

# Quantum Algorithm for 3-SAT Problem of 9 Variables by Quantum Fourier Transform with Repeat Qubits, and Weight Changes on QCEngine

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## Abstract

A quantum algorithm for the 3-SAT problem of 9 variables by the quantum Fourier transform with the repeat qubits, and the weight changes on the QCEngine, and its example are reported. When there are 3 literals with 2 'OR's in each clause, a weight of  $r$ -th clause is  $m_r$  [ $m_r$  is a natural number.  $1 \leq r \leq (\text{number of clauses}) = d$ .], the  $r$ -th clause is  $C_{u,r}(x_1, x_2, x_3, \dots, x_n)$  [ $u$  is  $2^0x_1 + 2^1x_2 + 2^2x_3 + \dots + 2^{n-1}x_n$ .  $x_1, x_2, x_3, \dots$ , and  $x_n$  are the variables, and the repeat qubits.], and  $S(u)$  is  $\sum_{r=1 \rightarrow d} m_r \times C_{u,r}(x_1, x_2, x_3, \dots, x_n)$ ,  $\text{mod}(S(u)_{\max})$  of  $S(u)$  [ $S(u)_{\max}$  is the maximum value of  $S(u)$ .] is computed, next, for  $u$ , the quantum Fourier transform is done. In this time, there are 9 variables, the repeat qubits from 0 to 6, and changed weights. The complexity of this method is able to be several times.

**Keywords:** Quantum algorithm, 3-SAT problem, 9 variables, quantum Fourier transform, repeat qubits, weight changes, QCEngine.

**AMS subject classification:** Primary 81-08; Secondary 81-10, 68Q12.

## Introduction

The complexity of the 3-SAT problem had been discussed by Cook. [1] Quantum computer's example of the 3-SAT problem is reported by Johnston, Harrigan, and Gimeno-Segovia with the QCEngine (free on-line quantum computer simulator). [2] Fujimura discussed a quantum algorithm for the 3-SAT problem by the Shor's Fourier transform with the RAM on the QCEngine. [3] Still more, Fujimura discussed a quantum algorithm for the 3-SAT problem of 7, and 8 variables by the quantum Fourier transform with the repeat qubits on the QCEngine. [4]

According to my advanced study, when the quantum Fourier transform with the repeat qubits, and the weight changes on the QCEngine for the 3-SAT problem of 9 variables is able to be several times. This method is equal to the Shor's Fourier transform.

Therefore, the quantum algorithm for the 3-SAT problem of 9 variables is examined by the quantum Fourier transform with the repeat qubits, and the weight changes on the QCEngine, and its result are reported.

### 3-SAT Problem

In the 3-SAT problem, it is assumed that (i) each value of  $n$  variables becomes "TRUE", or "FALSE", " $\sim$ " is "NOT", " $\vee$ " is "OR", " $\&$ " is "AND", (ii) " $\vee$ ", " $\sim$ ", and 3 different variables are included in each parentheses (= clause) that are connected by " $\&$ ". If a value of logical formula by the literals, and the logical connectives is "TRUE", it is decided whether there is at least one combination of values of the variables or not. [1-4]

### Quantum Algorithm

The following conditions are assumed. (I) Each value of variables  $x_1, x_2, x_3, \dots$ , and  $x_n$  becomes "TRUE" [= 1], or "FALSE" [= 0]. " $\sim$ " is "NOT". " $\vee$ " is "OR". " $\&$ " is "AND". For example, it is assumed in this algorithm that (1  $\vee$  1  $\vee$  1), (1  $\vee$  1  $\vee$  0), and (1  $\vee$  0  $\vee$  0) become 1, and (0  $\vee$  0  $\vee$  0) becomes 0. (II) " $\vee$ ", " $\sim$ ", and 3 different variables in  $x_1, x_2, x_3, \dots$ , and  $x_n$  are included in each clause, and then the clauses are connected by " $\&$ ". In these conditions, if a value of logical formula by the literals, and the operators is "TRUE", it is searched whether there is at least one combination of values of the variables or not. It is assumed that  $n$  is number of qubits,  $u$  is  $2^0x_1 + 2^1x_2 + 2^2x_3 + \dots + 2^{n-1}x_n$ , a weight of  $r$ -th clause is  $m_r$  [ $m_r$  is a natural number.  $1 \leq r \leq$  (number of clauses) =  $d$ .], the  $r$ -th clause is  $C_{u,r}(x_1, x_2, x_3, \dots, x_n)$ , and  $S(u)$  is  $\sum_{r=1}^d m_r \times C_{u,r}(x_1, x_2, x_3, \dots, x_n)$ , and  $S(u)_{\max}$  is [the maximum value of  $S(u)$ ] =  $k$ .

First of all, query quantum registers  $|x_i\rangle$  [ $1 \leq i \leq n$ .  $i$  is an integer.  $n$  is the number of variables in the logical formula, and the repeat qubits.], and work1 quantum registers  $|w_{1,j}\rangle$  [ $1 \leq j \leq t$ .  $j$ , and  $t$  are integers.  $t$  is a necessary number for  $S(u)_{\max} \leq 2^t$ .].

Step 1: The  $r$  data are introduced to the RAM [2].

Step 2: Each qubit of  $|x_i\rangle$ , and  $|w_{1,j}\rangle$  is set  $|0\rangle$ .

Step 3: The Hadamard gate  $\boxed{H}$  [2-9] acts on each qubit of  $|x_i\rangle$ . It changes them for entangled states.

Step 4: Each clause is presented by  $|x_i\rangle$ ,  $|w_{1,j}\rangle$ , add gate, and quantum operators. For  $|x_i\rangle$ , RAM[ $r - 1$ ] [RAM has  $m_r$  data of  $r = 1 \rightarrow d$ .] is incremented in  $|w_{1,j}\rangle$ . In a function,  $S(u) = \sum_{r=1}^d m_r \times C_{u,r}(x_1, x_2, x_3, \dots, x_n)$  is computed. This operation makes entangled data base. In this case,  $n$  is 9 variables in the logical formula, and the number of repeat qubits.

