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A Study on Heat and Mass Transfer of an Unsteady MHD Natural Convective Flow of a Nano Fluid in the Presence of Diffusion-Thermo with Inclined Magnetic Field and Thermal Radiation

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Abstract

In this paper the effect of Thermal radiation, inclined magnetic field in the presence of Diffusion thermo on MHD natural convective heat and mass transfer flow of a nanofluid past through a semi infinite flat plate has been investigated. The plate is moved with constant velocity u_0, temperature and concentration are assumed to be fluctuating with time harmonically from a constant mean at the plate. The dimensionless governing differential equations for this investigation are solved analytically using perturbation method. The effect of velocity profile, temperature profile, concentration profile for Cu water nanofluids are discussed through graphs. It is observed that the thermal radiation and diffusion thermo enhance the velocity and temperature profile. The fluid velocity is decreased with the increasing value of Magnetic field parameter, the angle of inclinations.

Keywords: Nanofluid; MHD; Diffusion thermo; Thermal radiation Porusmedium

1. INTRODUCTION

Nanofluids is a fluid in which Nanometer sized particles are collided with the convectional heat transfer fluids to enhance the thermal conductivity of the fluid.N.Bachok (2010) has investigated the Boundary layer flow of a Nanofluid over a moving surface in a flowing fluid. Natural convective boundary layer flow over a horizontal plate embedded in a porous medium saturated with a Nanofluid is studied by R.Gorla and A.Chamkha (2011). It has been undertaken to analyze the natural convective past an isothermal horizontal plate in a porous medium saturated by a Nanofluid .M.Turkyilmazoglu (2012) studied the Exact analytical solutions for heat and mass transfer of MHD slip flow in Nano fluids. It investigates the influence of slip on the behaviour of fluid flow and thermal transport of some electrically conducting Nanofluid over a permeable stretching / shrinking sheet. Nanofluid flow and heat transfer due to rotating disk by Mustafa Turkyilmazoglu (2014). It investigates the best performingNanofluids (Cu,Ag,Cuo,

Al2o3and Tio2) over a rotating disk in terms of heat transfer. Unsteady natural convective boundary layer flow of MHD Nanofluid over a stretching surfaces with chemical reaction using the spectral relaxation method. This model is studied by Nageeb A.H. Haroun, Sabyasachi Mondal

(2015). It investigates the heat and mass transfer in an unsteady MHD Nanofluid boundary layer flow due to a stretching surface. S.V.K.Varma ,P.Durga Prasad, Kiran kumar (2015) has investigated the Analytical study of heat and mass transfer enhancement in free convection flow with chemical reaction and constant heat source in Nanofluids. To analyze the development of unsteady free convection flow of a Nanofluid past a vertical permeable semi-infinite flat plate in a rotating frame. Motivated by the above literature, it is of interests to study the effect of inclined magnetic field and thermal radiation in the heat and mass transfer of nano fluids in the presence of Dufour effect. The governing equations are solved analytically using simple perturbations techniques. The objective of this work is to analyze the thermal radiation, chemical reaction, Inclined magnetic field and Dufour effect of the heat and mass transfer and also analyzed the characteristics of the flow.

2. MATHEMATICAL SOLUTION

We Consider an unsteady two dimensional MHD free convectional flow of a Nano fluid past through a vertical permeable semi infinite moving plate with constant heat source. Assume that the flow is in x direction which is taken along the plate and the axis of y is normal to the plate. The strength of inclined magnetic field B_0 is assumed to be acting along the y axis. Consider the fluid as a water based Nano fluids containing Cu Nano particles. It is assumed that the base fluid and the Nano particles are in thermal equilibrium state. Since the plate is of semi infinite length therefore the variation along the x axis will be negligible and the flow variables are the functions of y and the time t only.

From the physical description the governing equations are,

Continuity Equation

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \qquad (1)$$

Momentum Equations

$$\rho_{nf}\left(\frac{\partial u'}{\partial t'}+v'\frac{\partial u'}{\partial y'}\right)=\mu_{nf}\frac{\partial^2 u'}{\partial y'^2}+(\rho\beta)_{nf}g(T'-T'_\infty)-\frac{\mu_{nf}u'}{K'}-\sigma {B_0}^2 u' sin^2\theta \qquad (2)$$

Energy equations

$$\left(\frac{\partial T'}{\partial t'}+v'\frac{\partial T'}{\partial y'}\right)=\alpha_{nf}\frac{\partial^2 T'}{\partial y'^2}-\frac{Q'}{(\rho c_p)_{nf}}(T'-T'_\infty)+\frac{D_m K_T}{C_s(\rho c_\rho)_{nf}}\frac{\partial^2 c'}{\partial y'^2}-\frac{\partial Q_r}{\partial y'}\frac{1}{(\rho c_p)_{nf}} \qquad (3)$$

