phenomena in different areas in physics, engineering, finance, economics, and other areas. In particular, they serve as a model for asset price fluctuation in finance and is the driving force behind the famous Black-Scholes-Merton option pricing partial differential equation.

In his study on the linear stochastic price securities adjustments, [9] developed a logistical equation for security asset consideration. The study discovered that assets security prices rarely rise up exponentially because of the controlling factors that might inhibit the asset prices. [11] numerically investigated a linear Black-Scholes model using finite difference method based on full and semi-discrete schemes for European call and put option. The study revealed that as the price of the asset increases the call option value also increases.

[7] investigated the correlation trading strategies and found that an increase in volatility leads to an increase in the option value. The study also found that an increase in volatility lowers the value of the call option if the correlation coefficient is increased and vice versa. In their study on the numerical investigations of the effects of variable transaction costs on the value of an option, [10] analyzed an option pricing model of nonlinear Black-Scholes equation with stochastic transaction prices. The study indicated that the Black-Scholes model can be interpreted by transforming it fully from parabolic nonlinear equation into a parabolic quasi-linear equation for the second derivative of the price of an option. The study further revealed that as the price of the underlying asset increases the corresponding value of an option also increases.

[2] numerically investigated the generalized Black-Scholes-Merton pricing in an illiquid market with transaction costs. The study revealed that the existence of transactions costs, price slippage and large traders in a financial market that is not perfectly liquid impact heavily on option prices. The study also noted that an increase in the price of the underlying asset leads to an increase in the value of a call option and a decrease in the value of a put option. The study concluded that European options become more volatile due to a rise in transaction costs and the price impact from an illiquid market.

[8] developed a financial mathematical analysis model to obtain market options prices with changing volatilities. The results showed that the approximations by the Heston model perfectly performs but experiences a big challenge only if pricing options which take longer time to maturity and also yielding unrealistic prices when the moneyless is less than one. The study also revealed that if correlation coefficient is negative, then volatility will increase as the asset price return decreases. Conversely, if correlation coefficient is positive the volatility will increase leading to the decrease in asset price return.

[1] quantified the effects of the temporal and spatial analysis of the Black-Scholes equation on option pricing. He expanded the model by relaxing these assumptions and derived an option pricing formula in a more realistic financial environment. He introduced a new hedging strategy and the remodeling of stock prices to reflect some activities and consequences of modern trading markets. The study revealed that as volatility increases the value of both the call and put options also increases. The study also revealed that an increase in the risk-free interest rate results in an increase in the value of a call option but a decrease in the value of a put option.

[5] investigated the drawbacks and limitations of Black-Scholes model for option pricing. The study found that the most serious problem with the model is the issue of constant volatility, which is considerably disrupted in practice. The study also revealed that using stochastic and deterministic volatility yields the most accurate option contract price. The study recommended that the Black-Scholes model should be revised by including non-constant volatility, either deterministic or stochastic.

[6] studied pricing an option under the jump diffusion and multifactor stochastic processes. The study found that the price of an option increases with an increase in the volatility value and jump rate. Furthermore, the study discovered that price of an option reduces with the slow-scale rate and goes up with the fast-scale rate, and that the result of fast-scale volatility in the long run is lower than the effect of slow-scale volatility. Variance swap of the strike price was also found to be negatively correlated

with the time to maturity. The study recommended that it is important to carry out the investigation numerically to get more accurate solutions of higher dimensional partial differential equations governing the price of an option.

[3] studied simple formulas for pricing and hedging European options in the finite moment log-stable model. The study revealed that as stability parameters increase, the value of both the put and call options also increases. However, the study assumed that the market parameters such as risk-free interest rate, volatilities among other parameters are constant.

From the above literature on option pricing, it is noted that analysis of option pricing using one-dimensional Black-Scholes-Merton partial differential equation (BSMPDE) has been the focus of several researchers. Furthermore, most studies assumed constant market parameters. Thus, there is need to numerically investigate the effects of varying risk-free interest rate, correlation coefficient between the two assets prices, volatility of two asset prices and dividend payout on the value of an option considering a two-dimensional BSMPDE, which is the focus of the present study. The precise computation of the value of an option is essential in a structured and full-fledged financial economy.

