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Q-Cubic bi-quasi Ideals of Semigroups

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Abstract

In this paper, we introduce the notion of a Q-cubic bi-quasi ideal of semigroup and we characterize the regular semigroup in terms of a Q-cubic bi-quasi ideal of a semigroup.

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1. INTRODUCTION

In 1965, the fundamental concept of fuzzy sets was introduced by Zadeh [17]. At present, it is an important tool in science, engineering, computer science, control engineering, etc. In 1979, Kuroki [8, 9, 10] was given the idea of fuzzy ideal, fuzzy bi-ideals, and fuzzy interior ideals in semigroups. Later, concepts were expanded about interval-valued fuzzy sets that have many applications such as approximate reasoning, image processing, decision making, medicine, and mobile networks, etc. In 2006 [15], Narayanan and Manikanran initiated the notion of interval valued fuzzy ideal in semigroup. In 2012, Jun [6], introduced a new notion, called a cubic set, and investigated several properties and introduced cubic subsemigroups and cubic left (right) ideals of semigroups. Later, in 2015 Sadaf et al. [16], discussed cubic bi-ideal of a semigroup. In later years V. Chinnadurai and K. Bharathivelan[3], studied cubic ideal in Γ-semigroup and PO-Γ-semigroup.

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The idea of an intuitionistic Q-fuzzy set was first discussed by Atanassov [1, 2], as a generalization of the notion of a fuzzy set. Kyung Ho Kim[7] introduced on intuitionistic Q-Fuzzy semiprime ideals in Semigroups. Thillaigovindan et al.[12] discussed on interval-valued fuzzy quasi-ideals of semigroups. In 2020, the concept of fuzzy semigroups has been discussed in research on the prime fuzzy m-bi ideals in semigroups[14], Manahon et al. [13] studied on BF-semigroups and fuzzy BF-semigroups, and T. Gaketem[5] introduced cubic interior ideals in semigroups, etc.

The aim of this paper we define definition of Q-cubic bi-quasi ideal in semigroup and properties of Q-cubic bi-quasi ideals are investigated. Then we characterized regular semigroup in terms of Q-cubic bi-quasi ideal.

2. PRELIMINARIES

In this section, we give definitions that are used in this paper. By a subsemigroup of a semigroup S we mean a non-empty subset A of S such that $A^2 \subseteq A$, and by a left (right) ideal of S we mean a non-empty subset A of S such that $SA \subseteq A(AS \subseteq A)$. By a two-sided ideal or simply an ideal, we mean a non-empty subset of a semigroup S that is both a left and a right ideal of S. A non-empty subset A of S is called an interior ideal of S if $SAS \subseteq A$. A subsemigroup A of a semigroup S is called a bi-ideal of S if $ASA \subseteq A$. A non-empty subset A of a semigroup S is called a quasi-ideal of S if $AS \cap SA \subseteq A$. A subsemigroup A of a semigroup of a semigroup S is said to be left (right) bi-quasi ideal of S if $SA \cap ASA \subseteq A(AS \cap ASA \subseteq A)$. A subsemigroup A of a semigroup S is said to be bi-quasi ideal of S if it is both a left bi-quasi and right bi-quasi ideal of S.

Definition 1. Let X and Q be non-empty sets. A mapping $f : X \times Q \to [0, 1]$ is called a Q-fuzzy set of X over Q.

Definition 2. let X and Q be a non-empty set. A mapping $\bar{f} : X \times Q \to D[0, 1]$ is called interval Q-fuzzy set over Q, where $D[0, 1]$ denote the family of all closed sub interval of $[0, 1]$ and $\bar{f} = [f^-, f^+]$, where f^- and f^+ are Q-fuzzy sets of X such that $f^-(x, q) \leq f^+(x, q)$ for all $x \in X, q \in Q$.

Definition 3. Let X and Q be a non-empty sets. A Q-cubic set A is an object having the form $A = \{((x, q), \bar{f}(x, q), \omega(x, q)) : x \in X, q \in Q\}$ which is briefly denoted by $A = (\bar{f}, \omega)$ with respect to Q, where $\bar{f} : X \times Q \to D[0, 1]$ is an interval Q-fuzzy set over Q and $\omega : X \times Q \to [0, 1]$ is a Q-fuzzy set over Q.

Definition 4. Let $A = (\bar{f}_A, \omega_A)$ be a Q-cubic set in X. Define $U(A; \bar{t}, n) = \{x \in X | \bar{t} \subseteq \bar{f}(x, q), \omega(x, q) \leq n\}$, where $\bar{t} \in D[0, 1]$ and $n \in [0, 1]$ is called the Q-cubic level set of A.

For any non-empty subset I of a set X, the characteristic function of I is defined to be a structure $\chi_I = \{(x, \bar{f}_{\chi_I}(x, q), \omega_{\chi_I}(x, q)) : x \in X, q \in Q\}$ which is briefly denoted by $\chi_I = (\bar{f}_{\chi_I}, \omega_{\chi_I})$ where,

$$\bar{f}_{\chi_I} : X \to [0,1], x \mapsto \bar{f}_{\chi_I}(x,q) := \begin{cases} [1,1], & x \in I; \\ [0,0], & x \notin I. \end{cases}$$

$$\omega_{\chi_I} : X \to [0,1], x \mapsto \omega_{\chi_I}(x,q) := \begin{cases} 0, & x \in I; \\ 1, & x \notin I. \end{cases}$$

The whole cubic set S in a semigroup S is defined to be a structure

$$\chi_S = \{(x, \bar{f}_{\chi_S}(x,q), \omega_{\chi_S}(x,q)) : x \in S, q \in Q\},$$

with $\bar{f}_{\chi_S}(x,q) = [1,1]$ and $\omega_{\chi_S}(x,q) = 0$. It will be briefly denoted by $\chi_S = (\bar{f}_{\chi_S}, \omega_{\chi_S})$.