Step 5: For  $|x_i\rangle$ , the quantum Fourier transform (= QFT) [2-7] is done.

Step 6: For  $|x_i\rangle$ , and  $|w_{1,j}\rangle$ , the probes are done.

Step 7: For  $|x_i\rangle$ , the read is done.

Step 8: A number of spikes is estimated by the function (<https://oreilly-qc.github.io/?p=12-4> [2]), where the function estimate\_num\_spikes (spike, range) [spike: read value, range:  $2^n$ ] is used.

Step 9: From candidates of the number of spikes, the repeat period P is obtained.

Step 10: From  $u = P = 2^0x_1 + 2^1x_2 + 2^2x_3 + \dots + 2^{n-1}x_n$ , when there are  $S(P)_{\max}$  is  $\sum_{r=1}^d m_r \times C_{P,r}(x_1, x_2, x_3, \dots, x_n) = k$ , it is the answer [one combination of (value of logical formula) = 1].

### Example of Numerical Computation

4-1. 9 Variables, 6 Repeat Qubits, and RAM = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]  
For example at  $n = 15$  [9 variables, and 6 repeat qubits], it is assumed that a logical formula :  $(x_3 \vee x_4 \vee x_5) \& (\sim x_1 \vee x_2 \vee x_3) \& (\sim x_3 \vee x_4 \vee x_5) \& (x_3 \vee \sim x_4 \vee x_5) \& (\sim x_2 \vee x_3 \vee \sim x_5) \& (\sim x_3 \vee \sim x_4 \vee x_5) \& (\sim x_3 \vee x_4 \vee \sim x_5) \& (x_3 \vee \sim x_4 \vee \sim x_5) \& (\sim x_3 \vee \sim x_4 \vee \sim x_5) \& (x_4 \vee \sim x_5 \vee \sim x_6) \& (\sim x_5 \vee x_6 \vee \sim x_7) \& (x_6 \vee x_7 \vee \sim x_8) \& (x_7 \vee x_8 \vee \sim x_9)$ , each value of  $x_{1-9}$  :  $x_1 = x_2 = x_3 = x_4 = x_6 = x_7 = x_8 = x_9 = 0$ ,  $x_5 = 1$ ,  $m_1 = 1$ ,  $m_2 = 2$ ,  $m_3 = 3$ ,  $m_4 = 4$ ,  $m_5 = 5$ ,  $m_6 = 6$ ,  $m_7 = 7$ ,  $m_8 = 8$ ,  $m_9 = 9$ ,  $m_{10} = 10$ ,  $m_{11} = 11$ ,  $m_{12} = 12$ ,  $m_{13} = 13$ ,  $t = 7$ , and  $k = (13 + 1)13/2 = 91$ .

An example of program on the QCEngine is the following.

```
10 var a = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]; // RAM_a
20 var query_qubits = 15;
30 var work1_qubits = 7;
40 qc.reset(query_qubits + work1_qubits);
50 var query = qint.new(query_qubits, 'query');
60 var work1 = qint.new(work1_qubits, 'work1');
70 qc.label('q'); // set query
80 query.write(0);
90 query.hadamard();
100 qc.label(' ');
110 qc.label('w1'); // set work1
120 work1.write(0);
130 qc.print(' RAM before increment : ' + a + '\n');
140 var query16 = 16;
150 var work1_0 = 0;
160 qc.label('increment');
170 qc.not(query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
180 work1.add(a[0],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
190 qc.not(query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
200 qc.not(query.bits(0x2)|query.bits(0x4));
210 work1.add(a[1],query.bits(0x1)|query.bits(0x2)|query.bits(0x4));
220 qc.not(query.bits(0x2)|query.bits(0x4));
230 qc.not(query.bits(0x8)|query.bits(0x10));
240 work1.add(a[2],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
250 qc.not(query.bits(0x8)|query.bits(0x10));
260 qc.not(query.bits(0x4)|query.bits(0x10));
270 work1.add(a[3],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
```

```
280 qc.not(query.bits(0x4)|query.bits(0x10));
290 qc.not(query.bits(0x4));
300 work1.add(a[4],query.bits(0x2)|query.bits(0x4)|query.bits(0x10));
310 qc.not(query.bits(0x4));
320 qc.not(query.bits(0x10));
330 work1.add(a[5],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
340 qc.not(query.bits(0x10));
350 qc.not(query.bits(0x8));
360 work1.add(a[6],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
370 qc.not(query.bits(0x8));
380 qc.not(query.bits(0x4));
390 work1.add(a[7],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
400 qc.not(query.bits(0x4));
410 work1.add(a[8],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
420 qc.not(query.bits(0x8));
430 work1.add(a[9],query.bits(0x8)|query.bits(0x10)|query.bits(0x20));
440 qc.not(query.bits(0x8));
450 qc.not(query.bits(0x20));
460 work1.add(a[10],query.bits(0x10)|query.bits(0x20)|query.bits(0x40));
470 qc.not(query.bits(0x20));
480 qc.not(query.bits(0x20)|query.bits(0x40));
490 work1.add(a[11],query.bits(0x20)|query.bits(0x40)|query.bits(0x80));
500 qc.not(query.bits(0x20)|query.bits(0x40));
510 qc.not(query.bits(0x40)|query.bits(0x80));
520 work1.add(a[12],query.bits(0x40)|query.bits(0x80)|query.bits(0x100));
530 qc.not(query.bits(0x40)|query.bits(0x80));

540 qc.label('QFT');
550 query.QFT();
560 var prob16 = 0;
570 prob16 += query.peekProbability(query16);
580 // Print output query-Prob
590 qc.print(' Prob_query16: ' + prob16);
600 var prob0 = 0;
610 prob0 += work1.peekProbability(work1_0);
620 // Print output work1-Prob
630 qc.print(' Prob_work1_0: ' + prob0);
640 //read
650 qc.label('Rq');
660 var b2 = query.read();
670 // Print output result
680 qc.print(' Read query = ' + b2 + '.');
690 // end
```

When this program is copied on Programming Quantum Computers <https://oreilly-qc.github.io/#> [free on-line quantum computation simulator QCEngine] [2], you can run it. [Caution!: Please delate the line numbers.]

