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Quantum Algorithm for Maximum Integer Multiple-choice Generalized Knapsack Problem by Hybrid Method of Grover's Database Search and Shor's Data Decrease

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Abstract

A quantum algorithm for the maximum integer multiple-choice generalized knapsack problem by a hybrid method of Grover's database search and Shor's data decrease, and its example are reported. The maximum optimal combination of $\Sigma_{i = 1 \to n} a_{i, j(i)} x_{i, p(i)} \leq b$ [i = 1, 2, ⋯ , n. $j(i)$ = 1, 2, ⋯ , k. $p(i)$ = 0, 1, 2, ⋯ , m. $a_{i, j(i)}$: positive integer. $x_{i, p(i)}$ = 0, 1, 2, ⋯ , m. n, b, k, and m are positive integers.] is requested. The computational complexity of the classical computation is $(k(m + 1))^n$. The computational complexity becomes about $5n^2(\log_2 (k(m + 1)))^2$ by this quantum algorithm. Therefore, a decreased process becomes possible.

Keywords: Quantum algorithm, maximum integer multiple-choice generalized knapsack problem, Grover's database search, Shor's data decrease, computational complexity.

AMS subject classification: Primary 81-08; Secondary 68R05, 68W40.

1. Introduction

Arute et al. reported the quantum supremacy using a programmable superconducting processor [1]. The algorithms of the quantum computer by Deutsch-Jozsa, Shor, Grover, and so on are known [2-7]. Ambainis's quantum walk algorithms was the example to decrease the computational complexity [8]. When the feature of the problem isn't used, it is difficult to decrease the computational complexity. Bennett, Bernstein, Brassard, and Vazirani addressed the class NP cannot be solved on a quantum Turing machine in time $O(2^{n/2})$ [9]. However, they didn't eliminate the unnecessary data on the machine's way to the end. For this reason, Fujimura suggested that the probability amplitudes of the longest path problem are converged quickly by a hybrid method of Grover's database search and Shor's data decrease [3, 5-7, 10]. Its computational complexity is decreased. The maximum integer multiple-choice generalized knapsack problem [11] is examined by the hybrid method this time. Therefore, its result is reported.

2. Maximum Integer Multiple-choice Generalized Knapsack Problem

The maximum optimal combination of $\Sigma_{i=1\to n} a_{i,j(i)} x_i \leq b$, $i = 1, 2, \cdots, n$, $j(i) = 1, 2, \cdots, k$, and $0 \leq x_i$ [n, $a_{i,j(i)}$, b, and k are positive integers, and x_i is a non-negative integer.] is requested [11].

The problem may be generalized as follows:

It is assumed that there are $\Sigma_{i=1\to n} a_{i,j(i)} x_{i,p(i)} \leq b$, $i = 1, 2, \cdots, n$, $j(i) = 1, 2, \cdots, k$, $p(i) = 0, 1, 2, \cdots, m$, and $x_{i,p(i)} = 0, 1, 2, \cdots, m$ [n, $a_{i,j(i)}$, b, k, and m are positive integers.]. The computational complexity of the classical computation is $(k(m+1))^n$.

3. Quantum Algorithm

It is assumed that there are $\Sigma_{i=1\to n} a_{i,j(i)} x_{i,p(i)} \leq b$, $i = 1, 2, \cdots, n$, $j(i) = 1, 2, \cdots, k$, $p(i) = 0, 1, 2, \cdots, m$, $x_{i,p(i)} = 0, 1, 2, \cdots, m$ [n, $a_{i,j(i)}$, b, M_1[$< b$], k, and m are positive integers.], and $W_0 = (k(m+1))^n$.

Still more, it is assumed that a number of datum $U(V)$ is $(q(1)k^{n-1} + q(2)k^{n-2} + \cdots + q(n-1)k^1 + q(n)k^0)(m+1)^n + r(1)(m+1)^{n-1} + r(2)(m+1)^{n-2} + \cdots + r(n-1)(m+1)^1 + r(n)(m+1)^0$.