Species equation

$$\frac{\partial C'}{\partial t'}+v'\frac{\partial C'}{\partial y'}=D_B\frac{\partial^2 C'}{\partial y'^2}-K_l(C'-C'_\infty) \qquad (4)$$

From physical description the governing equations (2) to (4) becomes,

$$\frac{\partial v'}{\partial y'}=0 \qquad (5)$$

suction parameter normal to the plate is, v = -V_0 (constant),where v is independent of y.

$$\rho_{nf}\left(\frac{\partial u'}{\partial t'}-V_0\frac{\partial u'}{\partial y'}\right)=\mu_{nf}\frac{\partial^2 u'}{\partial y'^2}+(\rho\beta)_{nf}g(T'-T'_\infty)-\frac{\mu_{nf}u'}{K'}-\sigma {B_0}^2 u' sin^2\theta \qquad (6)$$

$$\left(\frac{\partial T'}{\partial t'}-V_0\frac{\partial T'}{\partial y'}\right)=\alpha_{nf}\frac{\partial^2 T'}{\partial y'^2}-\frac{Q'}{(\rho c_p)_{nf}}(T'-T'_\infty)+\frac{D_m K_T}{C_s(\rho c_\rho)_{nf}}\frac{\partial^2 c'}{\partial y'^2}-\frac{\partial Q_r}{\partial y'}\frac{1}{(\rho c_p)_{nf}} \qquad (7)$$

$$\frac{\partial C'}{\partial t'}-V_0\frac{\partial C'}{\partial y'}=D_B\frac{\partial^2 C'}{\partial y'^2}-K_l(C'-C'_\infty) \qquad (8)$$

Where $u'$and v' is the velocity components in x and y axes respectively.ρ_{nf} is the density of the nano fluid μ_{nf} is the viscosity of the nano fluid, $\beta_{nf}$is the coefficient of thermal expansion of Nano fluid,σ is the electric conductivity of the fluid and g is the acceleration due to gravity, $(\rho c_p)_{nf}$ is the heat capacitance of the nano fluid, K' is the permeability porous medium,T' is the temperature of the nano fluid, Q is the temperature dependent volumetric rate of heat source, α_{nf} is the thermal diffusivity of the nano fluid.$\varphi$is the solid volume fraction of the nano particles,K_{nf} and K_s are thermal conductivities of the base fluid , solid respectively. The viscosity, thermal diffusivity ,heat capacitance of the nano fluid is given by

$(\rho c_p)_{nf}=(1-\varphi)(\rho c_p)_f+\varphi(\rho c_p)_s$, $(\rho\beta)_{nf}=(1-\varphi)(\rho)_f+\varphi(\rho)_s$,

$(\rho\beta)_{nf}=(1-\varphi)(\rho\beta)_f+\varphi(\rho\beta)_s$ $K_{nf}=K_f\left(\frac{K_s+2K_f-2\varphi(K_f-K_s)}{K_s+2K_f+2\varphi(K_f-K_s)}\right.$,

$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}}$, $\alpha_{nf} = \frac{K_{nf}}{(\rho c_p)_{nf}}$,

The radiative heat term by using the Rosseland approximation is given by:

$q_r = \frac{-4\sigma^*}{3k^*}\frac{\partial T^4}{\partial z}$

$T^4 = 4T_\infty^3$ T- $3T_\infty^4$, $\frac{\partial y}{\partial x} = -16\frac{T_\infty^3\sigma^*}{3k^*}\frac{\partial^2 T}{\partial z'^2}$

The appropriate boundary conditions for the velocity ,temperature and concentrations fields are as follows,

$$y' = 0;\ u'(y',t') = U_0\ ,\quad T' = T_w' + (T_w' - T_\infty')\varepsilon e^{i\omega t},$$
$$C' = C_w' + (C_w' - C_\infty')\varepsilon e^{i\omega t} \qquad (9)$$

$$y' \to \infty\ ; u'(y',t') = 0\ ,\ T' = T_\infty'\ ,\ C' = C_\infty' \qquad (10)$$