The rest of the paper is organized as follows: section 2 presents the model description and mathematical analysis, section 3 presents the numerical technique used to solve the corresponding model, section 4 presents the results of the present study, and section 5 presents the conclusions drawn from the present study.

## 2. MATHEMATICAL FORMULATION

In this study, the underlying assets ( $S_1$  and  $S_2$ ) follow a log-normal random world with delta hedging done continuously to eliminate the risks. It is assumed that there are no transaction costs on the underlying assets and no arbitrage opportunities. Under the assumptions, the two-dimensional Black-Scholes-Merton partial differential equation (1) governing the value of an option is given by:

$$\frac{\partial V}{\partial t} + r \left[ S_1 \frac{\partial V}{\partial S_1} + S_2 \frac{\partial V}{\partial S_2} \right] + \frac{1}{2} \left[ \sigma_1^2 S_1^2 \frac{\partial^2 V}{\partial S_1^2} + \sigma_2^2 S_2^2 \frac{\partial^2 V}{\partial S_2^2} \right] + \rho \sigma_1 \sigma_2 S_1 S_2 \frac{\partial^2 V}{\partial S_1 \partial S_2} - rV = 0, \quad (1)$$

where  $V$  represents option value,  $r$  is the risk-free interest rate,  $\rho$  is the correlation coefficient between the two assets,  $\sigma_1$  and  $\sigma_2$  are the volatilities of the underlying

assets  $S_1$  and  $S_2$ , respectively. The derivation of equation (1) is shown in the appendix, considering the geometric Brownian motion (Wiener process). Equation (1) is subject to the following boundary conditions.

### 2.1. Boundary conditions for call option

$$\begin{cases} V(0, 0, t) = 0, & \text{given } 0 \leq t \leq T \\ V(x, y, t) = \max(x, y) - Ke^{-r(T-t)} & \text{as } x, y \rightarrow \infty \\ V(x, y, T) = \max(x - K, y - K, 0) & \text{when } t = T \\ K = 50, T = 1 \text{ year} \end{cases}$$

### 2.2. Boundary conditions for put option

$$\begin{cases} V(0, 0, t) = Ke^{-r(T-t)}, & \text{given } 0 \leq t \leq T \\ V(x, y, t) \rightarrow 0 & \text{as } x, y \rightarrow \infty \\ V(x, y, T) = \max(K - x, K - y, 0) & \text{when } t = T \\ K = 50, T = 1 \text{ year} \end{cases}$$

### 2.3. Dividend payout and change of variables

Suppose the underlying assets,  $S_1$  and  $S_2$ , receive constant dividend payout,  $d_1$  and  $d_2$ , respectively. This means that in a time  $dt$  each asset receives an amount of  $d_1 S_1 dt$  and  $d_2 S_2 dt$ , respectively. Thus, introducing a dividend payout on each of the underlying assets from equation (1) yields the standard two-dimensional BSMPDE:

$$\begin{aligned} & \frac{\partial V}{\partial t} + r \left[ S_1 \frac{\partial V}{\partial S_1} + S_2 \frac{\partial V}{\partial S_2} \right] - \left[ d_1 S_1 \frac{\partial V}{\partial S_1} + d_2 S_2 \frac{\partial V}{\partial S_2} \right] \\ & + \frac{1}{2} \left[ \sigma_1^2 S_1^2 \frac{\partial^2 V}{\partial S_1^2} + \sigma_2^2 S_2^2 \frac{\partial^2 V}{\partial S_2^2} \right] + \rho \sigma_1 \sigma_2 S_1 S_2 \frac{\partial^2 V}{\partial S_1 \partial S_2} - rV = 0 \end{aligned} \quad (2)$$

We introduce the following transformations of the independent variables according to [4]:

$$x = \ln(S_1) - \left( r - \frac{1}{2}\sigma_1^2 \right) t \quad \text{and} \quad y = \ln(S_2) - \left( r - \frac{1}{2}\sigma_2^2 \right) t$$

Thus, equation (2) reduces to

$$\begin{aligned} & \frac{\partial V}{\partial t} + r \left[ \frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} \right] - \left[ d_1 \frac{\partial V}{\partial x} + d_2 \frac{\partial V}{\partial y} \right] \\ & + \frac{1}{2} \left[ \sigma_1^2 \frac{\partial^2 V}{\partial x^2} + \sigma_2^2 \frac{\partial^2 V}{\partial y^2} \right] + \rho \sigma_1 \sigma_2 \frac{\partial^2 V}{\partial x \partial y} - rV = 0 \end{aligned} \quad (3)$$