However, 0-th datum is $q(1) = 0$, $q(2) = 0$, $\cdots$, $q(n) = 0$, $r(1) = 0$, $r(2) = 0$, $\cdots$, $r(n) = 0$ [$\rightarrow U(V = 0)$], $((k(m + 1))^n - 1)$-th datum is $q(1) = k - 1$, $q(2) = k - 1$, $\cdots$, $q(n) = k - 1$, $r(1) = m$, $r(2) = m$, $\cdots$, $r(n) = m$ [$\rightarrow U(V = (k(m + 1))^n - 1)$], $q(i) = 0, 1, 2, \cdots , k - 1$, and $r(i) = 0, 1, 2, \cdots , m$.

Therefore, the formula becomes $\Sigma_{i = 1 \rightarrow n}\, a_{i, (j(i) = q(i) + 1)}\, x_{i, (p(i) = r(i))} \leq b$.

g is the minimum integer that follows $(k(m + 1))^n/1 \leq 2^{2g} = 4^g$, because a number of combinations of answers is at least 1. $U(V = 0) = 0$, $U(V = (W_0/4) - 2)$, $U(V = (W_0/16) - 2)$, $\cdots$, $U(V = (W_0/4^{g-1}) - 2)$, and $U(V = (W_0/4^g) - 1)$ are computed. [$\rightarrow$ See Appendix-1]

Next, a quantum algorithm is shown as the following.

First of all, quantum registers $|q(1)\rangle$, $|q(2)\rangle$, $\cdots$, $|q(n)\rangle$, $|r(1)\rangle$, $|r(2)\rangle$, $\cdots$, $|r(n)\rangle$, $|s(1)\rangle$, $|s(2)\rangle$, $\cdots$, $|s(n)\rangle$, $|t(1)\rangle$, $|t(2)\rangle$, $\cdots$, $|t(n)\rangle$, $|c\rangle$, $|d\rangle$, and $|e\rangle$ are prepared. When F and G are the minimum integers that are $\log_2(4k)$ and $\log_2(4(m + 1))$ or more, respectively, each of $|q(i)\rangle$ and $|r(i)\rangle$ that i is an integer from 1 to n is consisted of F and G quantum bits [= qubits], respectively. [$\rightarrow$ See Appendix-2] States of $|q(1)\rangle$, $|q(2)\rangle$, $\cdots$, $|q(n)\rangle$, $|r(1)\rangle$, $|r(2)\rangle$, $\cdots$, $|r(n)\rangle$, $|s(1)\rangle$, $|s(2)\rangle$, $\cdots$, $|s(n)\rangle$, $|t(1)\rangle$, $|t(2)\rangle$, $\cdots$, $|t(n)\rangle$, $|c\rangle$, $|d\rangle$, and $|e\rangle$ are $q(1)$, $q(2)$, $\cdots$, $q(n)$, $r(1)$, $r(2)$, $\cdots$, $r(n)$, $s(1)$, $s(2)$, $\cdots$, $s(n)$, $t(1)$, $t(2)$, $\cdots$, $t(n)$, c, d, and e, respectively.

Step 1: Each qubit of $|q(1)\rangle$, $|q(2)\rangle$, $\cdots$, $|q(n)\rangle$, $|r(1)\rangle$, $|r(2)\rangle$, $\cdots$, $|r(n)\rangle$, $|s(1)\rangle$, $|s(2)\rangle$, $\cdots$, $|s(n)\rangle$, $|t(1)\rangle$, $|t(2)\rangle$, $\cdots$, $|t(n)\rangle$, $|c\rangle$, $|d\rangle$, and $|e\rangle$ is set $|0\rangle$.