We introduce the dimensionless variable as follows

$$u = \frac{u'}{U_0},\quad y = \frac{U_0 y'}{V_f},\quad t = \frac{U_0^2 t'}{V_f},\quad \omega = \frac{V_f \omega'}{U_0^2},\ \theta = \frac{(T' - T_\infty')}{(T_w' - T_\infty')}\ ,S = \frac{V_0}{U_0},$$
$$M = \frac{\sigma B_0^2 v_f sin^2\theta}{\rho_f U_0^2} \qquad (11)$$

$$Du = \frac{D_m K_T (C_w' - C_\infty')}{k_f C_S (T_w' - T_\infty')},\ R = \frac{-16 T_\infty^3 \sigma^*}{3KK^*},\quad Kr = \frac{K_l V_f}{U_0^2},\quad Sc = \frac{V_f}{D_B},$$
$$Q = \frac{Q' v_f^2}{K_f U_0^2} \qquad (12)$$

$$Pr = \frac{V_f}{K_f}\ ,\quad K = \frac{K' \rho_f U_0^2}{V_f^2},\quad Gr = \frac{(\rho\beta) g v_f (T_w' - T_\infty')}{\rho_f U_0^3}\ ,\psi = \frac{(C' - C_\infty')}{(C_w' - C_\infty')} \qquad (13)$$

Where Pr is the Prandtlnumber, S is the suction parameter, M is Magnetic parameter, Kr is the chemical reaction , Sc is the Schmidt number, Gr is the Grashof number, K is the Permeability parameter and Du is the Diffusion thermo parameter, R is the Radiation parameter.

By using non-dimensionless parameter quantities (13), the equations (6) to (8) reduces to the following non-dimensionless form,

$$A\left(\frac{\partial u}{\partial t} - S\frac{\partial u}{\partial y}\right) = D\frac{\partial^2 u}{\partial y^2} + BGr\theta \quad - \left(M + \frac{1}{K}\right)u = 0 \qquad (14)$$

$$C\left(\frac{\partial \theta}{\partial t} - S\frac{\partial \theta}{\partial y}\right) = \frac{1}{Pr}(E+R)\frac{\partial^2\theta}{\partial y^2} - \frac{q\theta}{Pr} + \frac{\partial^2\phi}{\partial y^2}\frac{Du}{Pr} \qquad (15)$$

$$\frac{\partial \phi}{\partial t} - S\frac{\partial \phi}{\partial y} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} - Kr\psi \quad (16)$$

The boundary conditions (9)and (10) in the dimensional form can be written as,

y = 0; u = 1, $\theta = 1 + \varepsilon e^{i\omega t}$,

$\psi = 1 + \varepsilon e^{i\omega t}$(17)

y→ ∞; u=0,$\theta = 0$,$\psi = 0$ (18)

3. SOLUTION OF THE PROBLEM

Equations (14) to(16) are coupled non linear partial differential equations and these equations can be solved in closed form. However , these equations can be reduced to a set of ordinary differential equations, which is solved analytically. The expressions for velocity, temperature and concentration are assumed as

$$u(y,t)=u_0 + \varepsilon u_1 e^{i\omega t} \quad (19)$$

$$\theta(y,t)=\theta_0 + \varepsilon \theta_1 e^{i\omega t} \quad (20)$$

$$\psi(y,t) = \psi_0 + \varepsilon\psi_1 e^{i\omega t} \quad (21)$$

Where ($\varepsilon \ll 1$) is a parameter

Substituting (19) to (21) in equations (14) to (16) and equating zeroth and first order equations we get the following set of ordinary differential equations

$$Du_0'' + SAu_0' - \left(M + \frac{1}{K}\right)u_0 = -BGr\theta_0 \quad (22)$$

$$Du_1'' + SAu_1' - \left(\left(M + \frac{1}{K}\right) + Ai\omega\right)u_1 = -BGr\theta_1 \quad (23)$$

$$(E + R)\theta_0'' + PrCS\theta_0' - Q\theta_0 = -Du\psi_0'' \quad (24)$$

$$(E + R)\theta_1'' + PrCS\theta_1' - (Q + PrCi\omega)\theta_1 = -Du\psi_1'' \quad (25)$$

$$\psi_0'' + SSc\psi_0' - KrSc\psi_0 = 0 \quad (26)$$

$$\psi_1'' + SSc\psi_1' - (i\omega + Kr)Sc\psi_1 = 0 \quad (27)$$

The boundary are given by

$u_0 = 1, u_1 = 0, \theta_0 = 1, \theta_1 = 1, \psi_0 = 1, ,\psi_1 = 1$ aty→0 (28)

$u_0 = 0, u_1 = 0, \theta_0 = 0, \theta_1 = 0, \psi_0 = 0 ,\psi_1 = 0$ aty→ ∞ (29)