A result of this program is the following.

The probability probe value of  $|w_{1,j}\rangle = 0 : \approx 0.0019531 (\approx 1/512)$ .

The probability probe value of  $|x_i\rangle = 16 : \approx 0.0000000$ .

The example of 50 times test : The read value of  $|x_i\rangle$  ;  $R_q = 23552, 2560, 13312, 5120, 0, 7168, 960, 21248, 4096, 768, 27648, 28672, 31744, 31488, 1024, 10176, 1024, 8192, 1536, 1024, 1536, 1472, 31744, 7680, 0, 4096, 27648, 1024, 29696, 31744, 8192, 0, 6976, 0, 32384, 8832, 2048, 3840, 31360, 3072, 384, 27648, 25472, 22528, 27584, 0, 12288, 18432, 11264, 15424$ . (=spike)

The candidates of number of spikes are estimated by the function [the function estimate\_num\_spikes (spike, range) [spike : read value, range :  $2^n = 2^{15} = 32768$ ]] :  $R_q \rightarrow$  candidates ;  $23552 \rightarrow 4, 7, 14, 18, \dots$  ;  $2560 \rightarrow 13, 26, \dots$  ;  $13312 \rightarrow 3, 5, 10, 15, 17, \dots$  ;  $5120 \rightarrow 6, 13, 19, \dots$  ;  $0 \rightarrow$  nothingness ;  $7168 \rightarrow 5, 9, 18, \dots$  ;  $960 \rightarrow 34, \dots$  ;  $21248 \rightarrow 3, 6, 9, 11, 14, 17, \dots$  ;  $4096 \rightarrow 8, 16, \dots$  ;  $768 \rightarrow 43, \dots$  ;  $27648 \rightarrow 6, 13, 19, \dots$  ;  $28672 \rightarrow 8, 16, \dots$  ;  $31744 \rightarrow 32, \dots$  ;  $31488 \rightarrow 26, \dots$  ;  $1024 \rightarrow 32, \dots$  ;  $10176 \rightarrow 3, 6, 10, 13, 16, \dots$  ;  $8192 \rightarrow 4, 8, 12, 16, \dots$  ;  $1536 \rightarrow 21, \dots$  ;  $1472 \rightarrow 22, \dots$  ;  $7680 \rightarrow 4, 9, 13, 17, \dots$  ;  $27648 \rightarrow 6, 13, 19, \dots$  ;  $29696 \rightarrow 11, 21, \dots$  ;  $32384 \rightarrow 85, \dots$  ;  $6976 \rightarrow 5, 9, 14, 28, \dots$  ;  $8832 \rightarrow 4, 7, 11, 15, 26, \dots$  ;  $2048 \rightarrow 16, \dots$  ;  $3840 \rightarrow 9, 17, \dots$  ;  $31360 \rightarrow 23, \dots$  ;  $3072 \rightarrow 11, 21, \dots$  ;  $384 \rightarrow 85, \dots$  ;  $27648 \rightarrow 6, 13, 19, \dots$  ;  $25472 \rightarrow 5, 9, 18, \dots$  ;  $22528 \rightarrow 3, 6, 10, 13, 16, \dots$  ;  $27584 \rightarrow 6, 13, 19, \dots$  ;  $12288 \rightarrow 3, 5, 8, 16, \dots$  ;  $18432 \rightarrow 2, 5, 7, 9, 16, \dots$  ;  $11264 \rightarrow 3, 6, 9, 12, 15, 17, \dots$  ;  $15424 \rightarrow 2, 4, 6, 9, 11, 13, 15, 17, \dots$ .

When  $u$  is 16 ( $2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 + 2^8x_9 + 2^9x_{10} + 2^{10}x_{11} + 2^{11}x_{12} + 2^{12}x_{13} + 2^{13}x_{14} + 2^{14}x_{15} = 2^0 \times 0 + 2^1 \times 0 + 2^2 \times 0 + 2^3 \times 0 + 2^4 \times 1 + 2^5 \times 0 + 2^6 \times 0 + 2^7 \times 0 + 2^8 \times 0 + 2^9 \times 0 + 2^{10} \times 0 + 2^{11} \times 0 + 2^{12} \times 0 + 2^{13} \times 0 + 2^{14} \times 0 = 16$ ), the value of logical formula is 1. Therefore, it is the answer.

4-2. 9 Variables, 5 Repeat Qubits, and RAM = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]

For example at  $n = 14$  [9 variables, and 5 repeat qubits], it is assumed that the logical formula, each value of  $x_{1\sim 9}$ , each value of  $m_{1\sim 13}$ ,  $t$ , and  $k$  are same in the section 4-1.

A result of this problem is the following.

The probability probe value of  $|w_{1,j}\rangle = 0 : \approx 0.0019531 (\approx 1/512)$ .

The probability probe value of  $|x_i\rangle = 16 : \approx 0.0000000$ .

