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f-Biharmonic Submanifolds in S-Space Form

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Abstract

Complex and contact structures are generalized in the form of S-manifolds. In this paper we have studied f-biharmonic submanifolds in S-space form and have derived condition on second fundamental form for f-biharmonic submanifolds.
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1. INTRODUCTION

Harmonic maps are important field of research being the critical points of energy fuctional. Due to both geometric and analytical aspects, harmonic maps are attractive field of research.

The idea behind the biharmonic maps is old and is attractive subject of research. They have been studied since 1862 by Maxwell and Airy to describe a mathematical model of elasticity. Biharmonic maps are generalization of harmonic maps and first regular studied by J. Eells and L. Lemair in 1983 [4]. In 1986 G. Y. Jiang [6] discussed first and second variations formulas for bienergy functional. After this many authors studied biharmonic maps and biharmonic submanifolds etc. [2], [5], [10],[11],[12].

Lichnerowicz gave idea of f-biharmonic maps [7]. Many authors studied the f-biharmonic submanifolds for complex space form and generalized Sasakian space forms separately [8], but after the generalization of complex and contact space forms as S-space forms [1], it is natural to study f-biharmonic maps on S-space forms. After introduction, second section contains basics of f-biharmonic maps and S-space forms. Third section consists of main results.

2. PRELIMINARIES

In this section, we recall some well known facts concerning harmonic maps, biharmonic maps and S-manifolds.

Let $F : (M, g) \longrightarrow (N, h)$ be a smooth map between two Riemannian manifolds of dimensions m and n respectively. The energy density of F is a smooth function $e(F) : M \longrightarrow [0, \infty)$ given by [4],

$$e(F)_p = \frac{1}{2}Tr_g(F^*h)(p) = \frac{1}{2}\sum_{i=1}^{m} h(F_{*p}u_i, F_{*p}u_i),$$

for any $p \in M$ and any orthonormal basis $\{u_1, \ldots, u_m\}$ of T_pM. If M is a compact Riemannian manifold, the energy $E(F)$ of F is the integral of its energy density:

$$E(F) = \int_M e(F)\upsilon_g,$$

where υ_g is the volume measure associated with the metric g on M. A map $F \in C^\infty(M, N)$ is said to be harmonic if it is a cricital point of the energy functional E on the set of all maps between (M, g) and (N, h). Now, let (M, g) be a compact Riemannian manifold. If we look at the Euler-Lagrange equations for the corresponding variational problem, a map $F : M \longrightarrow N$ is harmonic if and only if $\tau(F) \equiv 0$, where $\tau(F)$ is the tension field which is defined by

$$\tau(F) = Tr_g\widetilde{\nabla}dF,$$

where $\widetilde{\nabla}$ is the connection induced by the Levi-Civita connection on M and the F-pullback connection of the Levi Civita connection on N.

A map is called biharmonic if it is a critical point of the bi-energy functional

$$E_2(F) = \int_M |\tau(F)|^2\upsilon_g,$$

on the space of smooth maps between two Riemannian manifolds. Critical points of E_2 are called biharmonic maps and they are solutions of the Euler-Lagrange equation;

$$\tau_2(F) = -\Delta^F\tau(F) - trace_gR^N(dF, \tau(F))dF = 0. \tag{1}$$

For a positive, well defined and C^∞ differentiable function $f : M \to R$, f-biharmonic maps are critical points of the f-bienergy functional for maps $F : (M, g) \to (N, h)$, between Riemannian manifolds,

$$E_{2,f}(F) = \int_M f|\tau(F)|^2\upsilon_g,$$

Euler-Lagrange equation for f-biharmonic maps is as:

$$\tau_{2,f}(F) = f\tau_2(F) + \Delta f \tau(F) + 2\nabla^F_{gradf}\tau(F) = 0. \tag{2}$$

As a generalization of both almost complex (in even dimension) and almost contact (in odd dimension) structures, Yano introduced in [14] the notion of f-structure on a smooth manifold of dimension $2n+s$, i.e. a tensor field of type (1,1) and rank $2n$ satisfying $f^3 + f = 0$. The existence of such a structure is equivalent to a reduction of the structural group of the tangent bundle to $U(n) \times O(s)$. Let N be a $(2n+s)$-dimensional manifold with an f-structure of rank $2n$. If there exist s global vector fields $\xi_1, \xi_2, \ldots, \xi_s$ on N such that:

$$f\xi_\alpha = 0, \qquad \eta_\alpha \circ f = 0, \qquad f^2 = -I + \sum \xi_\alpha \otimes \eta_\alpha, \tag{3}$$

where η_α are the dual 1-forms of ξ_α, we say that the f-structure has complemented frames. For such a manifold there exists a Riemannian metric g such that

$$g(X,Y) = g(fX, fY) + \sum \eta_\alpha(X)\eta_\alpha(Y)$$

for any vector fields X and Y on N. See [1].