Step 2: The Hadamard gate $\boxed{H}$ acts on each qubit of $|q(1)\rangle$, $|q(2)\rangle$, $\cdots$, $|q(n - 1)\rangle$, $|q(n)\rangle$, $|r(1)\rangle$, $|r(2)\rangle$, $\cdots$, $|r(n - 1)\rangle$, and $|r(n)\rangle$ [3, 4]. It changes them for entangled states. The total states are $(2^F)^n(2^G)^n$. [$|q(i)\rangle$ is consisted of F qubits, and $|r(i)\rangle$ is consisted of G qubits. Each qubit is acted on by $\boxed{H}$. Therefore, $(F + G)n$ of $\boxed{H}$ are necessary.]

Step 3: It is assumed that a quantum gate (A) changes $|s(i)\rangle$ for $|1\rangle$ in $q(i) < k$, or it changes $|s(i)\rangle$ for $|0\rangle$ in the others of $q(i)$. And a quantum gate (B) changes $|t(i)\rangle$ for $|1\rangle$ in $r(i) < m + 1$, or it changes $|t(i)\rangle$ for $|0\rangle$ in the others of $r(i)$. As target states for $|s(i)\rangle$ and $|t(i)\rangle$ are 1, quantum phase inversion gates (PI) and quantum inversion about mean gates (IM) act on $s(i)$ and $t(i)$ [Grover's database search. The same gates action is shown in the following.] [3, 6, 7]. When Q_1 is the minimum even integer that is $(2^F/k)^{1/2}$ or more, the total number that (PI) and (IM) act on $|s(i)\rangle$ is Q_1, because they are a couple. When Q_2 is the minimum even integer that is $(2^G/(m + 1))^{1/2}$ or more, the total number that (*PI*) and (*IM*) act on |*t*(*i*)› is Q_2, because they are a couple. Next, an observation gate (*OB*) observes |*s*(*i*)› and |*t*(*i*)› [Shor's data decrease. The same gate action is shown in the following.] [3, 5]. [→ See Appendix-2] These actions are repeated sequentially from |*q*(1)› and |*r*(1)› to |*q*(*n*)› and |*r*(*n*)›, respectively. Therefore, each state of |*q*(*i*)› and |*r*(*i*)› is 0, 1, ⋯ , *k* – 1, and 0, 1, ⋯ , *m*, respectively, and the total states become $(k(m + 1))^n$ [= W_0].

Step 4: It is assumed that a quantum gate (C_1) changes |*c*› and |*d*› for $|c + q(1)k^{n-1}(m + 1)^n + r(1)(m + 1)^{n-1}\rangle$ and $|d + a_{1, q(1)+1}\, x_{1, r(1)}\rangle$, respectively, from *q*(1) and *r*(1). Similarly, (C_i) [$2 \le i \le n - 1$. *i* is an integer.] changes |*c*› and |*d*› for $|c + q(i)k^{n-i}(m + 1)^n + r(i)(m + 1)^{n-i}\rangle$ and $|d + a_{i, q(i)+1}\, x_{i, r(i)}\rangle$, respectively, from *q*(*i*) and *r*(*i*). This action is repeated sequentially from *q*(2) and *r*(2) to *q*(*n* – 1) and *r*(*n* – 1), respectively. (C_n) changes |*c*› and |*d*› for $|c + q(n)k^0(m + 1)^n + r(n)(m + 1)^0\rangle$ and $|d + a_{n, q(n)+1}\, x_{n, r(n)}\rangle$, respectively, from *q*(*n*) and *r*(*n*). Therefore, |*c*› and |*d*› become |*U*(*V*)› and $|\Sigma_{i=1\to n}\, a_{i, q(i)+1}\, x_{i, r(i)}\rangle$, respectively.