The example of 50 times test : The read value of  $|x_i\rangle$  ;  $R_q = 192, 13504, 256, 4000, 15392, 15872, 11264, 13824, 1024, 9728, 512, 1568, 13184, 8192, 0, 7680, 15872, 4352, 13056, 15232, 0, 14336, 1600, 640, 7424, 512, 512, 15872, 9248, 14848, 15872, 13824, 15936, 1280, 15360, 1024, 544, 8704, 11264, 14016, 0, 14848, 12800, 512, 15872, 14048, 2368, 15488, 1024, 14848$ . (=spike)

The candidates of number of spikes are estimated by the function [the function estimate\_num\_spikes (spike, range) [spike : read value, range :  $2^n = 2^{14} = 16384$ ]] :  $R_q \rightarrow$  candidates ;  $192 \rightarrow 85, \dots$  ;  $13504 \rightarrow 6, 11, 17, \dots$  ;  $256 \rightarrow 64, \dots$  ;  $4000 \rightarrow 4, 8, 12, 16, \dots$  ;  $15392 \rightarrow 17, \dots$  ;  $15872 \rightarrow 32, \dots$  ;  $11264 \rightarrow 3, 6, 10, 13, 16, \dots$  ;  $13824 \rightarrow 6, 13, 19, \dots$  ;  $1024 \rightarrow 16, \dots$  ;  $9728 \rightarrow 3, 5, 10, 15, 17, \dots$  ;  $512 \rightarrow 32, \dots$  ;  $1568$

→ 10, 21, ... ; 13184 → 5, 10, 15, 20, ... ; 8192 → 2, 4, 6, 8, 10, 12, 14, 16, ... ; 0 → nothingness ; 7680 → 2, 4, 6, 9, 11, 13, 15, 17, ... ; 4352 → 4, 8, 11, 15, 30, ... ; 13056 → 5, 10, 15, 20, ... ; 15232 → 14, 28, ... ; 14336 → 8, 16, ... ; 1600 → 10, 20, ... ; 640 → 26, ... ; 7424 → 2, 4, 7, 9, 11, 22, ... ; 512 → 32, ... ; 9248 → 2, 5, 7, 14, 16, ... ; 14848 → 11, 21, ... ; 13824 → 6, 13, 19, ... ; 15936 → 37, ... ; 1280 → 13, 26, ... ; 15360 → 16, ... ; 544 → 30, ... ; 8704 → 2, 4, 6, 9, 11, 13, 15, 17, ... ; 14016 → 7, 14, 21, ... ; 12800 → 5, 9, 18, ... ; 14048 → 7, 14, 21, ... ; 2368 → 7, 14, 21, ... ; 15488 → 18, ... .

When  $u$  is 16 ( $2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 + 2^8x_9 + 2^9x_{10} + 2^{10}x_{11} + 2^{11}x_{12} + 2^{12}x_{13} + 2^{13}x_{14} = 2^0 \times 0 + 2^1 \times 0 + 2^2 \times 0 + 2^3 \times 0 + 2^4 \times 1 + 2^5 \times 0 + 2^6 \times 0 + 2^7 \times 0 + 2^8 \times 0 + 2^9 \times 0 + 2^{10} \times 0 + 2^{11} \times 0 + 2^{12} \times 0 + 2^{13} \times 0 = 16$ ), the value of logical formula is 1. Therefore, it is the answer.

4-3. 9 Variables, 4 Repeat Qubits, and RAM = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]  
For example at  $n = 13$  [9 variables, and 4 repeat qubits], it is assumed that the logical formula, each value of  $x_{1-9}$ , each value of  $m_{1-13}$ ,  $t$ , and  $k$  are same in the section 4-1.

A result of this problem is the following.

The probability probe value of  $|w_{1,j}\rangle = 0 : \approx 0.0019531 (\approx 1/512)$ .

The probability probe value of  $|x_i\rangle = 16 : \approx 0.0027948$ .

The example of 50 times test : The read value of  $|x_i\rangle ; R_q = 3840, 7168, 0, 7936, 6208, 7680, 0, 64, 208, 7424, 64, 7936, 7680, 2624, 1920, 8128, 5376, 8000, 3584, 6080, 5632, 7104, 640, 6400, 7680, 0, 2176, 7936, 7104, 7168, 80, 8000, 3328, 8032, 8176, 7680, 1024, 7488, 8000, 0, 512, 0, 7840, 1600, 1600, 4864, 0, 576, 848, 7424$ . (=spike)

The candidates of number of spikes are estimated by the function [the function estimate\_num\_spikes (spike, range) [spike : read value, range :  $2^n = 2^{13} = 8192$ ]] :  $R_q \rightarrow$  candidates ; 3840 → 2, 4, 6, 9, 11, 13, 15, 17, ... ; 7168 → 8, 16, ... ; 0 → nothingness ; 7936 → 32, ... ; 6208 → 4, 8, 12, 17, ... ; 7680 → 16, ... ; 64 → 128, ... ; 208 → 39, ... ; 7424 → 11, 21, ... ; 2624 → 3, 6, 9, 13, 16, ... ; 1920 → 4, 9, 13, 17, ... ; 8128 → 128, ... ; 5376 → 3, 6, 9, 12, 15, 17, ... ; 8000 → 43, ... ; 3584 → 2, 5, 7, 9, 16, ... ; 6080 → 4, 8, 12, 16, ... ; 5632 → 3, 6, 10, 13, 16, ... ; 7104 → 8, 15, 30, ... ; 640 → 13, 26, ... ; 6400 → 5, 9, 18, ... ; 2176 → 4, 8, 11, 15, 30, ... ; 7936 → 32, ... ; 80 → 102 ; 8000 → 43, ... ; 3328 → 3, 5, 10, 15, 17, ... ; 8032 → 51, ... ; 8176 → 512, ... ; 1024 → 8, 16, ... ; 7488 → 12, 23, ... ; 516 → 16, ... ; 7840 → 23, ... ; 1600 → 5, 10, 15, 20, ... ; 4864 → 3, 5, 10, 15, 17, ... ; 576 → 14, 28, ... ; 848 → 10, 19, ... ; 7424 → 11, 21, ... .