An f-structure f is normal, if it has complemented frames and

$$[f,f] + 2\sum \xi_\alpha \otimes d\eta_\alpha = 0,$$

where $[f,f]$ is Nijenhuis torsion of f.

Let Ω be the fundamental 2-form defined by $\Omega(X,Y) = g(X, fY)$, X, $Y \in T(N)$. A normal f-structure for which the fundamental form Ω is closed, $\eta_1 \wedge \cdots \wedge \eta_s \wedge (d\eta_\alpha)^n \neq 0$ for any α, and $d\eta_1 = \cdots = d\eta_s = \Omega$ is called to be an S-structure. A smooth manifold endowed with an S-structure will be called an S-manifold. These manifolds were introduced by Blair in [1].

We have to remark that if we take $s = 1$, S-manifolds are natural generalizations of Sasakian manifolds. In the case $s \geq 2$ some interesting examples are given in [1].

If *N* is an S-manifold, then the following formulas are true (see [1]):

$$\overline{\nabla}_X \xi_\alpha = -fX, \qquad X \in T(N), \quad \alpha = 1, \ldots, s, \tag{4}$$

$$(\overline{\nabla}_X f)Y = \sum \{g(fX, fY)\xi_\alpha + \eta_\alpha(Y) f^2 X\}, \quad X, Y \in T(N), \tag{5}$$

where $\overline{\nabla}$ is the Riemannian connection of g. Let *L* be the distribution determined by the projection tensor $-f^2$ and let *M* be the complementary distribution which is determined by f^2+I and spanned by $\xi_1,\ldots,\xi_s$. It is clear that if $X\in L$ then $\eta_\alpha(X)=0$ for any α, and if $X\in M$, then $fX=0$. A plane section π on N is called an invariant f-section if it is determined by a vector $X\in L(x)$, $x\in N$, such that $\{X,fX\}$ is an orthonormal pair spanning the section. The sectional curvature of π is called the f-sectional curvature. If N is an S-manifold of constant f-sectional curvature k, then its curvature tensor has the form

$$\begin{aligned}\widetilde{R}(X,Y)Z &= \sum_{\alpha,\beta}\{\eta^\alpha(X)\eta^\beta(Z)f^2Y-\eta^\alpha(Y)\eta^\beta(Z)f^2X-g(fX,fZ)\eta^\alpha(Y)\xi_\beta \\ &\quad +g(fY,fZ)\eta^\alpha(X)\xi_\beta\}+\frac{k+3s}{4}\{-g(fY,fZ)f^2X+g(fX,fZ)f^2Y\} \\ &\quad +\frac{k-s}{4}\{g(X,fZ)fY-g(Y,fZ)fX+2g(X,fY)fZ\}, \qquad (6)\end{aligned}$$

X, Y, Z $\in T(N)$. Such a manifold $N(k)$ will be called an S-space form. The Euclidean space E^{2n+s} and the hyperbolic space H^{2n+s} are examples of S-space forms.

Let M be an m-dimensional submanifold immersed in N. Then M is an invariant submnaifold if $\xi_\alpha\in TM$ for any α and $fV\in TM$ for any $V\in TM$. It is said to be anti-invariant submanifold if $fV\in TM^\perp$ for any $V\in TM$. For a vector field $X\in TM^\perp$, it can be written as $fX=tX+SX$, where tX is tangent component of fX, SX is normal component of f X. If S does not vanishes, then its an f-structure [3]. For a vector field $Y\in TM$, it can be written as $fY=TY+NY$, where TY is tangent component of fY, NY is normal component of f Y.

Consider the structure vector fields $\xi_1,\xi_2,\ldots,\xi_s$ are tangent to M, $dim(M)\geq s$. Then M is CR-submaifold of N if there are two differentiable distributions D and $D^\perp$ on M, $TM=D+D^\perp$ such that

- D and $D^\perp$ are mutually orthogonal to each other.
- The distribution D is invariant under f, i.e. $fD_p=D_p$, for any $p\in M$
- The distribution $D^\perp$ is anti invariant under f, i.e. $fD_p^\perp\subseteq T_pM^\perp$ for any $p\in M$.

It can be prove that each hypersurface of N which is tangent to $\xi_1,\xi_2,\ldots,\xi_s$, have the structure of CR-submnaifold of N, for detail see [3].