Step 5: It is assumed that a quantum gate (D_1) changes |*e*› for |1› in $U(V = 0) = 0 \le c \le U(V = (W_0/4) - 2)$ or $M_1 \le d \le b$, or it changes |*e*› for |0› in the others of *c* and *d*. As the target state for |*e*› is 1, (*PI*) and (*IM*) act on |*e*›. The number of the data that is included in $0 \le c \le U(V = (W_0/4) - 2)$ or $M_1 \le d \le b$ is $W_1 \approx W_0/4$. [→ See Appendix-1] When T_1 is the minimum even integer that is $(W_0/W_1)^{1/2}$ or more, the total number that (*PI*) and (*IM*) act on |*e*› is $T_1 \approx 2$. Next, (*OB*) observes |*e*›, and the data of W_1 remain. Similarly, (D_i) [$2 \le i \le g - 1$. *i* is the integer.] changes |*e*› for |1› in $0 \le c \le U(V = (W_0/4^i) - 2)$ or $M_1 \le d \le b$, or it changes |*e*› for |0› in the others of *c* and *d*. As the target state for |*e*› is 1, (*PI*) and (*IM*) act on |*e*›. The number of the data that is included in $0 \le c \le U(V = (W_0/4^i) - 2)$ or $M_1 \le d \le b$ is $W_i \approx W_0/4^i$. When T_i is the minimum even integer that is $(W_{i-1}/ W_i)^{1/2}$ or more, the total number that (*PI*) and (*IM*) act on |*e*› is $T_i \approx 2$. Next, (*OB*) observes |*e*›, and the data of W_i remain. These actions are repeated sequentially from 2 to *g* – 1 at *i*. (D_g) changes |*e*› for |1› in $0 \le c \le U(V = (W_0/4^g) - 1)$ or $M_1 \le d \le b$, or it changes |*e*› for |0› in the others of *c* and *d*. As the target state for |*e*› is 1, (*PI*) and (*IM*) act on |*e*›. The number of the data that is included in $0 \le c \le U(V = (W_0/4^g) - 1)$ or $M_1 \le d \le b$ is $W_g \approx 1$. When T_g is the minimum even integer that is $(W_{g-1}/W_g)^{1/2}$ or more, the total number that (*PI*) and (*IM*) act on |*e*› is $T_g \approx 2$. Next, (*OB*) observes |*q*(1)›, |*q*(2)›, ⋯ , |*q*(*n*)›, |*r*(1)›, |*r*(2)›, ⋯ , |*r*(*n*)›, |*s*(1)›, |*s*(2)›, ⋯ , |*s*(*n*)›, |*t*(1)›, |*t*(2)›, ⋯ , |*t*(*n*)›, |*c*›, |*d*›, and |*e*›, and one of the data of W_g remains. Therefore, one example of combinations that are $M_1 \le d = \Sigma_{i=1\to n}\, a_{i,}$

$_{q(i)+1} x_{i, r(i)} \leq b$ is obtained.

Step 6: When the state of |*e*› is 1 or 0, M_1 is assumed to be M_2 [> M_1] or M_2 [< M_1], respectively, these computations from step 1 to step 6 are repeated. It is assumed that the maximum weight M_{max} for *b* is obtained by repeating about $\log_2 (k(m+1))^n$ [12].

An example is shown as the next section. However, this algorithm is applied as far as the effect of Grover's database search and Shor's data decrease.