Assuming  $d_1 = d_2 = d$  represents the dividend payout on both assets and  $\sigma_1 = \sigma_2 = \sigma$ , equation (3) reduces to:

$$\begin{aligned} & \frac{\partial V}{\partial t} + (r - d) \left[ \frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} \right] + \frac{\sigma^2}{2} \left[ \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right] \\ & + \rho \sigma^2 \frac{\partial^2 V}{\partial x \partial y} - rV = 0 \end{aligned} \quad (4)$$

Equation (4) is solved numerically using finite difference method based on Crank-Nicolson scheme.

### 3. NUMERICAL PROCEDURE

The initial boundary value problem (IBVP) in equation (4) is solved numerically using Crank-Nicolson finite difference scheme. In this scheme, the spatial computational domain defined by  $\Omega = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1\}$  is partitioned into  $n_x$  and  $n_y$  equal sub-intervals based on a linear Cartesian mesh and uniform grid. The discrete approximation of  $V(x, y, t)$  at the grid point  $(i\Delta x, j\Delta y, n\Delta t)$  is denoted by  $V_{i,j}^n$  for  $i = 1, 2, \dots, n_x$ ;  $j = 1, 2, \dots, n_y$ ;  $n = 0, 1, 2, \dots$ , where  $\Delta x = \frac{1}{n_x}$  is the grid size in  $x$ -direction,  $\Delta y = \frac{1}{n_y}$  is the grid size in  $y$ -direction, and  $\Delta t$  represents the increment in time. The nodes at  $i = 1, j = 1$  and  $i = n_x, j = n_y$  define the boundary. The time derivative is approximated using forward difference scheme while the spatial derivatives in  $x$  and  $y$  directions are approximated by the average of the central difference approximations at  $n^{\text{th}}$  and  $(n + 1)^{\text{th}}$  levels. The values of the nodes at the  $n^{\text{th}}$  level are known while the values of the nodes at the  $(n + 1)^{\text{th}}$  level are unknown.

Thus using the Crank-Nicolson scheme, we have the proposed averages for the derivatives involved in equation (4) as:

$$V = \frac{1}{2} [V_{i,j}^n + V_{i,j}^{n+1}] \quad (5)$$

$$\frac{\partial V}{\partial t} = \frac{V_{i,j}^{n+1} - V_{i,j}^n}{\Delta t} \quad (6)$$

$$\frac{\partial V}{\partial x} = \frac{1}{2} \left[ \frac{V_{i+1,j}^n - V_{i-1,j}^n}{2(\Delta x)} + \frac{V_{i+1,j}^{n+1} - V_{i-1,j}^{n+1}}{2(\Delta x)} \right] \quad (7)$$

$$\frac{\partial V}{\partial y} = \frac{1}{2} \left[ \frac{V_{i,j+1}^n - V_{i,j-1}^n}{2(\Delta y)} + \frac{V_{i,j+1}^{n+1} - V_{i,j-1}^{n+1}}{2(\Delta y)} \right] \quad (8)$$

$$\begin{aligned} \frac{\partial^2 V}{\partial x^2} &= \frac{1}{2} \left[ \frac{V_{i+1,j}^n - 2V_{i,j}^n + V_{i-1,j}^n}{(\Delta x)^2} \right] \\ &+ \frac{1}{2} \left[ \frac{V_{i+1,j}^{n+1} - 2V_{i,j}^{n+1} + V_{i-1,j}^{n+1}}{(\Delta x)^2} \right] \end{aligned} \quad (9)$$

$$\begin{aligned} \frac{\partial^2 V}{\partial y^2} &= \frac{1}{2} \left[ \frac{V_{i,j+1}^n - 2V_{i,j}^n + V_{i,j-1}^n}{(\Delta y)^2} \right] \\ &+ \frac{1}{2} \left[ \frac{V_{i,j+1}^{n+1} - 2V_{i,j}^{n+1} + V_{i,j-1}^{n+1}}{(\Delta y)^2} \right] \end{aligned} \quad (10)$$