When  $u$  is 16 ( $2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 + 2^8x_9 + 2^9x_{10} + 2^{10}x_{11} + 2^{11}x_{12} + 2^{12}x_{13} = 2^0 \times 0 + 2^1 \times 0 + 2^2 \times 0 + 2^3 \times 0 + 2^4 \times 1 + 2^5 \times 0 + 2^6 \times 0 + 2^7 \times 0 + 2^8 \times 0 + 2^9 \times 0 + 2^{10} \times 0 + 2^{11} \times 0 + 2^{12} \times 0 = 16$ ), the value of logical formula is 1. Therefore, it is the answer.

4-4. 9 Variables, 3 Repeat Qubits, and RAM = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]  
For example at  $n = 12$  [9 variables, and 3 repeat qubits], it is assumed that the logical formula, each value of  $x_{1-9}$ , each value of  $m_{1-13}$ ,  $t$ , and  $k$  are same in the section 4-1.

A result of this problem is the following.

The probability probe value of  $|w_{1,j}\rangle = 0 : \approx 0.0019531 (\approx 1/512)$ .

The probability probe value of  $|x_i\rangle = 16 : \approx 0.0016164$ .

The example of 50 times test : The read value of  $|x_i\rangle$  ;  $R_q = 272, 3992, 1920, 3376, 4064, 640, 3904, 2144, 4048, 336, 0, 2776, 480, 3904, 3896, 3840, 3328, 2688, 1280, 256, 3920, 3968, 2048, 4000, 480, 384, 616, 3712, 3008, 448, 3816, 2560, 128, 512, 2560, 1392, 3984, 4016, 1408, 1408, 3712, 2456, 1536, 3960, 128, 2432, 448, 112, 1824, 1472$ . (=spike)

The candidates of number of spikes are estimated by the function [the function estimate\_num\_spikes (spike, range) [spike : read value, range :  $2^n = 2^{12} = 4096$ ]] :  $R_q \rightarrow$  candidates ;  $272 \rightarrow 15, 30, \dots$  ;  $3992 \rightarrow 39, \dots$  ;  $1920 \rightarrow 2, 4, 6, 9, 11, 13, 15, 17, \dots$  ;  $3376 \rightarrow 6, 11, 17, \dots$  ;  $4064 \rightarrow 128, \dots$  ;  $640 \rightarrow 6, 13, 19, \dots$  ;  $3904 \rightarrow 21, \dots$  ;  $2144 \rightarrow 2, 4, 6, 8, 11, 13, 15, 17, \dots$  ;  $4048 \rightarrow 85, \dots$  ;  $336 \rightarrow 12, 24, \dots$  ;  $0 \rightarrow$  nothingness ;  $2048 \rightarrow 2, 4, 6, 8, 10, 12, 14, 16, \dots$  ;  $4000 \rightarrow 43, \dots$  ;  $480 \rightarrow 9, 17, \dots$  ;  $384 \rightarrow 11, 21, \dots$  ;  $616 \rightarrow 7, 13, 20, \dots$  ;  $3712 \rightarrow 11, 21, \dots$  ;  $3008 \rightarrow 4, 8, 11, 15, 30, \dots$  ;  $448 \rightarrow 9, 18, \dots$  ;  $3816 \rightarrow 15, 29, \dots$  ;  $2560 \rightarrow 3, 5, 8, 16, \dots$  ;  $128 \rightarrow 32, \dots$  ;  $512 \rightarrow 8, 16, \dots$  ;  $1392 \rightarrow 3, 6, 9, 12, 15, 18, \dots$  ;  $3984 \rightarrow 37, \dots$  ;  $4016 \rightarrow 51, \dots$  ;  $1408 \rightarrow 3, 6, 9, 12, 15, 17, \dots$  ;  $3712 \rightarrow 11, 21, \dots$  ;  $2456 \rightarrow 3, 5, 10, 15, 20, \dots$  ;  $1536 \rightarrow 3, 5, 8, 16, \dots$  ;  $3960 \rightarrow 30, \dots$  ;  $2432 \rightarrow 3, 5, 10, 15, 17, \dots$  ;  $112 \rightarrow 37, \dots$  ;  $1824 \rightarrow 2, 5, 7, 9, 18, \dots$  ;  $1472 \rightarrow 3, 6, 8, 11, 14, 25, \dots$ .

When  $u$  is  $16 (2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 + 2^8x_9 + 2^9x_{10} + 2^{10}x_{11} + 2^{11}x_{12} = 2^0 \times 0 + 2^1 \times 0 + 2^2 \times 0 + 2^3 \times 0 + 2^4 \times 1 + 2^5 \times 0 + 2^6 \times 0 + 2^7 \times 0 + 2^8 \times 0 + 2^9 \times 0 + 2^{10} \times 0 + 2^{11} \times 0 = 16)$ , the value of logical formula is 1. Therefore, it is the answer.

4-5. 9 Variables, 2 Repeat Qubits, and RAM = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]

For example at  $n = 11$  [9 variables, and 2 repeat qubits], it is assumed that the logical formula, each value of  $x_{1\sim 9}$ , each value of  $m_{1\sim 13}$ ,  $t$ , and  $k$  are same in the section 4-1.

A result of this problem is the following.

The probability probe value of  $|w_{1,j}\rangle = 0 : \approx 0.0019531 (\approx 1/526)$ .

The probability probe value of  $|x_i\rangle = 16 : \approx 0.0059283$ .