By substituting the boundary conditions (28) and (29) in equations(22) to (27) we get the following results

$$u_0 = B_5 e^{-m_5 y} + B_3 e^{-m_3 y} + B_4 e^{-m_1 y}$$

$u_1 = B_8 e^{-m_6 y} + B_6 e^{-m_4 y} + B_7 e^{-m_2 y}$

$\theta_0 = B_1 e^{-m_3 y} + A_1 e^{-m_1 y}$

$\theta_1 = B_2 e^{-m_4 y} + A_2 e^{-m_2 y}$

$\psi_0 = e^{-m_1 y}$

$\psi_1 = e^{-m_2 y}$ (30)

Substituting the values of (30) in equation (19) to (21), we get

$$u(y,t) = (B_5 e^{-m_5 y} + B_3 e^{-m_3 y} + B_4 e^{-m_1 y}) + \varepsilon(B_8 e^{-m_6 y} + B_6 e^{-m_4 y} + B_7 e^{-m_2 y})e^{i\omega t} \quad (31)$$

$$\theta(y,t) = (B_1 e^{-m_3 y} + A_1 e^{-m_1 y}) + \varepsilon(B_2 e^{-m_4 y} + A_2 e^{-m_2 y})e^{i\omega t} \quad (32)$$

$$\psi(y,t) = (e^{-m_1 y}) + \varepsilon\,(e^{-m_2 y})e^{i\omega t} \quad (33)$$

Shearing stress

Shearing stress

The shearing stress at the plate in dimensional form is given by

$$\tau = \left(\frac{\partial u}{\partial t}\right)_{y=0} \quad (34)$$

$$\tau = (-B_5 m_5 - B_3 m_3 - B_4 m_1) + \varepsilon\,(-B_8 m_6 - B_6 m_4 - B_7 m_2)e^{i\omega t} \quad (35)$$

Nusslet Number

The non- dimensional co-efficient of heat transfer defined by Nusslet number is given by

$$Nu = -\left(\frac{\partial \theta}{\partial t}\right)_{y=0} \quad (36)$$

$$Nu = (B_1 m_3 + A_1 m_1) + \varepsilon\,(B_2 m_4 + A_2 m_2)e^{i\omega t} \quad (37)$$

Sherwood number :

The non-dimensional co-efficient of mass transfer defined by Sherwood number is given by

$$Sh = -\left(\frac{\partial \psi}{\partial t}\right)_{y=0} \quad (38)$$

$$Sh = m_1 + \varepsilon(m_2)e^{i\omega t} \quad (39)$$

4. RESULTS AND DISCUSSION

We have calculated the velocity field ,

Temperature field,Concentration, the

Shearing stress,the rate of heat transfer in terms of nusselt number (Nu) and the rate of Mass transfer in terms os Sherwood number (Sh).By mentioning the specific values to the different parameter such as Radiative parameter(R),Suctioneffect(S) ,Magnetic Field (M),Dufour effect (Du), Schmidt number (Sc), Grash of number (Gr) ,Chemical reaction (Kr),Prandtl number(Pr),Permeability parameter Figure 1; Velocity profile for Du fractions of nano particles φ.The following are the value that we are used throughout the graphs in our present study Sc=0.60 , Kr=0.5, S=0.1, Q=2, K=4, φ=0.20,M=0.5,Gr=2,Du=2 all the graphs that uses these values unless it is specifically mentioned on the corresponding graphs.