$$\begin{aligned} \frac{\partial^2 V}{\partial x \partial y} &= \frac{1}{2} \left[ \frac{V_{i+1,j+1}^n - V_{i-1,j+1}^n - V_{i+1,j-1}^n + V_{i-1,j-1}^n}{4(\Delta x)(\Delta y)} \right] \\ &+ \frac{1}{2} \left[ \frac{V_{i+1,j+1}^{n+1} - V_{i-1,j+1}^{n+1} - V_{i+1,j-1}^{n+1} + V_{i-1,j-1}^{n+1}}{4(\Delta x)(\Delta y)} \right] \end{aligned} \quad (11)$$

Substituting equations (5)-(11) into equation (4), multiplying through by  $\Delta t$  and rearranging yields

$$\begin{aligned}
 & \left\{ 1 - \frac{r(\Delta t)}{2} \right\} V_{i,j}^{n+1} + \\
 & \frac{(r-d)\Delta t}{4} \left[ \frac{V_{i+1,j}^{n+1} - V_{i-1,j}^{n+1}}{(\Delta x)} + \frac{V_{i,j+1}^{n+1} - V_{i,j-1}^{n+1}}{(\Delta y)} \right] + \\
 & \frac{\sigma^2 \Delta t}{4} \left[ \frac{V_{i+1,j}^{n+1} - 2V_{i,j}^{n+1} + V_{i-1,j}^{n+1}}{(\Delta x)^2} + \frac{V_{i,j+1}^{n+1} - 2V_{i,j}^{n+1} + V_{i,j-1}^{n+1}}{(\Delta y)^2} \right] \\
 & + \frac{\rho\sigma^2 \Delta t}{8(\Delta x)(\Delta y)} \left[ V_{i+1,j+1}^{n+1} - V_{i-1,j+1}^{n+1} - V_{i+1,j-1}^{n+1} + V_{i-1,j-1}^{n+1} \right] \\
 = & -\frac{\rho\sigma^2 \Delta t}{8(\Delta x)(\Delta y)} \left[ V_{i+1,j+1}^n - V_{i-1,j+1}^n - V_{i+1,j-1}^n + V_{i-1,j-1}^n \right] \\
 & -\frac{\sigma^2 \Delta t}{4} \left[ \frac{V_{i+1,j}^n - 2V_{i,j}^n + V_{i-1,j}^n}{(\Delta x)^2} + \frac{V_{i,j+1}^n - 2V_{i,j}^n + V_{i,j-1}^n}{(\Delta y)^2} \right] \\
 & -\frac{(r-d)\Delta t}{4} \left[ \frac{V_{i+1,j}^n - V_{i-1,j}^n}{(\Delta x)} + \frac{V_{i,j+1}^n - V_{i,j-1}^n}{(\Delta y)} \right] \\
 & + \left\{ 1 + \frac{r(\Delta t)}{2} \right\} V_{i,j}^n \tag{12}
 \end{aligned}$$

The corresponding finite difference approximations of the boundary conditions are given by:

**3.0.1 Call option**

$$\begin{cases} V_{0,0}^{n+1} = 0, & \text{for all } n \\ V_{n_x, n_y}^{n+1} = \max(n_x \Delta x, n_y \Delta y) - Ke^{-r(n+1)\Delta t} & \text{for all } n \end{cases}$$

**3.0.2 Put option**

$$\begin{cases} V_{0,0}^{n+1} = Ke^{-r(n+1)\Delta t}, & \text{for all } n \\ V_{n_x, n_y}^{n+1} = 0 & \text{for all } n \end{cases}$$

For  $n = 0$  and  $i = 1, 2, \dots, n_x - 1; j = 1, 2, \dots, n_y - 1$ , equation (12) yields a system of  $(n_x - 1) \times (n_y - 1)$  linear equations for the  $(n_x - 1) \times (n_y - 1)$  unknown values in the first time row in terms of the initial values and the boundary values. Similarly,