For two Q-cubic sets $A = (\bar{f}, \omega)$, $B = (\bar{g}, \upsilon)$ in a semigroup S, we define $A \sqsubseteq B$ if and only if $\bar{f}\bar{\subseteq}\bar{g}$ and $\omega \succeq \upsilon$, where $\bar{f}\bar{\subseteq}\bar{g}$ means that $\bar{f}(x,q) \subseteq \bar{g}(x,q)$ and $\omega \succeq \upsilon$ means that $\omega(x,q) \geq \upsilon(x,q)$ for all $x \in S, q \in Q$.

The Q-cubic product of $A = (\bar{f}, \omega)$ and $B = (\bar{g}, \upsilon)$ is defined to be a Q-cubic set $A \,\tilde{\circ}\, B = \{((x,q), (\bar{f} \circ \bar{g})(x,q), (\omega \circ \upsilon)(x,q)) : x \in S, q \in Q\}$

$$(\bar{f} \circ \bar{g})(x,q) = \begin{cases} \bigcup\limits_{x=yz} \{\bar{f}(y,q) \cap \bar{g}(z,q)\} & \text{for some } x,y,z \in S, q \in Q; \\ 0 & \text{otherwise.} \end{cases}$$

$$(\omega \circ \upsilon)(x,q) = \begin{cases} \bigwedge\limits_{y=yz} \{\omega(y,q) \vee \upsilon(z,q)\} & \text{for some } x,y,z \in S, q \in Q; \\ 1 & \text{otherwise.} \end{cases}$$

Let $A = (\bar{f}, \omega)$ and $B = (\bar{g}, \upsilon)$ be two Q-cubic sets in S. The intersection of A and B denoted by $A \sqcap B$ is the Q-cubic set. $A \sqcap B = (\bar{f}\bar{\cap}\bar{g}, \omega \curlyvee \upsilon)$ with respect to Q, where $(\bar{f}\bar{\cap}\bar{g})(x,q) = \bar{f}(x,q) \cap \bar{g}(x,q)$ and $(\omega \curlyvee \upsilon)(x,q) = \omega(x,q) \vee \upsilon(x,q)$.

The union of A and B denoted by $A \sqcup B$ is the Q-cubic set. $A \sqcup B = (\bar{f}\bar{\cup}\bar{g}, \omega \curlywedge \upsilon)$ with respect to Q, where $(\bar{f}\bar{\cup}\bar{g})(x,q) = \bar{f}(x,q) \cup \bar{g}(x,q)$ and $(\omega \curlywedge \upsilon)(x,q) = \omega(x,q) \wedge \upsilon(x,q)$.

Definition 5. A Q-cubic set $A = (\bar{f}, \omega)$ of S is called a Q-cubic subsemigroup of S if it satisfies the following conditions:

1. $\bar{f}(x,q) \cap \bar{f}(y,q) \subseteq \bar{f}(xy,q)$,
2. $\omega(xy,q) \leq \omega(x,q) \vee \omega(y,q)$

for all $x, y \in S$ and $q \in Q$.

Definition 6. A Q-cubic set $A = (\bar{f}, \omega)$ of S is called a Q-cubic left(resp.right) ideal of S if it satisfies the following conditions:

1. $\bar{f}(y,q) \subseteq \bar{f}(xy,q)(\bar{f}(x,q) \subseteq \bar{f}(xy,q))$,
2. $\omega(xy,q) \leq \omega(y,q)(\omega(xy,q) \leq \omega(x,q))$

for all $x, y \in S$ and $q \in Q$.

A Q-cubic set $A = (\bar{f}, \omega)$ of S is called a Q-cubic ideal of S if it is both Q-cubic left ideal and Q-cubic right ideal of S.

3. Q-CUBIC BI-QUASI IDEALS OF SEMIGROUPS

In this section we define Q-cubic bi-quasi ideals in semigroup and investigation properties of Q-cubic bi-quasi ideals.

Definition 7. A Q-cubic subsemigroup $A = (\bar{f}, \omega)$ of S is called a Q-cubic left(right) bi-quasi ideal of S if it satisfies the following conditions:

1. $\bar{f}_{\chi S} \circ \bar{f} \bar{\cap} \bar{f} \circ \bar{f}_{\chi S} \circ \bar{f} \bar{\subseteq} \bar{f}$ ($\bar{f} \circ \bar{f}_{\chi S} \bar{\cap} \bar{f} \circ \bar{f}_{\chi S} \circ \bar{f} \bar{\subseteq} \bar{f}$),
2. $\omega \preceq \omega_{\chi S} \circ \omega \curlyvee \omega \circ \omega_{\chi S} \circ \omega$ ($\omega \preceq \omega \circ \omega_{\chi S} \curlyvee \omega \circ \omega_{\chi S} \circ \omega$),

A Q-fuzzy set $A = (\bar{f}, \omega)$ of semigroup S is called a Q-cubic bi-quasi ideal if it is both Q-cubic left bi-quasi ideal and Q-cubic right bi-quasi ideal of S.

Theorem 8. *Every Q-cubic left ideal of a semigroup S is a Q-cubic left bi-quasi ideal of S.*

Proof. Let $A = (\bar{f}, \omega)$ be a Q-cubic left ideal of a semigroup S. Let $x \in S$ and $q \in Q$. Then

$$\begin{aligned}(\bar{f}_{\chi S} \circ \bar{f})(x,q) &= \bigcup_{x=yz} \{\bar{f}_{\chi S}(y,q) \cap \bar{f}(z,q)\} \\ &= \bigcup_{x=yz} \{\bar{f}(z,q)\} \\ &\subseteq \bigcup_{x=yz} \{\bar{f}(yz,q)\} \\ &= \bigcup_{x=yz} \{\bar{f}(x,q)\} \\ &= \bar{f}(x,q).\end{aligned}$$

Thus $\bar{f}_{\chi S} \circ \bar{f} \bar{\cap} \bar{f} \circ \bar{f}_{\chi S} \circ \bar{f} \bar{\subseteq} \bar{f}$.