4. Numerical Computation

It is assumed that there are $n = 3$, $b = 58$, $k = 4$, $m = 2$, $M_1 = 50$, $a_{1,1} = 1$, $a_{1,2} = 4$, $a_{1,3} = 7$, $a_{1,4} = 10$, $a_{2,1} = 2$, $a_{2,2} = 5$, $a_{2,3} = 8$, $a_{2,4} = 11$, $a_{3,1} = 3$, $a_{3,2} = 6$, $a_{3,3} = 9$, $a_{3,4} = 12$, $x_{1,0} = 0$, $x_{1,1} = 1$, $x_{1,2} = 2$, $x_{2,0} = 0$, $x_{2,1} = 1$, $x_{2,2} = 2$, $x_{3,0} = 0$, $x_{3,1} = 1$, $x_{3,2} = 2$, $g = 6$ [$(4 \cdot 3)^3/1 = 1728 \leq 4^6 = 4096$], $U(V = 0) = 0$, $U(V = (1728/4) - 2 = 432 - 2 = 430) = 430$, $U(V = (1728/16) - 2 = 108 - 2 = 106) = 106$, $U(V = (1728/64) - 2 = 27 - 2 = 25) = 25$, $U(V = (1728/256) - 2 = 6.75 - 2 = 4.75 \approx 5) \approx 5$, $U(V = (1728/1024) - 2 \approx 1.69 - 2 = -0.31 \approx 0) \approx 0$, and $U(V = (1728/4096) - 1 \approx 0.42 - 1 = -0.58 \approx 0) \approx 0$.

First of all, |*q*(1)›, |*q*(2)›, |*q*(3)›, |*r*(1)›, |*r*(2)›, |*r*(3)›, |*s*(1)›, |*s*(2)›, |*s*(3)›, |*t*(1)›, |*t*(2)›, |*t*(3)›, |*c*›, |*d*›, and |*e*› are prepared. When *F* and *G* are the minimum integers that are $\log_2 16 = 4 \leq 4 = F$ and $\log_2 12 \approx 3.58 \leq 4 = G$, respectively, each of |*q* (*i*) › and |*r* (*i*) › that *i* is the integer from 1 to 3 is consisted of $F = G = 4$ qubits. States of |*q*(1)›, |*q*(2)›, |*q*(3)›, |*r*(1)›, |*r*(2)›, |*r*(3)›, |*s*(1)›, |*s*(2)›, |*s*(3)›, |*t*(1)›, |*t*(2)›, |*t*(3)›, |*c*›, |*d*›, and |*e*› are *q*(1), *q*(2), *q*(3), *r*(1), *r*(2), *r*(3),*s*(1), *s*(2), *s*(3), *t*(1), *t*(2), *t*(3), *c*, *d*, and *e*, respectively.

Step 1: Each qubit of |*q*(1)›, |*q*(2)›, |*q*(3)›, |*r*(1)›, |*r*(2)›, |*r*(3)›, |*s*(1)›, |*s*(2)›, |*s*(3)›, |*t*(1)›, |*t*(2)›, |*t*(3)›, |*c*›, |*d*›, and |*e*› is set |0›.

Step 2: $\boxed{H}$ acts on each qubit of |*q*(1)›, |*q*(2)›, |*q*(3)›, |*r*(1)›, |*r*(2)›, and |*r*(3)›. It changes them for entangled states. The total states are $(2^4)^3(2^4)^3 = 16777216$. [|*q*(*i*)› is consisted of 4 qubits, and |*r*(*i*)› is consisted of 4 qubits. Each qubit is acted on by $\boxed{H}$. Therefore, 24 of $\boxed{H}$ are necessary.]

Step 3: (*A*) changes |*s*(*i*)› for |1› in $q(i) < k = 4$, or it changes |*s*(*i*)› for |0› in the others of *q*(*i*). And (*B*) changes |*t*(*i*)› for |1› in $r(i) < m + 1 = 3$, or it changes |*t*(*i*)› for |0› in the others of *r*(*i*). As the target state for |*s*(*i*)› and |*t*(*i*)› are 1, (*PI*) and (*IM*) act on *s*(*i*) and *t*(*i*). When Q_1 and Q_2 are the minimum even integer that are $(2^4/4)^{1/2} = 2 \leq 2 = Q_1$ and $(2^4/3)^{1/2} \approx 2.31 \leq 4 = Q_2$, respectively, the total number that (*PI*) and (*IM*) act on

$|s(i)\rangle$ and $|t(i)\rangle$ is $Q_1 = 2$ and $Q_2 = 4$. Next, (*OB*) observes $|s(i)\rangle$ and $|t(i)\rangle$. These actions are repeated sequentially from $|q(1)\rangle$ and $|r(1)\rangle$ to $|q(3)\rangle$ and $|r(3)\rangle$, respectively. Therefore, each state of $|q(i)\rangle$ and $|r(i)\rangle$ is 0, 1, 2, 3, and 0, 1, 2, respectively, and the total states become $(4(2 + 1))^3 = 1728$ [$= W_0$].

