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Study on Maximum – Cut Via Metric Vertex Weight Tree and its Density

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Abstract

In this paper we will compute maximum-cut for metric tree, then we will illustrate the density of such cuts of metric tree.

Keywords: Maximum-cut , metric tree , density of graph.

1. INTRODUCTION

For V is set , a distance function on Y is a map $\lambda : Y \times Y \rightarrow R+$ is symmetric, and satisfies the condition $d(i, i) = 0 \ \forall i \in X$.

The distance is said to be a metric if the triangle inequality holds, i.e., $d(i, j) \leq d(i, k) + d(k, j) \ \forall i, j, k \in X$.

Definition1 :A metric M on V is a cut metric, if ∃

$S \subseteq V \ni$ for all $i, j \in V$,

$d_{ij} = 1$ if $\{i, j\} \in \partial S$,

$d_{ij} = 0$, otherwise . δS is metric.

Note that: Every metric tree is metric if and only if satisfying triangle inequality.

2. MAIN RESULTS

For connected weighted tree T(V , E) a vertex weight function

W: V $\rightarrow R_+$ for two vertices v_1 , v_2 , v_1- v_2 cut problem we take maximum weight cut that separates v_1 and v_2 in separated metrics , that , v_1- v maximum cut is:

Max$\{\sum W_v t_v$, t is elementary cut metric э $t_{v1\ v2} \geq 1$ and $v \epsilon V\}$

Theorem2.1:

For metric tree of n-vertices , the maximum-cut obtained by taking the maximum absolute difference between two cuts t_1 , $t_2, \ldots\ldots t_n$ contains at least two vertices connected by one edge.

Lemma 2.2:

If T is nontrivial connected metric-tree of order n , then T has at most (n-2) cuts vertices.

Proof:

For metric – tree of order n : it has at least two vertices, these vertices are not splittable vertices(namely leaves , then any sub-tree t_1 has at most (n-2) cut vertices.

Example 2.3:

For metric-tree in Fig.(1-a) with 6 vertices , to compute maximum cut of vertices we follow the following steps:

1. Edge 1↔2 results in $d_{1\text{-}2}$ = 150 – 10 = 140
2. Edge 2↔3 results in $d_{2\text{-}3}$ = 150 – 10 = 140
3. Edge 2↔5 results in $d_{2\text{-}5}$ = 120 – 40 = 80
4. Edge 4↔5 results in $d_{4\text{-}5}$ = 110 – 50 = 60
5. Edge 5↔6 results in $d_{5\text{-}6}$ = 100 – 60 = 40

Since max. absolute difference is 140 makes only one vertex separated (v_1 or v_3) then

The maximum cut allowed is 80 (cuts of v_2, v_5) and the output cut shown in Fig.(1-b).

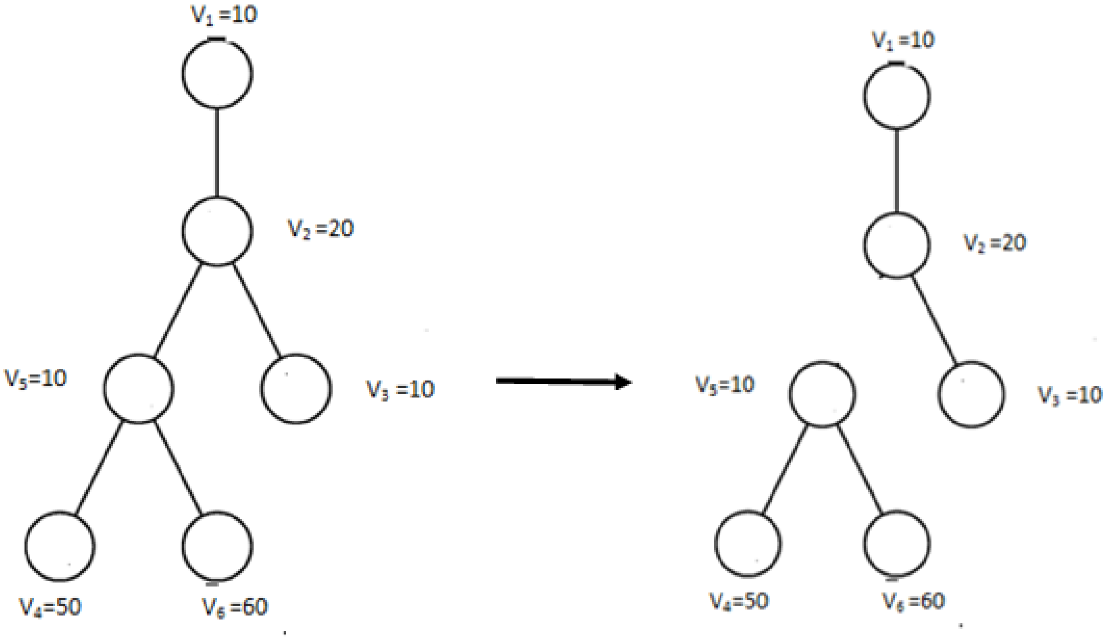

Fig.(1-a) Fig.(1-b)