And

$$\begin{aligned}(\omega_{\chi_S}\circ\omega)(x,q) &= \bigwedge_{x=yz}\{\omega_{\chi_S}(y,q)\vee\omega(z,q)\}\\ &= \bigwedge_{x=yz}\{\omega_{\chi_S}(y,q)\vee\omega(z,q)\}\\ &= \bigwedge_{x=yz}\{\omega(z,q)\}\\ &\geq \bigwedge_{x=yz}\{\omega(yz,q)\}\\ &= \bigwedge_{x=yz}\{\omega(x,q)\}\\ &= \omega(x,q).\end{aligned}$$

Then $\omega_{\chi_S}\circ\omega \curlyvee \omega\circ\omega_{\chi_S}\circ\omega \succeq \omega$.

Hence $A=(\bar{f},\omega)$ be a Q-cubic left bi-quasi ideal of the semigroup S. □

Theorem 9. *Every Q-cubic left ideal of a semigroup S is a Q-cubic right bi-quasi ideal of S.*

Proof. Let $A=(\bar{f},\omega)$ be a Q-cubic left ideal of a semigroup S. Let $x\in S$ and $q\in Q$. We have $(\bar{f}_{\chi_S}\circ\bar{f})(x,q)\subseteq\bar{f}(x,q)$ and $(\omega_{\chi_S}\circ\omega)(x,q)\geq\omega(x,q)$. Then

$$\begin{aligned}(\bar{f}\circ\bar{f}_{\chi_S}\circ\bar{f})(x,q) &= \bigcup_{x=abc}\{\bar{f}(a,q)\cap(\bar{f}_{\chi_S}\circ\bar{f})(bc,q)\}\\ &\subseteq \bigcup_{x=abc}\{\bar{f}(a,q)\cap\bar{f}(bc,q)\}\\ &\subseteq \bar{f}(x,q).\end{aligned}$$

Thus $\bar{f}\circ\bar{f}_{\chi_S}\bar{\cap}\bar{f}\circ\bar{f}_{\chi_S}\circ\bar{f}\bar{\subseteq}\bar{f}$.

And

$$\begin{aligned}(\omega\circ\omega_{\chi_S}\circ\omega)(x,q) &= \bigwedge_{x=abc}\{\omega(a,q)\vee(\omega_{\chi_S}\circ\omega)(bc,q)\}\\ &\geq \bigwedge_{x=abc}\{\omega(a,q)\vee\omega(bc,q)\}\\ &\geq \omega(x,q).\end{aligned}$$

Now $\omega\circ\omega_{\chi_S}\curlyvee\omega\circ\omega_{\chi_S}\circ\omega\succeq\omega$.

Hence $A=(\bar{f},\omega)$ be a Q-cubic right bi-quasi ideal of the semigroup S. □

Theorem 10. *Every Q-cubic right ideal of a semigroup S is a Q-cubic right bi-quasi ideal of S.*

Proof. Let $A = (\bar{f}, \omega)$ be a Q-cubic right ideal of a semigroup S. Let $x \in S$ and $q \in Q$. Then

$$\begin{aligned}(\bar{f} \circ \bar{f}_{\chi S})(x, q) &= \bigcup_{x=yz} \{\bar{f}(y, q) \cap \bar{f}_{\chi S}(z, q)\} \\ &= \bigcup_{x=yz} \{\bar{f}(y, q)\} \\ &\subseteq \bigcup_{x=yz} \{\bar{f}(yz, q)\} \\ &= \bigcup_{x=yz} \{\bar{f}(x, q)\} \\ &= \bar{f}(x, q).\end{aligned}$$

Thus $\bar{f} \circ \bar{f}_{\chi S} \bar{\cap} \bar{f} \circ \bar{f}_{\chi S} \circ \bar{f} \bar{\subseteq} \bar{f}$.

And

$$\begin{aligned}(\omega \circ \omega_{\chi S})(x, q) &= \bigwedge_{x=yz} \{\omega(y, q) \vee \omega_{\chi S}(z, q)\} \\ &= \bigwedge_{x=yz} \{\omega(z, q)\} \\ &\geq \bigwedge_{x=yz} \{\omega(yz, q)\} \\ &= \bigwedge_{x=yz} \{\omega(x, q)\} \\ &= \omega(x, q).\end{aligned}$$

Then $\omega \circ \omega_{\chi S} \curlyvee \omega \circ \omega_{\chi S} \circ \omega \succeq \omega$.

Hence $A = (\bar{f}, \omega)$ be a Q-cubic right bi-quasi ideal of the semigroup S. □

Corollary 11. *Every Q-cubic right ideal of a semigroup S is a Q-cubic left bi-quasi ideal of S.*

Corollary 12. *Every Q-cubic right(left) ideal of a semigroup S is a Q-cubic bi-quasi ideal of S.*

Theorem 13. *Let S be a semigroup and $A = (\bar{f}, \omega)$ be a non-empty Q-fuzzy set of S. A Q-fuzzy set $A = (\bar{f}, \omega)$ is a Q-cubic left bi-quasi ideal of a semigroup S if and only if the Q-cubic level set $U(A; \bar{t}, n)$ of A is a left bi-quasi ideal of a semigroup S for every $\bar{t} \in D[0, 1], n \in [0, 1]$, where $U(A; \bar{t}, n) \neq \emptyset$.*

Proof. Assume that $A = (\bar{f}, \omega)$ is a Q-cubic left bi-quasi ideal of a semigroup S, $U(A; \bar{t}, n) \neq \emptyset, \bar{t} \in D[0,1], n \in [0,1]$.

Let $x \in SU(A; \bar{t}, n) \cap U(A; \bar{t}, n)SU(A; \bar{t}, n)$. Then $x = ba = cde$ where $b, d \in S$ and $a, c, e \in U(A; \bar{t}, n)$. Then $\bar{t} \subseteq (\bar{f}_{\chi_S} \circ \bar{f})(x, q)$ and $\bar{t} \subseteq (\bar{f} \circ \bar{f}_{\chi_S} \circ \bar{f})(x, q)$ implies that $\bar{t} \subseteq \bar{f}(x, q)$ and $(\omega_{\chi_S} \circ \omega)(x, q) \leq n$ and $(\omega \circ \omega_{\chi_S} \circ \omega)(x, q) \leq n$ implies that $\omega(x, q) \leq n$. Then $x \in U(A; \bar{t}, n)$. Therefore $U(A; \bar{t}, n)$ is a left bi-quasi ideal of the semigroup S.