Step 4: (C_1) changes $|c\rangle$ and $|d\rangle$ for $|c + q(1)4^2(2 + 1)^3 + r(1)(2 + 1)^2\rangle$ and $|d + a_{1,\, q(1)+1}\, x_{1,\, r(1)}\rangle$, respectively, from $q(1)$ and $r(1)$. Similarly, (C_2) changes $|c\rangle$ and $|d\rangle$ for $|c + q(2)4^1(2 + 1)^3 + r(2)(2 + 1)^1\rangle$ and $|d + a_{2,\, q(2)+1}\, x_{2,\, r(2)}\rangle$, respectively, from $q(2)$ and $r(2)$. (C_3) changes $|c\rangle$ and $|d\rangle$ for $|c + q(3)4^0(2 + 1)^3 + r(3)(2 + 1)^0\rangle$ and $|d + a_{3,\, q(3)+1}\, x_{3,\, r(3)}\rangle$, respectively, from $q(3)$ and $r(3)$. Therefore, $|c\rangle$ and $|d\rangle$ become $|U(V) = q(1)4^2{\cdot}3^3 + r(1)3^2 + q(2)4{\cdot}3^3 + r(2)3 + q(3)3^3 + r(3)\rangle$ and $|\Sigma_{\,i = 1\to 3}\, a_{i,\, q(i)+1}\, x_{i,\, r(i)}\rangle$, respectively.

Step 5: (D_1) changes $|e\rangle$ for $|1\rangle$ in $U(V = 0) = 0 \le c \le U(V = (1728/4) - 2 = 430) = 430$ or $M_1 = 50 \le d \le b = 58$, or it changes $|e\rangle$ for $|0\rangle$ in the others of c and d. As the target state for $|e\rangle$ is 1, (*PI*) and (*IM*) act on $|e\rangle$. The number of the data that is included in $0 \le c \le 430$ or $50 \le d \le 58$ is $W_1 \approx 1728/4 = 432$. When T_1 is the minimum even integer that is $(1728/\,(1728/4))^{1/2} = 2 \le 2 \approx T_1$, the total number that (*PI*) and (*IM*) act on $|e\rangle$ is $T_1 \approx 2$. Next, (*OB*) observes $|e\rangle$, and the data of W_1 remain. Similarly, (D_i) [$2 \le i \le 6 - 1 = 5$. i is the integer.] changes $|e\rangle$ for $|1\rangle$ in $0 \le c \le U(V = (W_0/4^i) - 2)$ or $50 \le d \le 58$, or it changes $|e\rangle$ for $|0\rangle$ in the others of c and d. As the target state for $|e\rangle$ is 1, (*PI*) and (*IM*) act on $|e\rangle$. The number of the data that is included in $0 \le c \le U(V = (W_0/4^i) - 2)$ or $50 \le d \le 58$ is $W_i \approx W_0/4^i$. When T_i is the minimum even integer that is $(W_{i-1}/W_i)^{1/2} \approx ((W_0/4^{i-1})/\,(W_0/4^i))^{1/2} = 2 \le 2 \approx T_i$, the total number that (*PI*) and (*IM*) act on $|e\rangle$ is $T_i \approx 2$. Next, (*OB*) observes $|e\rangle$, and the data of W_i remain. These actions are repeated sequentially from 2 to 5 at i. (D_6) changes $|e\rangle$ for $|1\rangle$ in $0 \le c \le U(V = (W_0/4^6) - 1) \approx 0$ or $50 \le d \le 58$, or it changes $|e\rangle$ for $|0\rangle$ in the others of c and d. As the target state for $|e\rangle$ is 1, (*PI*) and (*IM*) act on $|e\rangle$. The number of the data that is included in $0 \le c \le U(V = (W_0/4^6) - 1) \approx 0$ or $50 \le d \le 58$ is $W_6 \approx 1$. When T_6 is the minimum even integer that is $(W_5/W_6)^{1/2} = 2 \le 2 \approx T_6$, the total number that (*PI*) and (*IM*) act on $|e\rangle$ is $T_6 \approx 2$. Next, (*OB*) observes $|q(1)\rangle$, $|q(2)\rangle$, $|q(3)\rangle$, $|r(1)\rangle$, $|r(2)\rangle$, $|r(3)\rangle$, $|s(1)\rangle$, $|s(2)\rangle$, $|s(3)\rangle$, $|t(1)\rangle$, $|t(2)\rangle$, $|t(3)\rangle$, $|c\rangle$, $|d\rangle$, and $|e\rangle$, and one of the data of W_6 remains. For example, when $q(1)$, $q(2)$, $q(3)$, $r(1)$, $r(2)$, $r(3)$, $s(1)$, $s(2)$, $s(3)$, $t(1)$, $t(2)$, $t(3)$, c, d, and e are 3, 3, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1699 [$= U(V = q(1)4^2(2 + 1)^3 + r(1)(2 + 1)^2 + q(2)4^1(2 + 1)^3 + r(2)(2 + 1)^1 + q(3)4^0(2 + 1)^3 + r(3)(2 + 1)^0 = 3{\cdot}4^2{\cdot}3^3 + 2{\cdot}3^2 + 3{\cdot}4{\cdot}3^3 + 2{\cdot}3^1 + 2{\cdot}3^3 + 1)$], 51 [$= a_{1,4}\, x_{1,2} + a_{2,4}\, x_{2,2} + a_{3,3}\, x_{3,1} = 10{\cdot}2 + 11{\cdot}2 + 9{\cdot}1$], and 1, respectively.