3. MAIN RESULTS

Since tension field of the isometric immersion $F : (M^m, g) \to (N^n, h)$ is given as

$$\tau(F) = Tr\Delta dF = TrB = mH, \tag{7}$$

from (7) and (1), we have

$$\tau_2(F) = -m\Delta^F H - mTrace_g R^N(dF, H)dF. \tag{8}$$

We have expression for $\Delta^F H$ as

$$\Delta^F H = \Delta^\perp H + \frac{m}{2} grad|H|^2 + Tr(B(., A_H.)) + 2Tr(A_{\Delta^\perp H}(.)). \tag{9}$$

Substituting (9) into (8) we have

$$\begin{aligned}\tau_2(F) &= -m\Delta^\perp H - m\frac{m}{2} grad|H|^2 - mTr(B(., A_H.)) \\ &\quad -2mTr(A_{\Delta^\perp H}(.)) - mTrace_g R^N(dF, H)dF.\end{aligned} \tag{10}$$

For isometric immersion (2) becomes

$$\tau_2(F) + m\frac{\Delta f}{f} H - 2mA_H grad(lnf) + 2m\nabla^\perp_{grad(lnf)} H = 0. \tag{11}$$

Substituting (10) into (11), we have

$$\begin{aligned}&-\Delta^\perp H - \frac{m}{2} grad|H|^2 - Tr(B(., A_H.)) - 2Tr(A_{\Delta^\perp H}(.)) - Trace_g R^N(dF, H)dF \\ &+\frac{\Delta f}{f} H - 2A_H grad(lnf) + 2\nabla^\perp_{gradlnf} H = 0.\end{aligned} \tag{12}$$

Theorem 3.1. *Let M^m be a submanifold of S-space form N^{2n+s}. Then M^m is biharmonic submanifold of S-space form N^{2n+s} if and only if*

$$\begin{aligned}&-\frac{m}{2} grad|H|^2 - 2tr(A_{\Delta^\perp H}(.)) - 2A_H grad(lnf) \\ &= \sum_\alpha \left(\frac{c+3s-4}{4}\right)(m-1)\eta_\alpha(H)\xi^t_\alpha + \frac{c-s}{4} 3TtH\end{aligned} \tag{13}$$

and

$$\begin{aligned}&-\Delta^\perp H - tr(B., A_H.) + \frac{\Delta f}{f} H + 2\left(\Delta^\perp_{grad(lnf)} H\right) = \sum_\alpha -\left(\frac{c+3s}{4}\right) mH \\ &+\left(\frac{c+3s-4}{4}\right)\left\{H|\xi^t_\alpha|^2 + m\eta_\alpha(H)\xi^\perp_\alpha\right\} + \frac{c-s}{4} 3NtH\end{aligned} \tag{14}$$

Proof. Consider the curvature tensor of S-space form (6) with constant sectional curvature k, for mean curvature vector H, we have

$$\begin{aligned}\widetilde{R}(e_i,H)e_i &= \sum_{\alpha,\beta}\{\eta^{\alpha}(e_i)\eta^{\beta}(e_i)f^2H-\eta^{\alpha}(H)\eta^{\beta}(e_i)f^2e_i-g(fe_i,fe_i)\eta^{\alpha}(H)\xi_{\beta}\\ &\quad +g(fH,fe_i)\eta^{\alpha}(e_i)\xi_{\beta}\}+\frac{k+3s}{4}\{-g(fH,fe_i)f^2e_i+g(fe_i,fe_i)f^2H\}\\ &\quad +\frac{k-s}{4}\{g(e_i,fe_i)fH-g(H,fe_i)fe_i+2g(e_i,fH)fe_i\}.\end{aligned}$$

After simplifying we have

$$\begin{aligned}tr\widetilde{R}(e_i,H)e_i &= \sum_{\alpha,\beta}\left(\frac{k+3s-4}{4}\right)\left\{H|\xi^t_{\alpha}|^2-\eta_{\alpha}(H)\xi^t_{\alpha}+m\eta_{\alpha}(H)\xi_{\alpha}\right\}\\ &\quad -\frac{k+3s}{4}mH+\frac{c-s}{4}3(TtH+NtH) \qquad (15)\end{aligned}$$

when M^m is f-biharmonic submanifold of N^n, substituting (15) in (12) and comparing tangential and normal components we have result. Where $TtH=g(H,Ne_i)Te_i$ and $NtH=g(H,Ne_i)Ne_i$.