Conversely suppose that $U(A; \bar{t}, n)$ is a left bi-quasi ideal of the semigroup S, for all $\bar{t} \in Im(\bar{f})$ and $n \in Im(\omega)$. Let $x, y \in S, q \in Q$. Then $\bar{f}(x, q) = \bar{t}_1, \bar{f}(y, q) = \bar{t}_2, \omega(x, q) = n_1, \omega(y, q) = n_2, \bar{t}_1 \supseteq \bar{t}_2$ and $n_1 \leq n_2$. Then $x, y \in U(A; \bar{t}, n)$. We have $SU(A; \bar{l}, m) \cap U(A; \bar{l}, m)SU(A; \bar{l}, m) \subseteq U(A; \bar{l}, m)$, for all $\bar{l} \in Im(\bar{f})$ and $m \in Im(\omega)$. Suppose $\bar{t} = \min\{Im(\bar{f})\}$ and $n = \max\{Im(\omega)\}$. Then $SU(A; \bar{t}, n) \cap U(A; \bar{t}, n)SU(A; \bar{t}, n) \subseteq U(A; \bar{t}, n)$. Therefor $\bar{f}_{\chi_S} \circ \bar{f} \bar{\cap} \bar{f} \circ \bar{f}_{\chi_S} \circ \bar{f} \bar{\subseteq} \bar{f}$ and $\omega \preceq \omega_{\chi_S} \circ \omega \curlyvee \omega \circ \omega_{\chi_S} \circ \omega$. Hence $A = (\bar{f}, \omega)$ is a Q-cubic left bi-quasi ideal of a semigroup S. □

Corollary 14. *Let S be a semigroup and $A = (\bar{f}, \omega)$ be a non-empty Q-fuzzy set of S. A Q-fuzzy set $A = (\bar{f}, \omega)$ is a Q-cubic right bi-quasi ideal of a semigroup S if and only if the Q-cubic level set $U(A; \bar{t}, n)$ of A is a right bi-quasi ideal of a semigroup S for every $\bar{t} \in D[0,1], n \in [0,1]$, where $U(A; \bar{t}, n) \neq \emptyset$.*

Corollary 15. *Let S be a semigroup and $A = (\bar{f}, \omega)$ be a non-empty Q-fuzzy set of S. A Q-fuzzy set $A = (\bar{f}, \omega)$ is a Q-cubic bi-quasi ideal of a semigroup S if and only if the Q-cubic level set $U(A; \bar{t}, n)$ of A is a bi-quasi ideal of a semigroup S for every $\bar{t} \in D[0,1], n \in [0,1]$, where $U(A; \bar{t}, n) \neq \emptyset$.*

Lemma 16. *For non-empty subsets G and H of a semigroup S, we have*

1. $\bar{f}_{\chi_G} \circ \bar{f}_{\chi_H} = \bar{f}_{\chi_{GH}}$,
2. $\bar{f}_{\chi_G} \bar{\cap} \bar{f}_{\chi_H} = \bar{f}_{\chi_{G \cap H}}$,
3. $\omega_{\chi_G} \circ \omega_{\chi_H} = \omega_{\chi_{GH}}$,
4. $\omega_{\chi_G} \curlyvee \omega_{\chi_H} = \omega_{\chi_{G \vee H}}$.

Proof. It is straightforward. □

Theorem 17. *Let I be a non-empty subset of a semigroup S and $\chi_I = (\bar{f}_{\chi_I}, \omega_{\chi_I})$ be the characteristic function of I. Then I is a left bi-quasi ideal of a semigroup S if and only if $\chi_I = (\bar{f}_{\chi_I}, \omega_{\chi_I})$ is a Q-cubic left bi-quasi ideal of a semigroup S.*

Proof. Suppose I is a left bi-quasi ideal of S. Then I is a subsemigroup of S and $SI \cap ISI \subseteq I$. Obviously $\chi_I = (\bar{f}_{\chi_I}, \omega_{\chi_I})$ is a Q-cubic subsemigroup of S. And

$$\begin{aligned}(\bar{f}_{\chi_S} \circ \bar{f}_{\chi_I} \bar{\cap} \bar{f}_{\chi_I} \circ \bar{f}_{\chi_S} \circ \bar{f}_{\chi_I})(x,q) &= (\bar{f}_{\chi_S} \circ \bar{f}_{\chi_I})(x,q) \cap (\bar{f}_{\chi_I} \circ \bar{f}_{\chi_S} \circ \bar{f}_{\chi_I})(x,q) \\ &= \bar{f}_{\chi_{SI}}(x,q) \cap \bar{f}_{\chi_{ISI}}(x,q) \\ &= \bar{f}_{\chi_{SI \cap ISI}}(x,q) \\ &\subseteq \bar{f}_{\chi_I}(x,q).\end{aligned}$$

Thus, $\bar{f}_{\chi_S} \circ \bar{f}_{\chi_I} \circ \bar{f}_{\chi_S} \bar{\cap} \bar{f}_{\chi_I} \circ \bar{f}_{\chi_S} \circ \bar{f}_{\chi_I} \bar{\subseteq} \bar{f}_{\chi_I}$. Similarly, we can show that $\omega_{\chi_S} \circ \omega \circ \omega_{\chi_S} \curlyvee \omega \circ \omega_{\chi_S} \circ \omega \succeq \omega$. Hence $\chi_I = (\bar{f}_{\chi_I}, \omega_{\chi_I})$ is a Q-cubic left bi-quasi ideal of S.