Step 6: In the example, the state of |*e*› is 1. Therefore, M_1 is assumed to be $M_2 = 54$ [> 51 > 50 = M_1], and these computations from step 1 to step 6 are repeated. It is assumed that the state of |*e*› is 1. When the state of |*e*› at $M_3 = 56$ is 1, and the state of |*e*› at $M_4 = 57$ is 0, the maximum weight M_{max} is 56 [= M_3]. Therefore, $q(1)$, $q(2)$, $q(3)$, $r(1)$, $r(2)$, $r(3)$, $s(1)$, $s(2)$, $s(3)$, $t(1)$, $t(2)$, $t(3)$, c, d, and e are 3, 3, 3, 1, 2, 2, 1, 1, 1, 1, 1, 1, 1718 [= $U(V = q(1)4^2(2 + 1)^3 + r(1)(2 + 1)^2 + q(2)4^1(2 + 1)^3 + r(2)(2 + 1)^1 + q(3)4^0(2 + 1)^3 + r(3)(2 + 1)^0 = 3\cdot4^2\cdot3^3 + 1\cdot3^2 + 3\cdot4^1\cdot3^3 + 2\cdot3^1 + 3\cdot4^0\cdot3^3 + 2\cdot3^0)$], 56 [= $a_{1,4}\, x_{1,1} + a_{2,4}\, x_{2,2} + a_{3,4}\, x_{3,2} = 10\cdot1 + 11\cdot2 + 12\cdot2$], and 1, respectively. As a result, $a_{1,4} = 10$, $a_{2,4} = 11$, $a_{3,4} = 12$, $x_{1,1} = 1$, $x_{2,2} = 2$, $x_{3,2} = 2$, and $M_{max} = 56$ are obtained.