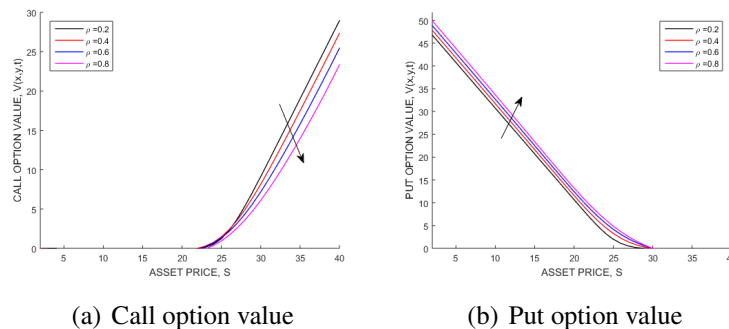
for  $n = 1, n = 2$ , and so on. Thus, for each time row, we have to solve a system of  $(n_x - 1) \times (n_y - 1)$  algebraic equations resulting from (12).

The numerical simulations are performed using uniform grid with a mesh width  $\Delta x = \Delta y = 0.1$  and time step-size  $\Delta t = 0.01$ , with the aid of MATLAB software. The steady state numerical solutions for different values of  $\sigma, \rho, r$  and  $d$  are obtained at specific values of  $n$ . The simulation results are presented graphically and discussed in the next section.

#### 4. RESULTS AND DISCUSSION

The simulation results, which are presented in form of graphs, focus on the effects of varying the correlation coefficient ( $\rho$ ), volatility ( $\sigma$ ), risk-free interest rate ( $r$ ), and dividend payout ( $d$ ) on both call and put option values.

##### 4.1. Effects of varying correlation coefficient on option value



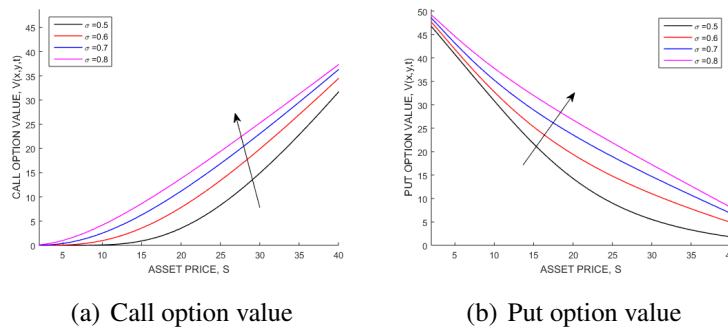
**Figure 1:** Graph of option value against asset price at varying correlation coefficient when  $K = 50, T = 1, \sigma = 0.1, r = 0.02$ , and  $d = 0.04$

Fig. 1(a) shows that as correlation coefficient between the assets prices increases, the call option value will decrease. This is because an increase in the correlation coefficient implies that the investor will not be comfortable buying an option with a closer relationship since this doesn't help in promoting diversification, which is very critical for any investor to realize high returns from the investments. For instance, if an investor holds assets having a positive correlation and are declining in the value, the investor will experience huge losses when he buys such an asset.

Fig. 1(b) shows that an increase in correlation coefficient between the two assets prices increases the put option value. This is because for a put option the investors will be

more comfortable dealing with options that are mostly related, which will help them in enjoying economies of scale especially for the options whose markets are readily available.

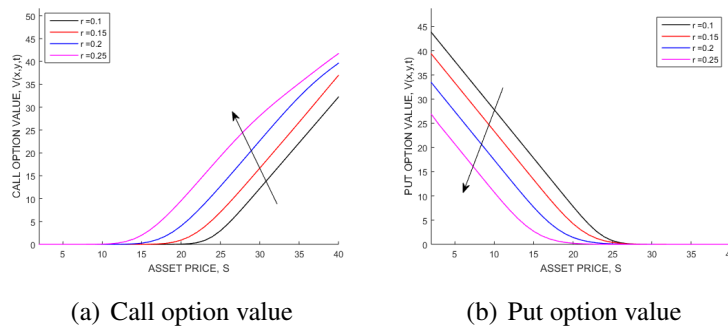
### 4.2. Effects of varying volatility on an option value



**Figure 2:** Graph of option value against asset price at varying volatility when  $K = 50, T = 1, \rho = 0.2, r = 0.02,$  and  $d = 0.04$

Fig. 2(a) shows that an increase in the volatility results in an increase in the call option value. Fig. 2(b) shows that an increase in the volatility results in an increase in the the put option value. These imply that there will be higher probability of price fluctuations. In reality, market uncertainties are also costly and therefore, its value will be higher than that of both the call and put options.