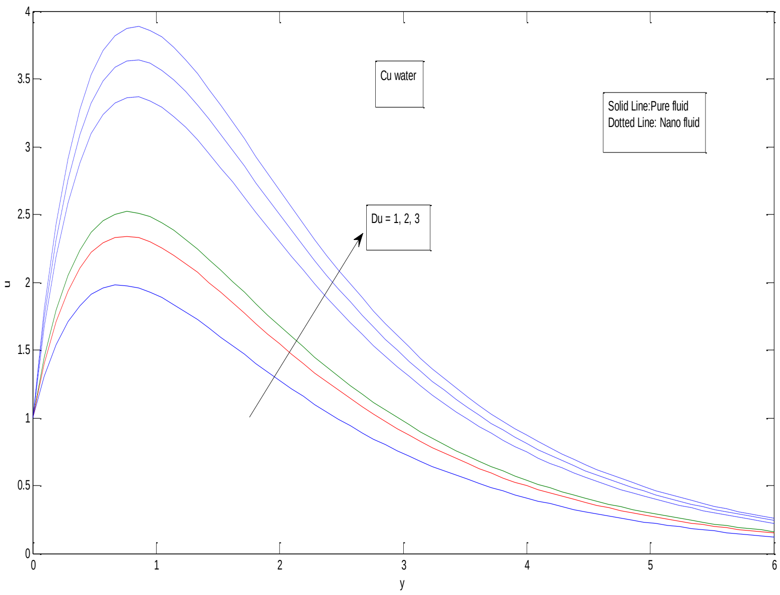

Figure 1: Velocity profile for Du

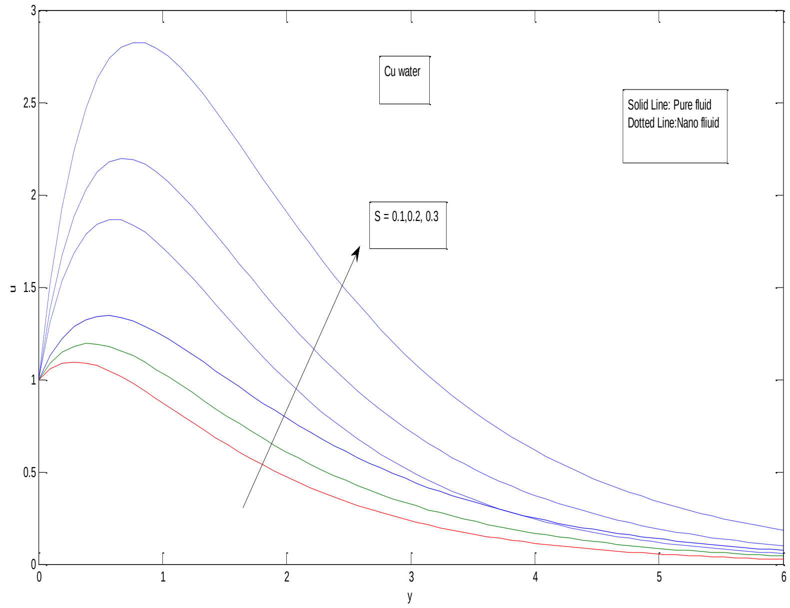

Figure 2: Velocity profile for S

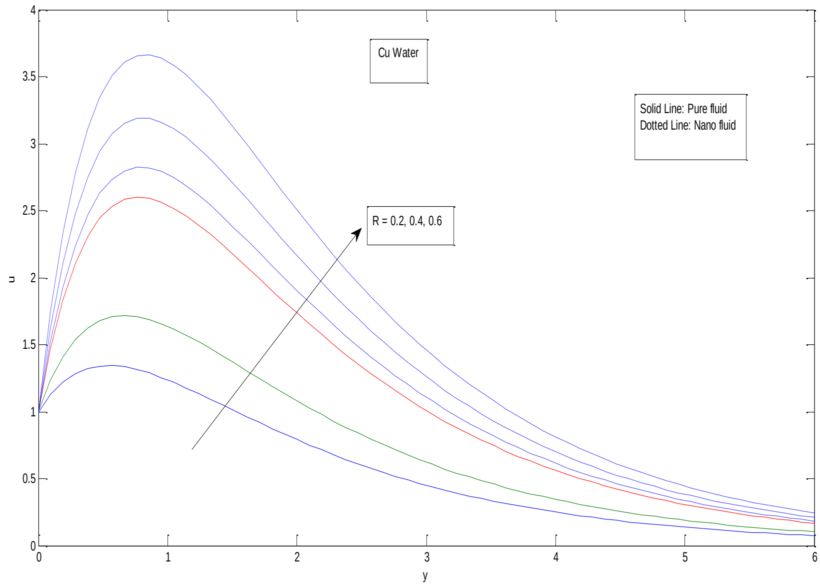

Figure 3: Velocity profile for R

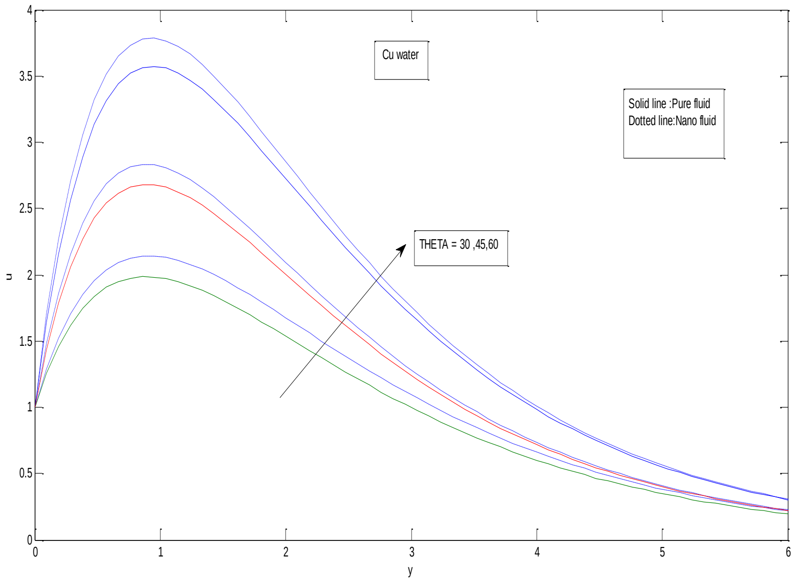

Figure 4: Velocity profile for theta

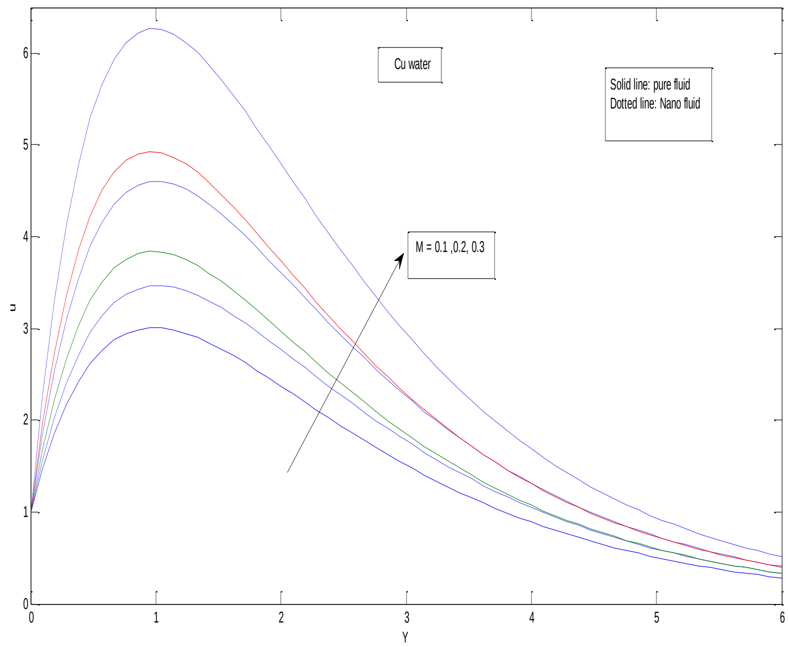

Figure 5: Velocity profile for M

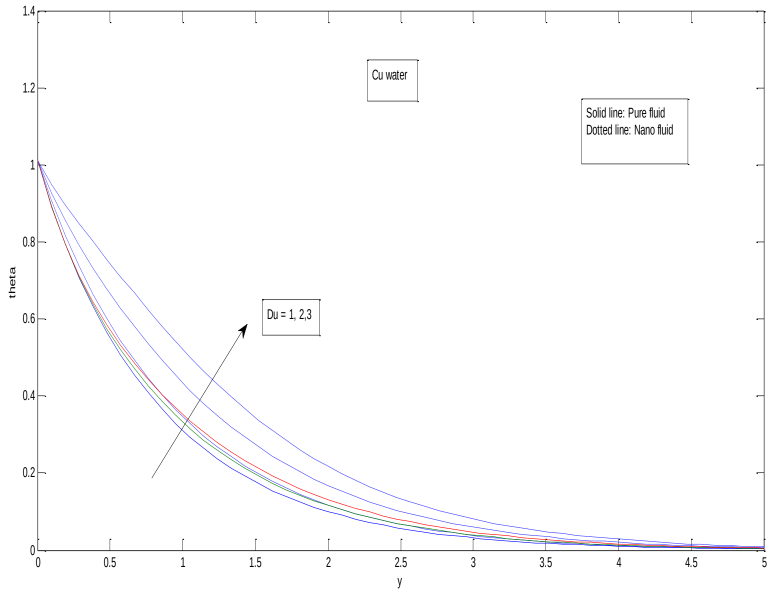

Figure 6: Velocity profile for Du

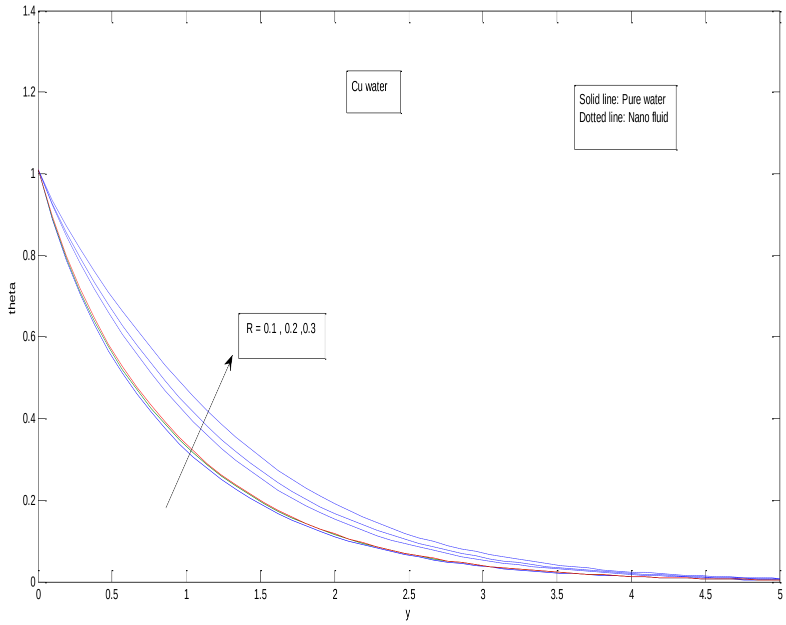

Figure 7: Temperture profile for R

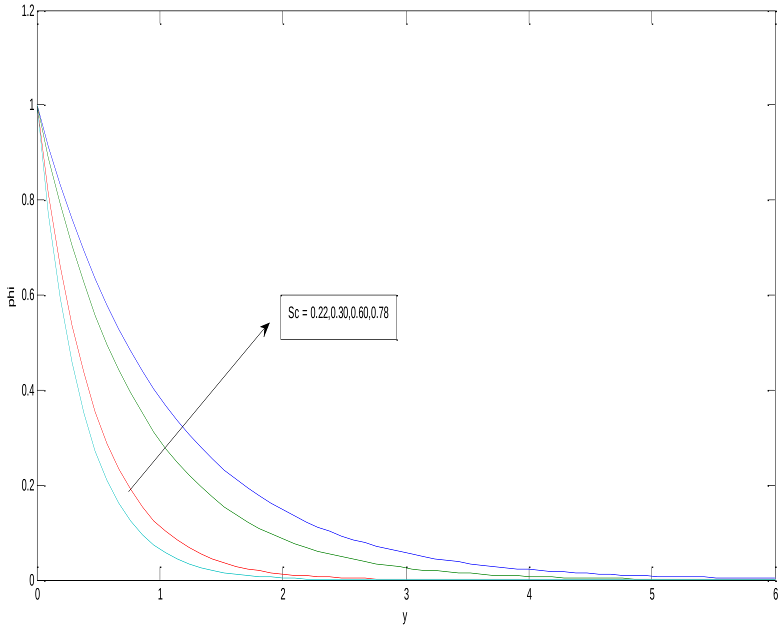

Figure 8: Concentration profile for Sc

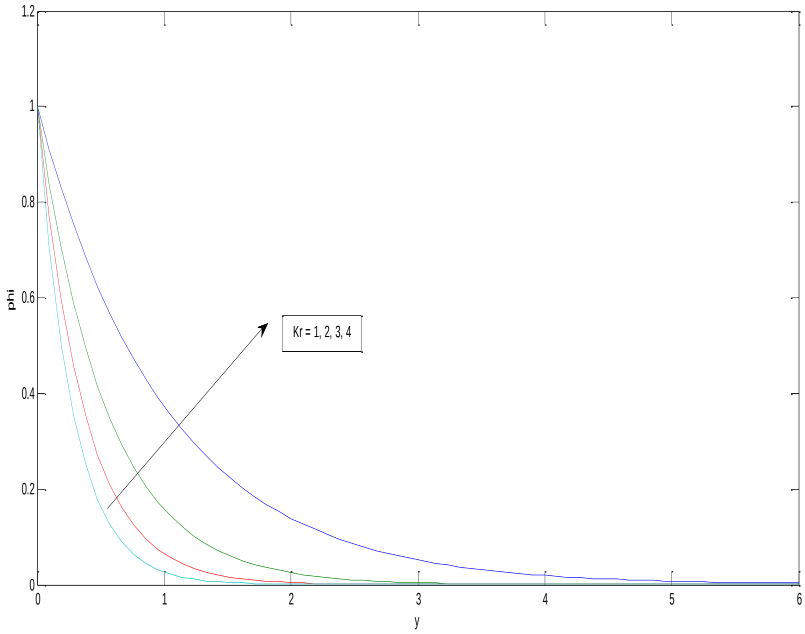

Figure 9: Concentration profile for Kr

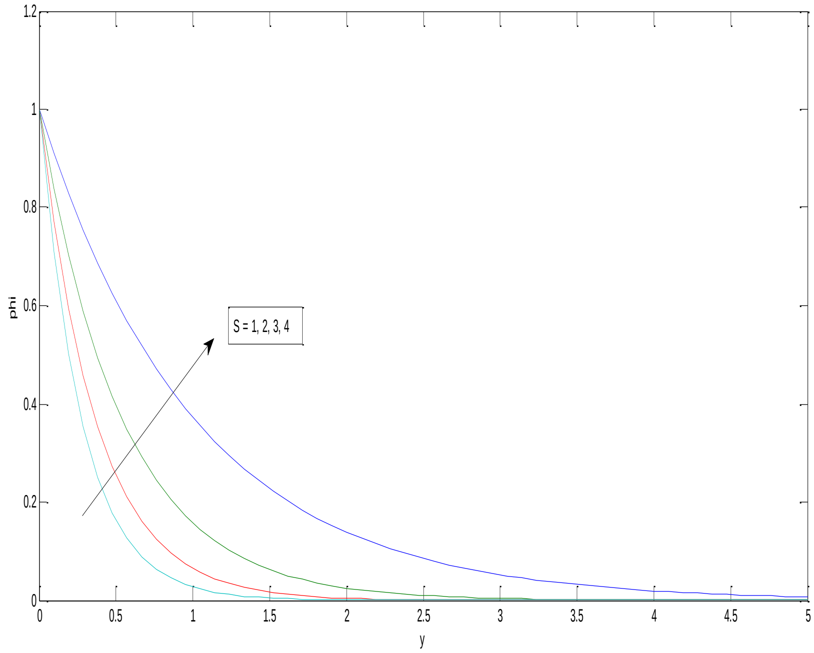

Figure 10: Concentration profile for S

Figure 1 represents the dimensionless velocity u for different values of Dufour number (Du). The analytical results show that the effect of Dufour parameter (Du) on the fluid velocity u for both Nano fluid ($\varphi \neq 0$) and regular fluid ($\varphi = 0$). The graph shows that the increasing values of dufour effect (Du) increase the velocity u for both regular and nano fluids.