Conversely suppose that $\chi_I = (\bar{f}_{\chi_I}, \omega_{\chi_I})$ is a Q-cubic left bi-quasi ideal of S. Then I is a subsemigroup of S. We have

$$\begin{aligned}&(\bar{f}_{\chi_S} \circ \bar{f}_{\chi_I})(x,q) \cap (\bar{f}_{\chi_I} \circ \bar{f}_{\chi_S} \circ \bar{f}_{\chi_I})(x,q) \subseteq \bar{f}_{\chi_I}(x,q) \\ &\Rightarrow \bar{f}_{\chi_{SI}}(x,q) \cap \bar{f}_{\chi_{ISI}}(x,q) \subseteq \bar{f}_{\chi_I}(x,q) \\ &\Rightarrow \bar{f}_{\chi_{SI \cap ISI}}(x,q) \subseteq \bar{f}_{\chi_I}(x,q).\end{aligned}$$

Thus $SI \cap ISI \subseteq I$. Hence I is a left bi-quasi ideal of a semigroup S. □

Corollary 18. *Let I be a non-empty subset of a semigroup S and $\chi_I = (\bar{f}_{\chi_I}, \omega_{\chi_I})$ be the characteristic function of I. Then I is a right bi-quasi ideal of a semigroup S if and only if $\chi_I = (\bar{f}_{\chi_I}, \omega_{\chi_I})$ is a Q-cubic right bi-quasi ideal of a semigroup S.*

Corollary 19. *Let I be a non-empty subset of a semigroup S and $\chi_I = (\bar{f}_{\chi_I}, \omega_{\chi_I})$ be the characteristic function of I. Then I is a bi-quasi ideal of a semigroup S if and only if $\chi_I = (\bar{f}_{\chi_I}, \omega_{\chi_I})$ is a Q-cubic bi-quasi ideal of a semigroup S.*

Theorem 20. *if $A = (\bar{f}, \omega)$ and $B = (\bar{g}, \upsilon)$ are Q-cubic bi-quasi ideals of a semigroup S, then $A \sqcap B = (\bar{f} \bar{\cap} \bar{g}, \omega \curlyvee \upsilon)$ is a Q-cubic left bi-quasi ideal of a semigroup S.*

Proof. Let $A = (\bar{f}, \omega)$ and $B = (\bar{g}, \upsilon)$ be Q-cubic bi-quasi ideals of a semigroup S. Then

$$\begin{aligned}(\bar{f}_{\chi_S} \circ \bar{f} \bar{\cap} \bar{g})(x,q) &= \bigcup_{x=ab} \{\bar{f}_{\chi_S}(a,q) \cap (\bar{f} \cap \bar{g})(b,q)\} \\ &= \bigcup_{x=ab} \{\bar{f}_{\chi_S}(a,q) \cap \bar{f}(b,q) \cap \bar{g}(b,q)\} \\ &= \bigcup_{x=ab} \{\{\bar{f}_{\chi_S}(a,q) \cap \bar{f}(b,q)\} \cap \{\bar{f}_{\chi_S}(a,q) \cap \bar{g}(b,q)\}\} \\ &= \bigcup_{x=ab} \{\bar{f}_{\chi_S}(a,q) \cap \bar{f}(b,q)\} \cap \bigcup_{x=ab} \{\bar{f}_{\chi_S}(a,q) \cap \bar{g}(b,q)\} \\ &= (\bar{f}_{\chi_S} \circ \bar{f})(x,q) \cap (\bar{f}_{\chi_S} \circ \bar{g})(x,q) \\ &= (\bar{f}_{\chi_S} \circ \bar{f} \bar{\cap} \bar{f}_{\chi_S} \circ \bar{g})(x,q).\end{aligned}$$

Therefore $\bar{f}_{\chi_S} \circ \bar{f} \bar{\cap} \bar{g} = \bar{f}_{\chi_S} \circ \bar{f} \bar{\cap} \bar{f}_{\chi_S} \circ \bar{g}$.

$$\begin{aligned}(\bar{f} \bar{\cap} \bar{g} \circ \bar{f}_{\chi_S} \circ \bar{f} \bar{\cap} \bar{g})(x,q) &= \bigcup_{x=abc} \{(\bar{f} \cap \bar{g})(a,q) \cap (\bar{f}_{\chi_S} \circ \bar{f} \cap \bar{g})(bc,q)\} \\ &= \bigcup_{x=abc} \{(\bar{f} \cap \bar{g})(a,q) \cap \{(\bar{f}_{\chi_S} \circ \bar{f} \cap \bar{f}_{\chi_S} \circ \bar{g})(bc,q)\}\} \\ &= \bigcup_{x=abc} \{(\bar{f} \cap \bar{g})(a,q) \cap \{(\bar{f}_{\chi_S} \circ \bar{f})(bc,q) \cap (\bar{f}_{\chi_S} \circ \bar{g})(bc,q)\}\} \\ &= \bigcup_{x=abc} \{\{\bar{f}(a,q) \cap (\bar{f}_{\chi_S} \circ \bar{f})(bc,q)\} \cap \{\bar{g}(a,q) \cap (\bar{f}_{\chi_S} \circ \bar{g})(bc,q)\}\} \\ &= (\bar{f} \circ \bar{f}_{\chi_S} \circ \bar{f})(x,q) \cap (\bar{g} \circ \bar{f}_{\chi_S} \circ \bar{g})(x,q) \\ &= (\bar{f} \circ \bar{f}_{\chi_S} \circ \bar{f} \bar{\cap} \bar{g} \circ \bar{f}_{\chi_S} \circ \bar{g})(x,q).\end{aligned}$$

Therefore $\bar{f} \bar{\cap} \bar{g} \circ \bar{f}_{\chi_S} \circ \bar{f} \bar{\cap} \bar{g} = \bar{f} \circ \bar{f}_{\chi_S} \circ \bar{f} \bar{\cap} \bar{g} \circ \bar{f}_{\chi_S} \circ \bar{g}$.