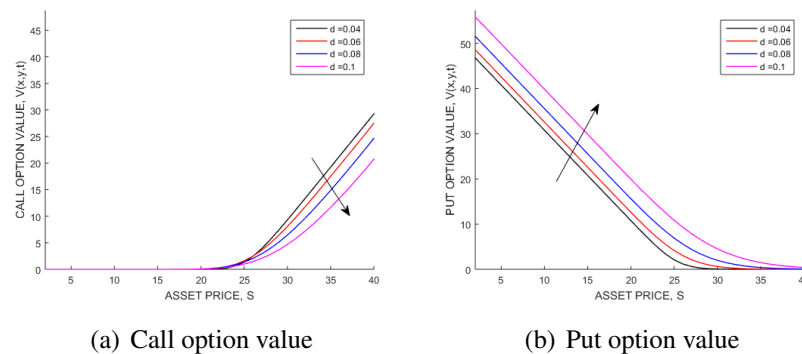
### 4.3. Effects of varying risk-free interest rate on an option value



**Figure 3:** Graph of option value against asset price at varying risk-free interest rate when  $K = 50, T = 1, \rho = 0.2, \sigma = 0.1,$  and  $d = 0.04$

Fig. 3(a) shows that an increase in the risk-free interest rate increases the call option value. This is because as the risk-free interest rate increases, the expected interest income and the associated costs of owning the underlying asset would also rise. Therefore, this would make the value of a call option to be more preferable and attractive. Fig. 3(b) shows that an increase in the risk-free interest rate decreases the put option value. This is because as the risk-free interest rate increases, the chances that the investor will make profits will significantly reduce.

#### 4.4. Effects of varying dividend payout on an option value



**Figure 4:** Graph of option value against asset price at varying dividend payout when  $K = 50$ ,  $T = 1$ ,  $\rho = 0.2$ ,  $\sigma = 0.1$ , and  $r = 0.02$

It is observed from Fig. 4(a) that an increase in the dividend payout results in a decrease in the call option value. This is because as the dividend payout for a call option increases, the expected benefits of acquiring the underlying asset will reduce. It is observed from Fig. 4(b) that an increase in the dividend payout increases the put option value. This is because as the dividend payout increases, the owning put options on dividend paying stocks will be more desirable.

## 5. CONCLUSION

This study leads to a conclusion that the market parameters such as volatilities, risk-free interest rate, correlation coefficient between the underlying asset prices, and dividend payout are pertinent when studying options as they lead to increased or decreased option value. Thus, these parameters shouldn't be assumed constant in any option pricing model.

## APPENDIX

### DERIVATION OF TWO-DIMENSIONAL BSMPDE

Consider the standard BSMPDE for an option with two assets with a dynamic market, ideal liquidity and payments of zero dividends before the option's maturity period. Using a European option whose pay-off depends on the prices of two underlying assets  $S_1$  and  $S_2$ , the geometric Brownian motion becomes

$$\begin{aligned} dS_1 &= \mu_1 S_1 dt + \sigma_1 S_1 dz_1 \\ dS_2 &= \mu_2 S_2 dt + \sigma_2 S_2 dz_2' \end{aligned} \quad (13)$$

where  $\mu_1, \mu_2$  represent the drift coefficients and  $z_1, z_2$  represent the Wiener processes.

Suppose that the random numbers  $dS_1$  and  $dS_2$  are correlated such that  $E[dS_1, dS_2] = \rho dt$ . Constructing the portfolio ( $\pi$ ) for the two underlying assets  $S_1$  and  $S_2$ , we have;

$$\pi = V - \Delta_1 S_1 - \Delta_2 S_2 \quad (14)$$

The change on the value of the portfolio (14) from  $t$  to  $dt$  is given by:

$$d\pi = dV - \Delta_1 dS_1 - \Delta_2 dS_2 \quad (15)$$

From *Itô's lemma* with two variables, we have

$$\begin{aligned} dV &= \frac{\partial V}{\partial S_1} dS_1 + \frac{\partial V}{\partial S_2} dS_2 \\ &+ \left[ \frac{\partial V}{\partial t} + \frac{1}{2} \sigma_1^2 S_1^2 \frac{\partial^2 V}{\partial S_1^2} + \rho \sigma_1 \sigma_2 S_1 S_2 \frac{\partial^2 V}{\partial S_1 \partial S_2} + \frac{1}{2} \sigma_2^2 S_2^2 \frac{\partial^2 V}{\partial S_2^2} \right] dt \end{aligned} \quad (16)$$