5. Discussion and Summary

The computational complexity of this quantum algorithm [= S] becomes the following. In the order of the actions by the gates, the number of them is $(F + G)n$ at $\boxed{H}$, n at (A), n at (B), $(Q_1 + Q_2)n = 4n$ or $6n$ or $8n$ at (PI) and (IM), $2n$ at (OB), n at (C_i) [$1 \leq i \leq n$. i is an integer.], g at (D_i) [$1 \leq i \leq$ g. i is an integer.], $\Sigma_{\,i = 1 \to g}\, T_i \approx 2g$ at (PI) and (IM), and g at (OB). These processes repeated about $\log_2 (k(m + 1))^n$. Therefore, S becomes $((F + G + Q_1 + Q_2 + 5)n + 4g) \log_2 (k(m + 1))^n$. In the example of the numerical computation at section 4, S is 324. The computational complexity of the classical computation [= Z] is $(k(m + 1))^n = (4(2 + 1))^3 = 1728$. After all, S/Z becomes about 1/5. When n is large enough, S becomes about $5n^2(\log_2 (k(m + 1)))^2$, where F is about $\log_2 (4k)$, G is about $\log_2 (4(m + 1))$, g is about $\log_2 (k(m + 1))^n$, and S/Z is about $5n^2(\log_2 (k(m + 1)))^2/ (k(m + 1))^n$. For example, as for $n = 50$, $k = 4$, and $m = 2$, S/Z is about $1/10^{49}$. Therefore, a decreased process becomes possible.

I hope that this result will be confirmed by many experiments.

Appendix-1

It is assumed that the number of data is N, the value of data of $N/4$ is Y, and values of data of $3N/4$ are the others. When the probability amplitudes of data of Y are marked a minus, the mean of probability amplitudes becomes

$$(N^{-1/2}(3N/4) - N^{-1/2}(N/4))/N = (1/2)N^{-1/2}.$$

When the inversion about mean is practiced, the probability amplitudes of data of Y are

$-(-N^{-1/2}) + (1/2)N^{-1/2}\times 2 = 2N^{-1/2}$,

and the probability amplitude of data of others are

$N^{-1/2} - (N^{-1/2} - (1/2)N^{-1/2})\times 2 = 0$.

Therefore, the sum of square of probability amplitude is

$(2N^{-1/2})^2(1/4)N + 0^2(3/4)N = 1 + 0 = 1$.

After all, the data of $N/4$ of Y remain [3, 6, 7, 10].

When this process is repeated, the number of data decreases and the probability amplitudes of necessary data increase.

Appendix-2

It is assumed that the state of $s(i)$ is 1, and there is $\log_2(4k) \leq F$. [$\rightarrow\ 4k \approx 2^F$] When the probability amplitudes of state of 1 are marked a minus, the mean of probability amplitudes becomes

$((2^F)^{-1/2}(2^F - k) - (2^F)^{-1/2}k)/2^F = (1 - (2k/2^F))(2^F)^{-1/2} \approx (1/2)(4k)^{-1/2}$.

When the inversion about mean is practiced, the probability amplitudes of state of 1 are

$-(-(2^F)^{-1/2}) + (1 - (2k/2^F))(2^F)^{-1/2}\times 2 = (3 - (4k/2^F))(2^F)^{-1/2} \approx 2(4k)^{-1/2}$,

and the probability amplitude of state of 0 are

$(2^F)^{-1/2} - ((2^F)^{-1/2} - (1 - (2k/2^F))(2^F)^{-1/2})\times 2 = (1 - (4k/2^F))(2^F)^{-1/2} \approx 0$.

Therefore, the sum of square of probability amplitude is

$((3 - (4k/2^F))(2^F)^{-1/2})^2 k + ((1 - (4k/2^F))(2^F)^{-1/2})^2(2^F - k) \approx 4(4k)^{-1}k + 0^2(4k - k) = 1$.

After all, the data of state of 1 remain [3, 6, 7, 10].

Similarly, this process is repeated for $t(i)$.

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