Then $\bar{f}_{\chi_S} \circ (\bar{f} \bar{\cap} \bar{g}) \bar{\cap} (\bar{f} \bar{\cap} \bar{g}) \circ \bar{f}_{\chi_S} \circ (\bar{f} \bar{\cap} \bar{g}) = \bar{f}_{\chi_S} \circ \bar{f} \bar{\cap} \bar{f} \circ \bar{f}_{\chi_S} \circ \bar{f} \bar{\cap} \bar{f}_{\chi_S} \circ \bar{g} \bar{\cap} \bar{g} \circ \bar{f}_{\chi_S} \circ \bar{g} \bar{\subseteq} \bar{f} \bar{\cap} \bar{g}$. Similarly, we can show that $\omega_{\chi_S} \circ \omega \curlyvee \upsilon \curlyvee \omega \curlyvee \upsilon \circ \omega_{\chi_S} \circ \omega \curlyvee \upsilon = \omega_{\chi_S} \circ \omega \circ \curlyvee \omega_{\chi_S} \circ \upsilon \circ \curlyvee \omega \circ \omega_{\chi_S} \circ \omega \curlyvee \upsilon \circ \omega_{\chi_S} \circ \upsilon \succeq \omega \curlyvee \upsilon$. Therefore $A \sqcap B = (\bar{f} \bar{\cap} \bar{g}, \omega \curlyvee \upsilon)$ is a Q-cubic left bi-quasi ideal of a semigroup S. □

Corollary 21. *If $A = (\bar{f}, \omega)$ and $B = (\bar{g}, \upsilon)$ are Q-cubic bi-quasi ideals of a semigroup S, then $A \sqcap B = (\bar{f} \bar{\cap} \bar{g}, \omega \curlyvee \upsilon)$ is a Q-cubic right bi-quasi ideal of a semigroup S.*

Corollary 22. *If $A = (\bar{f}, \omega)$ and $B = (\bar{g}, \upsilon)$ are Q-cubic bi-quasi ideals of a semigroup S, then $A \sqcap B = (\bar{f} \bar{\cap} \bar{g}, \omega \curlyvee \upsilon)$ is a Q-cubic bi-quasi ideal of a semigroup S.*

Theorem 23. *If $A = (\bar{f}, \omega)$ and $B = (\bar{g}, \upsilon)$ are Q-cubic right ideals and a Q-cubic left ideal of a semigroup S respectively. Then $A \sqcap B = (\bar{f} \bar{\cap} \bar{g}, \omega \curlyvee \upsilon)$ is a Q-cubic left bi-quasi ideal of a semigroup S.*

Proof. It following Theorem 20. □

Corollary 24. *If $A = (\bar{f}, \omega)$ and $B = (\bar{g}, \upsilon)$ are Q-cubic right ideals and a Q-cubic left ideal of a semigroup S respectively. Then $A \sqcap B = (\bar{f} \bar{\cap} \bar{g}, \omega \curlyvee \upsilon)$is a Q-cubic right bi-quasi ideal of a semigroup S.*

Corollary 25. *If $A = (\bar{f}, \omega)$ and $B = (\bar{g}, \upsilon)$ are Q-cubic right ideals and a Q-cubic left ideal of a semigroup S respectively. Then $A \sqcap B = (\bar{f} \bar{\cap} \bar{g}, \omega \curlyvee \upsilon)$ is a Q-cubic bi-quasi ideal of a semigroup S.*

Definition 26. A semigroup S is called regular if for all $a \in S$ there exists $x \in S$ such that $a = axa$.

Definition 27. A Q-cubic subsemigroup $A = (\bar{f}, \omega)$ of S is called a Q-cubic quasi ideal of S if it satisfies the following conditions:

1. $\bar{f}_{\chi_S} \circ \bar{f} \bar{\cap} \bar{f} \circ \bar{f}_{\chi_S} \bar{\subseteq} \bar{f}$,

2. $\omega \preceq \omega_{\chi_S} \circ \omega \curlyvee \omega \circ \omega_{\chi_S}$.

Theorem 28. *If $A = (\bar{f}, \omega)$ be a Q-cubic quasi ideal of a regular semigroup S. Then $A = (\bar{f}, \omega)$ is a Q-cubic ideal of a semigroup S.*

Proof. Assume that $A = (\bar{f}, \omega)$ is a Q-cubic quasi-ideal of S and let $x, y \in S, q \in Q$. Then

$$\begin{aligned}\bar{f}(xy,q) &\supseteq (\bar{f} \circ \bar{f}_{\chi_S})(xy,q) \cap (\bar{f}_{\chi_S} \circ \bar{f})(xy,q) \\ &= \bigcup_{xy=ab} \{\bar{f}(a,q) \cap \bar{f}_{\chi_S}(b,q)\} \cap \bigcup_{xy=ij} \{\bar{f}_{\chi_S}(i,q) \cap \bar{f}(j,q)\} \\ &\supseteq \bar{f}(x,q) \cap \bar{f}_{\chi_S}(y,q) \cap \bar{f}_{\chi_S}(x,q) \cap \bar{f}(y,q) \\ &= \bar{f}(x,q) \cap \bar{f}(y,q).\end{aligned}$$

Thus $\bar{f}(xy,q) \supseteq \bar{f}(x,q) \cap \bar{f}(y,q)$. And similarly we can show that $\omega(xy,q) \leq \omega(x,q) \vee \omega(y,q)$.
Hence $\mathcal{A} = (\bar{f}, \omega)$ is a Q-cubic subsemigroup of S. Let $x, y, z \in S, q \in Q$. Then

$$\begin{aligned}\bar{f}(xyz) &\supseteq (\bar{f} \circ \bar{f}_{\chi_S})(xyz,q) \cap (\bar{f}_{\chi_S} \circ \bar{f})(xyz,q) \\ &= \bigcup_{xyz=ab} \{\bar{f}(a,q) \cap \bar{f}_{\chi_S}(b,q)\} \cap \bigcup_{xyz=ij} \{\bar{f}_{\chi_S}(i,q) \cap \bar{f}(j,q)\} \\ &\supseteq \bar{f}(x,q) \cap \bar{f}_{\chi_S}(yz,q) \cap \bar{f}_{\chi_S}(xy,q) \cap \bar{f}(z,q) \\ &= \bar{f}(x,q) \cap \bar{f}(z,q).\end{aligned}$$

Thus $\bar{f}(xyz,q) \supseteq \bar{f}(x,q) \cap \bar{f}(z,q)$. And similarly we can show that $\omega(xyz,q) \leq \omega(x,q) \vee \omega(z,q)$. Hence $A = (\bar{f}, \omega)$ is a Q-cubic bi-ideal of S. Since S is regular, $A = (\bar{f}, \omega)$ is a Q-cubic bi-ideal of S and $x, y \in S$ we have $xy \in (xSx)S \subseteq xSx$. Thus there exists $k \in S$ such that $xy = xkx$. So

$$\bar{f}(xy,q) = \bar{f}(xkx,q) \supseteq \bar{f}(x,q) \cap \bar{f}(x,q) = \bar{f}(x,q).$$