Choosing  $\Delta_1 = \frac{\partial V}{\partial S_1}$ ,  $\Delta_2 = \frac{\partial V}{\partial S_2}$  and substituting equation (16) into equation (15), the portfolio changes by the amount

$$d\pi = \left[ \frac{\partial V}{\partial t} + \frac{1}{2} \sigma_1^2 S_1^2 \frac{\partial^2 V}{\partial S_1^2} + \rho \sigma_1 \sigma_2 S_1 S_2 \frac{\partial^2 V}{\partial S_1 \partial S_2} + \frac{1}{2} \sigma_2^2 S_2^2 \frac{\partial^2 V}{\partial S_2^2} \right] dt \quad (17)$$

This change is completely risk-less. Thus, it must be the same as the growth we would get if we put the equivalent amount of cash in a risk-free interest-bearing account:

$$d\pi = r\pi dt = r \left[ V - \frac{\partial V}{\partial S_1} S_1 - \frac{\partial V}{\partial S_2} S_2 \right] dt \quad (18)$$

Substituting equation (17) into (18) and rearranging yields the two-dimensional BSMPDE (1). This completes the proof.

## **CONFLICTS OF INTEREST**

The authors declare that there is no conflict of interest regarding the publication of this paper.

## **ACKNOWLEDGMENT**

The authors would like to thank the academic and technical support of Jomo Kenyatta University of Agriculture and Technology (JKUAT) especially Kisii CBD campus for providing the necessary learning resources to facilitate the successful completion of this research project. The library, computer and other facilities of the University have been indispensable. C. O. Ndede thanks the chairman, all other staff members of the department of pure and applied mathematics and his colleagues for their support and assistance, also for making his stay in the department pleasant and memorable.

## **REFERENCES**

- [1] Francis Agana. *Temporal and Spatial Analysis of the Black-Scholes Equation for Option Pricing*. PhD thesis, JKUAT-PAUSTI, 2018.
- [2] Francis Agana, Oluwole Daniel Makinde, and David Mwangi Theuri. Numerical treatment of a generalized black-scholes model for options pricing in an illiquid financial market with transactions costs. *Global Journal of Pure and Applied Mathematics*, 12(5):4349–4361, 2016.
- [3] Jean-Philippe Aguilar and Jan Korbel. Simple formulas for pricing and hedging european options in the finite moment log-stable model. *Risks*, 7(2):36, 2019.
- [4] Mauricio Contreras, Alejandro Llanquihuén, and Marcelo Villena. On the solution of the multi-asset black-scholes model: Correlations, eigenvalues and geometry. *arXiv preprint arXiv:1510.02768*, 2015.
- [5] Zuzana Janková. Drawbacks and limitations of black-scholes model for options pricing. 2018.
- [6] Shican Liu, Yanli Zhou, Yonghong Wu, and Xiangyu Ge. Option pricing under the jump diffusion and multifactor stochastic processes. *Journal of Function Spaces*, 2019, 2019.
- [7] Gunter Meissner. Correlation trading strategies—opportunities and limitations. *The Journal of Trading*, 11(4):14–32, 2016.

- [8] Mitun Kumar Mondal, Md Abdul Alim, Md Faizur Rahman, and Md Haider Ali Biswas. Mathematical analysis of financial model on market price with stochastic volatility. *Journal of Mathematical Finance*, 7(02):351, 2017.
- [9] SN Onyango. On the non-linear stochastic price adjustment of securities. *East African Journal of Statistics*, 1(1):109–116, 2005.
- [10] Daniel Ševčovič and Magdaléna Žitňanská. Analysis of the nonlinear option pricing model under variable transaction costs. *Asia-Pacific Financial Markets*, 23(2):153–174, 2016.
- [11] Md Kazi Salah Uddin, Mostak Ahmed, and Samir Kumar Bhowmilk. A note on numerical solution of a linear black-scholes model. *GANIT: Journal of Bangladesh Mathematical Society*, 33:103–115, 2013.

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