The example of 50 times test : The read value of  $|x_i\rangle$  ;  $R_q = 1792, 1652, 1920, 192, 1840, 1424, 200, 400, 56, 64, 1736, 1856, 128, 1984, 312, 672, 992, 1800, 2016, 0, 32, 0, 2000, 700, 236, 1984, 0, 1856, 1536, 1984, 1536, 32, 1968, 192, 84, 1984, 0, 1344, 256, 1856, 272, 128, 1984, 768, 1984, 64, 2016, 64, 1600, 1888$ . (=spike)

The candidates of number of spikes are estimated by the function [the function estimate\_num\_spikes (spike, range) [spike : read value, range :  $2^n = 2^{11} = 2048$ ]] :  $R_q \rightarrow$  candidates ;  $1792 \rightarrow 8, 16, \dots$  ;  $1652 \rightarrow 5, 10, 16, \dots$  ;  $1920 \rightarrow 16, \dots$  ;  $192 \rightarrow 11, 21, \dots$  ;  $1840 \rightarrow 10, 20, \dots$  ;  $1424 \rightarrow 3, 7, 10, 13, 23, \dots$  ;  $200 \rightarrow 10, 20, \dots$  ;  $400 \rightarrow 5, 10, 15, 20, \dots$  ;  $56 \rightarrow 37, \dots$  ;  $64 \rightarrow 32, \dots$  ;  $1736 \rightarrow 7, 13, 26, \dots$  ;  $1856 \rightarrow 11, 21, \dots$  ;  $128 \rightarrow 16, \dots$  ;  $1984 \rightarrow 32, \dots$  ;  $312 \rightarrow 7, 13, 26, \dots$  ;  $672 \rightarrow 3, 6, 9, 12, 15, 18, \dots$  ;  $992 \rightarrow 2, 4, 6, 8, 10, 12, 14, 17, \dots$  ;  $1800 \rightarrow 8, 17, \dots$  ;  $2016 \rightarrow 64, \dots$  ;  $0 \rightarrow$  nothingness ;  $32 \rightarrow 64, \dots$  ;  $2000 \rightarrow 43, \dots$  ;  $700 \rightarrow 3, 6, 9, 12, 15, 18, \dots$  ;  $236 \rightarrow 9, 17, \dots$  ;  $1984 \rightarrow 32, \dots$  ;  $1536 \rightarrow 4, 8, 12, 16, \dots$  ;  $1968 \rightarrow 26, \dots$  ;  $192 \rightarrow 11, 21, \dots$  ;  $84 \rightarrow 24, \dots$  ;  $1344 \rightarrow 3, 6, 9, 12, 15, 17, \dots$  ;  $256 \rightarrow 8, 16, \dots$  ;  $272 \rightarrow 8, 15, 30, \dots$  ;

128 → 16, ... ; 768 → 3, 5, 8, 16, ... ; 2016 → 64, ... ; 64 → 32, ... ; 1600 → 5, 9, 18, ... ; 1888 → 13, 26, ... .

When  $u$  is 16 ( $2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 + 2^8x_9 + 2^9x_{10} + 2^{10}x_{11} = 2^0 \times 0 + 2^1 \times 0 + 2^2 \times 0 + 2^3 \times 0 + 2^4 \times 1 + 2^5 \times 0 + 2^6 \times 0 + 2^7 \times 0 + 2^8 \times 0 + 2^9 \times 0 + 2^{10} \times 0 = 16$ ), the value of logical formula is 1. Therefore, it is the answer.

4-6. 9 Variables, 1 Repeat Qubit, and RAM = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]

For example at  $n = 10$  [9 variables, and 1 repeat qubit], it is assumed that the logical formula, each value of  $x_{1\sim 9}$ , each value of  $m_{1\sim 13}$ ,  $t$ , and  $k$  are same in the section 4-1.

A result of this problem is the following.

The probability probe value of  $|w_{1,j}\rangle = 0 : \approx 0.0019531 (\approx 1/512)$ .

The probability probe value of  $|x_i\rangle = 16 : \approx 0.014241$ .

The example of 50 times test : The read value of  $|x_i\rangle ; R_q = 40, 160, 518, 32, 1012, 410, 468, 224, 928, 102, 216, 116, 16, 1008, 128, 576, 0, 998, 0, 984, 98, 928, 988, 992, 32, 64, 352, 200, 868, 606, 48, 784, 32, 0, 40, 632, 800, 800, 618, 248, 16, 168, 88, 12, 238, 368, 48, 344, 168, 976. (=spike)$

The candidates of number of spikes are estimated by the function [the function estimate\_num\_spikes (spike, range) [spike : read value, range :  $2^n = 2^{10} = 1024$ ]] :  $R_q \rightarrow$  candidates ; 40 → 26, ... ; 160 → 6, 13, 19, ... ; 518 → 2, 4, 6, 8, 10, 12, 14, 16, ... ; 32 → 32, ... ; 1012 → 85, ... ; 410 → 3, 5, 10, 15, 20, ... ; 468 → 2, 4, 7, 9, 11, 22, ... ; 224 → 5, 9, 18, ... ; 928 → 11, 21, ... ; 102 → 10, 20, ... ; 216 → 5, 10, 14, 19, ... ; 116 → 9, 18, ... ; 16 → 64, ... ; 1008 → 64, ... ; 128 → 8, 16, ... ; 576 → 2, 5, 7, 9, 16, ... ; 0 → nothingness ; 998 → 39, ... ; 984 → 26, ... ; 98 → 10, 21, ... ; 928 → 11, 21, ... ; 988 → 28, ... ; 992 → 32, ... ; 32 → 32, ... ; 64 → 16, ... ; 352 → 3, 6, 9, 12, 15, 17, ... ; 200 → 5, 10, 15, 20, ... ; 868 → 7, 13, 26, ... ; 606 → 3, 5, 10, 15, 17, ... ; 48 → 21, ... ; 784 → 4, 9, 13, 17, ... ; 40 → 26, ... ; 632 → 3, 5, 8, 13, 26, ... ; 800 → 5, 9, 18, ... ; 618 → 3, 5, 10, 15, 20, ... ; 248 → 4, 8, 12, 17, ... ; 16 → 64, ... ; 168 → 6, 12, 18, ... ; 88 → 12, 23, ... ; 12 → 85 ; 238 → 4, 9, 13, 26, ... ; 368 → 3, 6, 8, 11, 14, 25, ... ; 344 → 3, 6, 9, 12, 15, 18, ... .