Corollary 3.1. *Let M^m be a submanifold of S-space form N^{2n+s}*

1. *If M^m is invariant then f-biharmonic if and only if*

$$\begin{aligned}&-\frac{m}{2}grad|H|^2-2tr(A_{\Delta^{\perp}H}(.))-2A_Hgrad(lnf)\\ &=\sum_{\alpha}\left(\frac{k+3s-4}{4}\right)(m-1)\eta_{\alpha}(H)\xi^t_{\alpha}+\frac{k-s}{4}3TtH \qquad (16)\end{aligned}$$

and

$$\begin{aligned}&-\Delta^{\perp}H-tr(B.,A_H.)+\frac{\Delta f}{f}H+2\left(\Delta^{\perp}_{grad(lnf)}H\right)=\sum_{\alpha}-\left(\frac{k+3s}{4}\right)mH\\ &+\left(\frac{k+3s-4}{4}\right)\left\{H|\xi^t_{\alpha}|^2+m\eta_{\alpha}(H)\xi^{\perp}_{\alpha}\right\} \qquad (17)\end{aligned}$$

2. *If M^m is anti-invariant then f-biharmonic if and only if*

$$\begin{aligned}&-\quad\frac{m}{2}grad|H|^2-2tr(A_{\Delta^{\perp}H}(.))-2A_Hgrad(lnf)\\ &=\quad\sum_{\alpha}\left(\frac{k+3s-4}{4}\right)(m-1)\eta_{\alpha}(H)\xi^t_{\alpha} \qquad (18)\end{aligned}$$

and

$$-\Delta^{\perp}H - tr(B.,A_H.) + \frac{\Delta f}{f}H + 2\left(\Delta^{\perp}_{grad(lnf)}H\right) = \sum_{\alpha} - \left(\frac{k+3s}{4}\right)mH$$
$$+\left(\frac{k+3s-4}{4}\right)\left\{H|\xi^t_\alpha|^2 + m\eta_\alpha(H)\xi^{\perp}_\alpha\right\} + \frac{k-s}{4}3NtH \tag{19}$$

Proof. 1. For invariant f-biharmonic submanifolds we take $N = 0$ in (13),(14) and obtain result.

2. For anti-invariant f-biharmonic submanifolds we take $T = 0$ in (13), (14) and obtain result.

Corollary 3.2. *Let M^m be a submanifold of S-space form N^{2n+s}*

1. *Let $\xi_\alpha : \forall\alpha = 1,\ldots,s$ be normal to M^m, then M^m is f-biharmonic if and only if*

$$-\frac{m}{2}grad|H|^2 - 2tr(A_{\Delta^{\perp}H}(.)) - 2A_H grad(lnf) = 0 \tag{20}$$

and

$$-\Delta^{\perp}H - tr(B.,A_H.) + \frac{\Delta f}{f}H + 2\left(\Delta^{\perp}_{grad(lnf)}H\right) = \sum_{\alpha} - \left(\frac{k+3s}{4}\right)mH$$
$$+\left(\frac{k+3s-4}{4}\right)m\eta_\alpha(H)\xi^{\perp}_\alpha + \frac{k-s}{4}3NtH \tag{21}$$

2. *Let $\xi_\alpha : \forall\alpha = 1,\ldots,s$ be tangent to M^m, then M^m is f-biharmonic if and only if*

$$-\frac{m}{2}grad|H|^2 - 2tr(A_{\Delta^{\perp}H}(.)) - 2A_H grad(lnf) = \sum_{\alpha} + \frac{k-s}{4}3TtH \tag{22}$$

and

$$-\Delta^{\perp}H - tr(B.,A_H.) + \frac{\Delta f}{f}H + 2\left(\Delta^{\perp}_{grad(lnf)}H\right) = \sum_{\alpha} - \left(\frac{k+3s}{4}\right)mH$$
$$+\left(\frac{k+3s-4}{4}\right)\left\{H|\xi^t_\alpha|^2\right\} + \frac{k-s}{4}3NtH \tag{23}$$

Proof. 1. Since $\xi_\alpha : \forall\alpha = 1,\ldots,s$ are normal to M^m, then tangential component of ξ_α vanishes and we take M^m as anti-invariant, $T = 0$. Taking $\xi^t = O$ and $T = 0$ in (13) and (14), we get result.

2. Since $\xi_\alpha : \forall\alpha = 1, \dots, s$ are tangent to M^m, therefore $\xi^\perp$ vanishes and taking $\eta_\alpha(H) = 0$ in (13) and (14), we get result.