Figure2 demonstrates the effect of Suction parameter (S) on fluid velocity u for both regular ($\varphi = 0$) and nano fluids ($\varphi \neq 0$) . It is seen that the velocity decreases by increasing the values of suction parameter (S).

It is observed from the figure 3 that the effect of thermal Radiation (R) on the fluid velocity u for both regular ($\varphi = 0$) and for nano fluids($\varphi \neq 0$). The increase in the value of thermal radiation increase the fluid velocity.

Figure 4 illustrates the effect of angle of inclination over the dimensionless velocity u for both regular($\varphi = 0$) and nano fluids ($\varphi \neq 0$). It indicates that the effect of increasing inclinations of the magnetic field reduces the velocity of the flow.

In figure 5 the velocity profile for Magnetic field parameter(M) is shown for both regular($\varphi = 0$) and nano fluids ($\varphi \neq 0$)

From the graph it is noted that the effect in of increasing magnetic field is to decrease the fluid velocity.

The effect of Dufour parameter (Du) in the Temperature profile for both regular $(\varphi = 0)$ and nanofluid($\varphi \neq 0$) is given in Figure 6.It shows that the increasing the value of Dufour effect the temperature profile .

Figure 7 displays the effect of thermal radiation Ron the temperature profile for both regular$\varphi = 0$ and nanofluid$(\varphi \neq 0)$ .We observed that the increase in the value of thermal radiation ,increases the temperature field.The increase in radiation parameter means the release of heat energy from the flow region and so the fluid temperature increases.

Figure 8 indicates the effect of Schmidt number (Sc) in concentration profile for both regular $(\varphi = 0)$ and nanofluid$(\varphi \neq 0)$ .The variation in the concentration boundary layer of the flow field for H_2,H_2O, vapour and ,it depicts that the concentration profile decreases with increasing the value of Schmidt number (Sc).

The influence of the chemical reaction parameter(Kr) in concentration profile for both regular$(\varphi = 0)$ and nanofluid$(\varphi \neq 0)$ is dicussed bin Figure 9.It observed that the increasing the value of chemical reaction parameter (Kr) decreases the concentration.

The Suction parameter (S) in concentration profile for both regular$(\varphi = 0)$ and nanofluid$(\varphi \neq 0)$ is discussed inFigure 10.It shows that the concentration decreases with decreases with increasing the value of suction parameter (S)

CONCLUSION

*****Fluid velocity decreases with the increasing value of magnectic field parameter ,the angle of inclinations and suction parameter for Cu water nanofluid and normal fluid while it increases with increasing value of Dufour number and thermal radiation.

* The effect of Thermal radiation and Dufour effect increases the temperature profile.

*The spices concentration decreases with the increasing value of suction parameter Chemical reaction and Schmidt number.

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