When  $u$  is 16 ( $2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 + 2^8x_9 + 2^9x_{10} = 2^0 \times 0 + 2^1 \times 0 + 2^2 \times 0 + 2^3 \times 0 + 2^4 \times 1 + 2^5 \times 0 + 2^6 \times 0 + 2^7 \times 0 + 2^8 \times 0 + 2^9 \times 0 = 16$ ), the value of logical formula is 1. Therefore, it is the answer.

4-7. 9 Variables, 0 Repeat Qubit, and RAM = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]

For example at  $n = 9$  [9 variables, and 0 repeat qubit], it is assumed that the logical formula, each value of  $x_{1\sim 9}$ , each value of  $m_{1\sim 13}$ ,  $t$ , and  $k$  are same in the section 4-1.

A result of this problem is the following.

The probability probe value of  $|w_{1,j}\rangle = 0 : \approx 0.0019531 (\approx 1/512)$ .

The probability probe value of  $|x_i\rangle = 16 : \approx 0.046167$ .

The example of 50 times test : The read value of  $|x_i\rangle ; R_q = 481, 73, 64, 208, 112, 496, 176, 346, 496, 52, 16, 504, 16, 464, 16, 468, 8, 272, 371, 496, 326, 456, 16, 82, 336, 0, 509, 40, 94, 4, 496, 416, 496, 437, 446, 192, 24, 400, 96, 384, 26, 16, 79, 48, 504, 448, 16, 38, 496, 480. (=spike)$

The candidates of number of spikes are estimated by the function [the function estimate\_num\_spikes (spike, range) [spike : read value, range :  $2^n = 2^9 = 512$ ]] :  $R_q \rightarrow$

candidates ; 482  $\rightarrow$  17, ... ; 73  $\rightarrow$  7, 14, 21, ... ; 64  $\rightarrow$  8, 16, ... ; 208  $\rightarrow$  3, 5, 10, 15, 17, ... ; 112  $\rightarrow$  5, 9, 18, ... ; 496  $\rightarrow$  32, ... ; 176  $\rightarrow$  3, 6, 9, 12, 15, 17, ... ; 346  $\rightarrow$  3, 6, 9, 12, 15, 19, ... ; 52  $\rightarrow$  10, 20, ... ; 16  $\rightarrow$  32, ... ; 504  $\rightarrow$  64, ... ; 464  $\rightarrow$  11, 21, ... ; 468  $\rightarrow$  12, 23, ... ; 8  $\rightarrow$  64, ... ; 272  $\rightarrow$  2, 4, 6, 9, 11, 13, 15, 17, ... ; 371  $\rightarrow$  4, 7, 11, 18, ... ; 326  $\rightarrow$  3, 6, 8, 11, 22, ... ; 456  $\rightarrow$  9, 18, ... ; 82  $\rightarrow$  6, 13, 19, ... ; 336  $\rightarrow$  3, 6, 9, 12, 15, 17, ... ; 0  $\rightarrow$  nothingness ; 509  $\rightarrow$  171, ... ; 40  $\rightarrow$  13, 26, ... ; 94  $\rightarrow$  5, 11, 22, ... ; 4  $\rightarrow$  128, ... ; 416  $\rightarrow$  5, 11, 16, ... ; 437  $\rightarrow$  7, 14, 20, ... ; 446  $\rightarrow$  8, 16, ... ; 192  $\rightarrow$  3, 5, 8, 16, ... ; 24  $\rightarrow$  21, ... ; 400  $\rightarrow$  5, 9, 18, ... ; 96  $\rightarrow$  5, 11, 16, ... ; 384  $\rightarrow$  4, 8, 12, 16, ... ; 26  $\rightarrow$  20, ... ; 16  $\rightarrow$  32, ... ; 79  $\rightarrow$  7, 13, 26, ... ; 48  $\rightarrow$  11, 21, ... ; 504  $\rightarrow$  64, ... ; 448  $\rightarrow$  8, 16, ... ; 38  $\rightarrow$  13, 27, ... ; 480  $\rightarrow$  16, ... .

When  $u$  is 16 ( $2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 + 2^8x_9 = 2^0 \times 0 + 2^1 \times 0 + 2^2 \times 0 + 2^3 \times 0 + 2^4 \times 1 + 2^5 \times 0 + 2^6 \times 0 + 2^7 \times 0 + 2^8 \times 0 = 16$ ), the value of logical formula is 1. Therefore, it is the answer.

### Discussion

In the 3-SAT problem, when [(the logical formula) = 1] is obtained, there is only one combination.

5-1. 9 Variables, Repeat Qubits from 0 to 6, and RAM = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]

In the section 4, there is RAM = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. And then, [(the logical formula) = 1] of combination of variables is selected, and the computation of repeats is done. When  $N$  is  $2^9$ , in the Grover's method, the complexity is  $N^{1/2} = 2^{9/2} \approx 23$ , in the new method, for (variables, repeat qubits) = (9, 0), it is  $50/8 \approx 6$ , for (9, 1), it is  $50/4 \approx 13$ , for (9, 2), it is  $50/9 \approx 6$ , for (9, 3), it is  $50/10 = 5$ , for (9, 4), it is  $50/12 \approx 4$ , for (9, 5), it is  $50/10 = 5$ , and for (9, 6), it is  $50/10 = 5$ .

In this range, the new method is less than the complexity of the Grover's method, and then, in for (9, 4), the probability is the maximum value 24%.

5-2. 9 Variables, Repeat Qubits from 0 to 6, and RAM = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 1]

There are RAM = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 1],  $t = 7$ , and  $k = 79$ . And then, [(the logical formula) = 1] of combination of variables is selected, and the computation of repeats is done. In the new method, for (9, 0), it is  $50/8 \approx 6$ , for (9, 1), it is  $50/12 \approx 4$ , for (9, 2), it is  $50/8 \approx 6$ , for (9, 3), it is  $50/5 = 10$ , for (9, 4), it is  $50/10 = 5$ , for (9, 5), it is  $50/13 \approx 4$ , and for (9, 6), it is  $50/13 \approx 4$ .