And similarly $\omega(xy,q) \leq \omega(x,q)$. Thus, $A = (\bar{f}, \omega)$ is a Q-cubic right ideal of S. Similarly, we can show that $\bar{f}(xy,q) \supseteq \bar{f}(y,q)$ and $\omega(xy,q) \leq \omega(y,q)$. Thus $A = (\bar{f}, \omega)$ is a Q-cubic left ideal of S. Hence $A = (\bar{f}, \omega)$ is a Q-cubic ideal of S. □

Theorem 29. *Let S be a regular semigroup. Then $A = (\bar{f}, \omega)$ is a Q-cubic left bi-quasi ideal of S if and only if $A = (\bar{f}, \omega)$ is a Q-cubic quasi ideal of S.*

Proof. Let $A = (\bar{f}, \omega)$ is a Q-cubic left bi-quasi ideal of S and $x \in S, q \in Q$. Thus, $(\bar{f}_{\chi_S} \circ \bar{f})(x,q) \cap (\bar{f} \circ \bar{f}_{\chi_S} \circ \bar{f})(x,q) \subseteq \bar{f}(x,q)$ and $\omega(x,q) \preceq (\omega_{\chi_S} \circ \omega)(x,q) \curlyvee (\omega \circ$

$\omega_{\chi_S} \circ \omega)(x, q)$. Suppose $(\bar{f}_{\chi_S} \circ \bar{f})(x, q) \supseteq \bar{f}(x, q)$. Since S is regular, there exists $y \in S$ such that $x = xyx$. Then

$$\begin{aligned}(\bar{f} \circ \bar{f}_{\chi_S} \circ \bar{f})(x, q) &= \bigcup_{x=xyx} \{\bar{f}(xy, q) \cap (\bar{f}_{\chi_S} \circ \bar{f})(x, q)\} \\ &\supseteq \bigcup_{x=xyx} \{\bar{f}(x, q) \cap \bar{f}(x, q)\} \\ &= \bar{f}(x, q).\end{aligned}$$

Which is a contradiction. Therefore $A = (\bar{f}, \omega)$ is a Q-cubic quasi ideal of S. By Theorem 28, converse is true. □

Corollary 30. *Let S be a regular semigroup. Then $A = (\bar{f}, \omega)$ is a Q-cubic right bi-quasi ideal of S if and only if $A = (\bar{f}, \omega)$ is a Q-cubic quasi ideal of S.*

Corollary 31. *Let S be a regular semigroup. Then $A = (\bar{f}, \omega)$ is a Q-cubic bi-quasi ideal of S if and only if $A = (\bar{f}, \omega)$ is a Q-cubic quasi ideal of S.*

Theorem 32. *Let S be a semigroup. S is a regular semigroup if and only if $B = SB \cap BSB$, for every bi-quasi ideal of S.*

Theorem 33. *Let S be a semigroup. Then S is a regular if and only if $\bar{f} = \bar{f}_{\chi_S} \circ \bar{f} \bar{\cap} \bar{f} \circ \bar{f}_{\chi_S} \circ \bar{f}$ and $\omega = \omega_{\chi_S} \circ \omega \curlyvee \omega \circ \omega_{\chi_S} \circ \omega$, for any Q-cubic left bi-quasi ideal of a semigroup S.*

Proof. Let $A = (\bar{f}, \omega)$ be a Q-cubic left bi-quasi ideal of the regular semigroup S. Then $\bar{f}_{\chi_S} \circ \bar{f} \bar{\cap} \bar{f} \circ \bar{f}_{\chi_S} \circ \bar{f} \bar{\subseteq} \bar{f}$ and $\omega \preceq \omega_{\chi_S} \circ \omega \curlyvee \omega \circ \omega_{\chi_S} \circ \omega$. Let $x \in S, q \in Q$. Since S is regular, there exists $a \in S$ such that $x = xax$. Thus

$$\begin{aligned}(\bar{f} \circ \bar{f}_{\chi_S} \circ \bar{f})(x, q) &= \bigcup_{x=xax} \{\bar{f}(x, q) \cap (\bar{f}_{\chi_S} \circ \bar{f})(ax, q)\} \\ &= \bigcup_{x=xax} \{\bar{f}(x, q) \cap \bigcup_{ax=yz} \{\bar{f}_{\chi_S}(y, q) \cap \bar{f}(z, q)\}\} \\ &\supseteq \bigcup_{x=xax} \{\bar{f}(x, q) \cap \bar{f}(x, q)\} \\ &= \bar{f}(x, q).\end{aligned}$$

Similarly, $(\bar{f}_{\chi_S} \circ \bar{f})(x, q) \supseteq \bar{f}(x, q)$, $\omega(x, q) \geq (\omega_{\chi_S} \circ \omega)(x, q)$ and $\omega(x, q) \geq (\omega \circ \omega_{\chi_S} \circ \omega)(x, q)$. Therefore $\bar{f} = \bar{f}_{\chi_S} \circ \bar{f} \bar{\cap} \bar{f} \circ \bar{f}_{\chi_S} \circ \bar{f}$ and $\omega = \omega_{\chi_S} \circ \omega \curlyvee \omega \circ \omega_{\chi_S} \circ \omega$.