Proposition 3.1. *Let M^{2n-1+s} be a hypersurface of S-space form N^{2n+s} with non zero constant mean curvature H and $\forall\ \xi_\alpha : \alpha = 1, \dots, s$ tangent to M. Then M is proper f-biharmonic if and only if*

$$|B|^2 = \frac{\Delta f}{f} + \left(\frac{k+3s}{4}\right)(2n-1+s) - \left(\frac{k+3s-4}{4}\right)(s) \\ +3\left(\frac{k-s}{4}\right) \tag{24}$$

and

$$Scal_M = (-2(2n+s-2)+1)\, s\left(\frac{k+3s-4}{4}\right) + \left(\frac{k+3s}{4}\right)(2n-1+s)(2n+s-3) \\ + \left(\frac{k-1}{4}\right)3(2n-2) - (2n-1+s)H^2 - \frac{\Delta f}{f}$$

Proof. Let M^{2n-1+s} be a f-biharmonic hypersurface of M^{2n+s} and $\forall\ \xi_\alpha : \alpha = 1, \dots, s$ tangent to M, then $m = 2n-1+s$ and $\eta_\alpha(H) = 0$ and since

$$\begin{aligned} f^2H &= -H + \eta_\alpha(H)\xi_\alpha = -H \\ f(fH) &= -H, \end{aligned}$$

this implies

$$\begin{aligned} TtH &= -TSH \\ NtH &= -H - NSH. \end{aligned}$$

Therefore taking $m = 2n-1+s$, $\eta_\alpha(H) = 0$ and $SH = 0$ in (13) and (14), we have

$$-\frac{2n-1+s}{2} grad|H|^2 - 2tr(A_{\Delta^\perp H}(.)) - 2A_H grad(lnf) = 0 \tag{25}$$

and

$$-\Delta^\perp H - tr(B., A_H.) + \frac{\Delta f}{f}H + 2\left(\Delta^\perp_{grad(lnf)}H\right) = \sum_\alpha -\left(\frac{k+3s}{4}\right)(2n-1+s)H \\ + \left(\frac{k+3s-4}{4}\right)sH - \frac{k-s}{4}3H. \tag{26}$$

For constant mean curvature H, we have

$$A_H grad(lnf) = 0 \tag{27}$$

and

$$-tr(B., A_H.) + \frac{\Delta f}{f}H = \sum_{\alpha} -\left(\frac{k+3s}{4}\right)(2n-1+s)H$$
$$+\left(\frac{k+3s-4}{4}\right)sH - \frac{k-s}{4}3H. \tag{28}$$

From equation (28), we have

$$tr(B., A_H.) = \frac{\Delta f}{f}H + \sum_{\alpha}\left(\frac{k+3s}{4}\right)(2n-1+s)H$$
$$-\left(\frac{k+3s-4}{4}\right)sH + \frac{k-s}{4}3H. \tag{29}$$

Therefore

$$|B|^2 = \frac{\Delta f}{f} + \left(\frac{k+3s}{4}\right)(2n-1+s) - \left(\frac{k+3s-4}{4}\right)(s)$$
$$+3\left(\frac{k-s}{4}\right) \tag{30}$$

Using Gauss equation we have

$$Scal_M = \sum_{i,j} h(\widetilde{R}(e_i,e_j)e_j,e_i) - |B|^2 - (2n-1+s)H^2$$
$$= -2(2n+s-2)s\left(\frac{k+3s-4}{4}\right) + \left(\frac{k+3s}{4}\right)(2n-1+s)(2n+s-2)$$
$$+\left(\frac{k-1}{4}\right)3(2n-1) - |B|^2 - (2n-1+s)H^2.$$

Therefore

$$Scal_M = -2(2n+s-2)s\left(\frac{k+3s-4}{4}\right) + \left(\frac{k+3s}{4}\right)(2n-1+s)(2n+s-2)$$
$$+\left(\frac{k-1}{4}\right)3(2n-1) - \frac{\Delta f}{f} - \left(\frac{k+3s}{4}\right)(2n-1+s) + \left(\frac{k+3s-4}{4}\right)(s)$$
$$-3\left(\frac{k-s}{4}\right) - (2n-1+s)H^2.$$

After simplifying,

$$Scal_M = (-2(2n+s-2)+1)\,s\left(\frac{k+3s-4}{4}\right) + \left(\frac{k+3s}{4}\right)(2n-1+s)(2n+s-3)$$
$$+\left(\frac{k-1}{4}\right)3(2n-2) - (2n-1+s)H^2 - \frac{\Delta f}{f}$$

Remark 3.1. *1. For $s = 0$. we get the results of Theorem 3.1, Corollary 3.1, Corollary 3.2 and Proposition 3.1 for Khaler manifolds*

2. For $s = 1$. we get the results of Theorem 3.1, Corollary 3.1 and Corollary 3.2 and Proposition 3.1 for Sasakian manifolds

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