Theorem 2.4:

For vertex-weight metric tree $T_m =(V^\backslash , E)$ is called metric max-cut tree if satisfies the condition, for any two vertices $v_1\ v_2 \in T$

Then: $\lambda_T (v_1 , v_2) = \max C_T(S_C)$ for $C \in P_{v1 , v2}$ which is consider the uniqe path from v_1 , v_2 ,

S_C is connected component of T containing S obtained by deleting e , $C_T(S_C)$ is the number of outgoing edges from S_C.

Lemma 2.5:

A vertex-weight cut metric tree $T (v^\backslash , E , C^\backslash)$ will be constructed by satisfying

$C^\backslash (v) = G_T (s_v)\ \ \forall v \in v^\backslash$.

The connectivity between v_1 , v_2 can be obtained by computing max. $C^\backslash (v)$ value over vertices $v \in P_{v1 , v2}$.

Theorem 2.6:

For $n \geq 1$, any metric tree $T_m = (v_m , e_m)$ with at least n vertices , can be cut into $T_m = (T_{m1} , T_{m2} , \ldots\ldots T_{mn})$ of disjoint sub-trees,

$T_{mi} = (v_{mi} , e_{mi})$.

So, for each sub-tree contains at least n vertices , at most 3n vertices.

Proof:

We will prove ∀ h= 3n+1 ∃ cuts into 2 sub-trees , each with at least n vertices and at most 2n vertices.

For vϵT_m, the maximum-cut of T contain one edge incident to V i.e :

∃ v ϵT so that v-sub-tree has at most 2n vertices.

Initially, root T has v vertices, then ∃ v-sub-tree with at least 2n+1 vertices, then the max. number of vertices in v-sub-tree decrease at least 1 vertex.

If ∃ v-sub-tree with n+1 vertex , then v-sub-tree and union of v-sub-trees is cut into 2 parts.

Else, every v-sub-tree has at most n vertices, then ∃ collection of n-sub-trees and the total number of vertices between n, 2n, The union of all and of other v-sub-trees is a cut of tree.

3. DENSITY OF MAX.-CUT OF METRIC TREE

If T_m is metric tree with v nods and e edges $T_m(v,e)$, then the density of it can be computed by:

$$D_T = \frac{\sum e}{(n\ 2)} = \frac{\sum e}{|v|(|v|-1)},$$

if metric tree cuts into C_1 , C_2 , cuts, then we can compute density for every individual cut.

Note- The density of graph $D \leq 1$.

Theorem 3.1:

For metric tree T_m, the density of T_m is less than the density of every individual cut $C_1 , C_2 , \ldots$..

and $D_T < D_{C1} + D_{C2} + \ldots\ldots\ldots$

Proof:

For metric tree of v vertices , e edges T_m(v, e) , if we cut it into two metric trees T_{m1}(v_1,e_1), T_{m2}(v_2 , e_2) , then:

$D_{Tm1} = \frac{\sum e1}{|v1|(\,|v1|-1\,)}$, $D_{Tm} = \frac{\sum e}{|v|(\,|v|-1\,)}$

$D_{Tm2} = \frac{\sum e2}{|v2|(\,|v2|-1\,)}$

Since $\sum e > \sum e_1$, $\sum e > \sum e_2$,

I v I > v_1, I v I > v_2 ,

Then $D_{Tm} < D_{T1}$, $D_{Tm} < D_{T2}$

and $D_{Tm} < D_{T1} + D_{T2}$

Example 3.2:

For tree shown in Fig.(1-a) ,= 1/6

The density $D_t = \frac{5}{6 \, x \, 5}$

For cut 1 , D_{c1} = 2/3x2 = 1/3.

For cut 2 D_{c2} = 2/3x2 = 1/3.

Then $D_T < D_{C1} + D_{C2}$

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