Conversely suppose that let B be a left bi-quasi ideal of a semigroup S. Then by Theorem 17, $\chi_B = (\bar{f}_{\chi_B}, \omega_{\chi_B})$ be a Q-cubic bi-interior ideal of the semigroup S. Thus

$$\begin{aligned}\bar{f}_{\chi_B}(x, q) &= (\bar{f}_{\chi_S} \circ \bar{f}_{\chi_B})(x, q) \cap (\bar{f}_{\chi_B} \circ \bar{f}_{\chi_S} \circ \bar{f}_{\chi_B})(x, q) \\ &= \bar{f}_{\chi_{SB}}(x, q) \cap \bar{f}_{\chi_{BSB}}(x, q) \\ &= \bar{f}_{\chi_{SB \cap BSB}}(x, q).\end{aligned}$$

Therefore $B = SB \cap BSB$. By Theorem 32, S is regular semigroup. □

Corollary 34. *Let S be a semigroup. Then S is a regular if and only if $\bar{f} = \bar{f}_{\chi_S} \circ \bar{f} \bar{\cap} \bar{f} \circ \bar{f}_{\chi_S} \circ \bar{f}$ and $\omega = \omega_{\chi_S} \circ \omega \curlyvee \omega \circ \omega_{\chi_S} \circ \omega$, for any Q-cubic right bi-quasi ideal of a semigroup S.*

Corollary 35. *Let S be a semigroup. Then S is a regular if and only if $\bar{f} = \bar{f}_{\chi_S} \circ \bar{f} \bar{\cap} \bar{f} \circ \bar{f}_{\chi_S} \circ \bar{f}$ and $\omega = \omega_{\chi_S} \circ \omega \curlyvee \omega \circ \omega_{\chi_S} \circ \omega$ or $\bar{f} = \bar{f} \circ \bar{f}_{\chi_S} \bar{\cap} \bar{f} \circ \bar{f}_{\chi_S} \circ \bar{f}$ and $\omega = \omega \circ \omega_{\chi_S} \curlyvee \omega \circ \omega_{\chi_S} \circ \omega$, for any Q-cubic bi-quasi ideal of a semigroup S.*

Theorem 36. *Let S be a semigroup. Then S is a regular if and only if $\bar{f} \bar{\cap} \bar{g} \bar{\subseteq} \bar{g} \circ \bar{f} \bar{\cap} \bar{f} \circ \bar{g} \circ \bar{f}$ and $\omega \curlyvee \upsilon \succeq \upsilon \circ \omega \curlyvee \omega \circ \upsilon \circ \omega$, for every Q-cubic left bi-quasi ideal $A = (\bar{f}, \omega)$ and every Q-cubic ideal $B = (\bar{g}, \upsilon)$ of a semigroup S.*

Proof. Let S be a regular semigroup and $x \in S$. Then there exists $y \in S$ such that $x = xyx$.

$$
\begin{aligned}
(\bar{f} \circ \bar{g} \circ \bar{f})(x, q) &= \bigcup_{x=xyx} \{(\bar{f} \circ \bar{g})(xy, q) \cap \bar{f}(x, q)\} \\
&= \bigcup_{x=xyx} \{ \bigcup_{xy=xyxy} \{\bar{f}(x, q) \cap \bar{g}(yxy, q)\} \cap \bar{f}(x, q)\} \\
&\supseteq \{\bar{f}(x, q) \cap \bar{g}(x, q)\} \cap \bar{f}(x, q) \\
&= \bar{f}(x, q) \cap \bar{g}(x, q) \\
&= (\bar{f} \bar{\cap} \bar{g})(x, q).
\end{aligned}
$$

And

$$
\begin{aligned}
(\bar{g} \circ \bar{f})(x, q) &= \bigcup_{x=xyx} \{\bar{g}(xy, q) \cap \bar{f}(x, q)\} \\
&\supseteq \bar{g}(x, q) \cap \bar{f}(x, q) \\
&= (\bar{g} \bar{\cap} \bar{f})(x, q).
\end{aligned}
$$

Similary we can prove $(\omega \circ \upsilon \circ \omega)(x, q) \preceq (\upsilon \curlyvee \omega)(x, q)$ and $(\upsilon \circ \omega)(x, q) \preceq (\upsilon \curlyvee \omega)(x, q)$. Hence $\bar{f} \bar{\cap} \bar{g} \bar{\subseteq} \bar{g} \circ \bar{f} \bar{\cap} \bar{f} \circ \bar{g} \circ \bar{f}$ and $\omega \curlyvee \upsilon \succeq \upsilon \circ \omega \curlyvee \omega \circ \upsilon \circ \omega$.

Conversely suppose that the condition holds. Let $A = (\bar{f}, \omega)$ be a Q-cubic left bi-quasi ideal. We have $\bar{f} \bar{\cap} \bar{f}_{\chi_S} \bar{\subseteq} \bar{f}_{\chi_S} \circ \bar{f} \bar{\cap} \bar{f} \circ \bar{f}_{\chi_S} \circ \bar{f}$ and $\omega \curlyvee \omega_{\chi_S} \preceq \omega \circ \omega_{\chi_S} \curlyvee \omega \circ \omega_{\chi_S} \circ \omega$ implies that $\bar{f} \bar{\subseteq} \bar{f}_{\chi_S} \circ \bar{f} \circ \bar{\cap} \bar{f} \circ \bar{f}_{\chi_S} \circ \bar{f}$ and $\omega \succeq \omega \circ \omega_{\chi_S} \curlyvee \omega \circ \omega_{\chi_S} \circ \omega$. By Theorem 33, S is a regular semigroup. □

Corollary 37. *Let S be a semigroup. Then S is a regular if and only if $\bar{f} \bar{\cap} \bar{g} \subseteq \bar{g} \circ \bar{f} \bar{\cap} \bar{f} \circ \bar{g} \circ \bar{f}$ and $\omega \curlyvee \upsilon \succeq \upsilon \circ \omega \curlyvee \omega \circ \upsilon \circ \omega$, for every Q-cubic right bi-quasi ideal $A = (\bar{f}, \omega)$ and every Q-cubic ideal $B = (\bar{g}, \upsilon)$ of a semigroup S.*

Corollary 38. *Let S be a semigroup. Then S is a regular if and only if $\bar{f} \bar{\cap} \bar{g} \subseteq \bar{g} \circ \bar{f} \bar{\cap} \bar{f} \circ \bar{g} \circ \bar{f}$ and $\omega \curlyvee \upsilon \preceq \upsilon \circ \omega \curlyvee \omega \circ \upsilon \circ \omega$, for every Q-cubic bi-quasi ideal $A = (\bar{f}, \omega)$ and every Q-cubic ideal $B = (\bar{g}, \upsilon)$ of a semigroup S.*

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