In this range, the new method is less than the complexity of the Grover's method, and then, in for (9, 5), and (9, 6), the probability is the maximum value 26%.

5-3. 9 Variables, Repeat Qubits from 0 to 6, and RAM = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 1, 2]

There are RAM = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 1, 2],  $t = 7$ , and  $k = 69$ . And then, [(the logical formula) = 1] of combination of variables is selected, and the computation of repeats is done. In the new method, for (9, 0), it is  $50/5 = 10$ , for (9, 1), it is  $50/9 \approx 6$ , for (9, 2), it is  $50/10 = 5$ , for (9, 3), it is  $50/9 \approx 6$ , for (9, 4), it is  $50/6 \approx 8$ , for (9, 5), it is  $50/13 \approx 4$ , and for (9, 6), it is  $50/9 \approx 6$ .

In this range, the new method is less than the complexity of the Grover's method, and then, in for (9, 5), the probability is the maximum value 26%.

5-4. 9 Variables, Repeat Qubits from 0 to 6, and RAM = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 1, 2, 3]

There are RAM = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 1, 2, 3],  $t = 6$ , and  $k = 61$ . And then, [(the logical formula) = 1] of combination of variables is selected, and the computation of repeats is done. In the new method, for (9, 0), it is  $50/13 \approx 4$ , for (9, 1), it is  $50/6 \approx 8$ , for (9, 2), it is  $50/7 \approx 7$ , for (9, 3), it is  $50/6 \approx 8$ , for (9, 4), it is  $50/10 = 5$ , for (9, 5), it is  $50/14 \approx 4$ , and for (9, 6), it is  $50/14 \approx 4$ .

In this range, the new method is less than the complexity of the Grover's method, and then, in for (9, 5), and (9, 6), the probability is the maximum value 28%.

5-5. 9 Variables, Repeat Qubits from 0 to 6, and RAM = [1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 2, 3, 4]

There are RAM = [1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 2, 3, 4],  $t = 6$ , and  $k = 55$ . And then, [(the logical formula) = 1] of combination of variables is selected, and the computation of repeats is done. In the new method, for (9, 0), it is  $50/8 \approx 6$ , for (9, 1), it is  $50/7 \approx 7$ , for (9, 2), it is  $50/10 = 5$ , for (9, 3), it is  $50/10 = 5$ , for (9, 4), it is  $50/13 \approx 4$ , for (9, 5), it is  $50/8 \approx 6$ , and for (9, 6), it is  $50/8 \approx 6$ .

In this range, the new method is less than the complexity of the Grover's method, and then, in for (9, 4), the probability is the maximum value 26%.

5-6. 9 Variables, Repeat Qubits from 0 to 6, and RAM = [1, 2, 3, 4, 5, 6, 7, 8, 1, 2, 3, 4, 5]

There are RAM = [1, 2, 3, 4, 5, 6, 7, 8, 1, 2, 3, 4, 5],  $t = 6$ , and  $k = 51$ . And then, [(the logical formula) = 1] of combination of variables is selected, and the computation of repeats is done. In the new method, for (9, 0), it is  $50/10 = 5$ , for (9, 1), it is  $50/8 \approx 6$ , for (9, 2), it is  $50/4 \approx 13$ , for (9, 3), it is  $50/7 \approx 7$ , for (9, 4), it is  $50/9 \approx 6$ , for (9, 5), it is  $50/9 \approx 6$ , and for (9, 6), it is  $50/6 \approx 8$ .

In this range, the new method is less than the complexity of the Grover's method, and then, in for (9, 0), the probability is the maximum value 20%.

5-7. 9 Variables, Repeat Qubits from 0 to 6, and RAM = [1, 2, 3, 4, 5, 6, 7, 1, 2, 3, 4, 5, 6]

There are RAM = [1, 2, 3, 4, 5, 6, 7, 1, 2, 3, 4, 5, 6],  $t = 6$ , and  $k = 49$ . And then, [(the logical formula) = 1] of combination of variables is selected, and the computation of repeats is done. In the new method, for (9, 0), it is  $50/10 = 5$ , for (9, 1), it is  $50/10 = 5$ , for (9, 2), it is  $50/11 \approx 5$ , for (9, 3), it is  $50/8 \approx 6$ , for (9, 4), it is  $50/12 \approx 4$ , for (9, 5), it is  $50/9 \approx 6$ , and for (9, 6), it is  $50/10 \approx 5$ .

In this range, the new method is less than the complexity of the Grover's method, and then, in for (9, 4), the probability is the maximum value 24%.

5-8. 9 Variables, Repeat Qubits from 0 to 6, and RAM = [1, 2, 3, 4, 5, 6, 1, 2, 3, 4, 5, 6, 1]

There are  $RAM = [1, 2, 3, 4, 5, 6, 1, 2, 3, 4, 5, 6, 1]$ ,  $t = 6$ , and  $k = 43$ . And then, [(the logical formula) = 1] of combination of variables is selected, and the computation of repeats is done. In the new method, for (9, 0), it is  $50/8 \approx 6$ , for (9, 1), it is  $50/7 \approx 7$ , for (9, 2), it is  $50/7 \approx 7$ , for (9, 3), it is  $50/5 = 10$ , for (9, 4), it is  $50/6 \approx 8$ , for (9, 5), it is  $50/11 \approx 5$ , and for (9, 6), it is  $50/10 = 5$ .

In this range, the new method is less than the complexity of the Grover's method, and then, in for (9, 5), the probability is the maximum value 22%.

5-9. 9 Variables, Repeat Qubits from 0 to 6, and  $RAM = [1, 2, 3, 4, 5, 1, 2, 3, 4, 5, 1, 2, 3]$

There are  $RAM = [1, 2, 3, 4, 5, 1, 2, 3, 4, 5, 1, 2, 3]$ ,  $t = 6$ , and  $k = 36$ . And then, [(the logical formula) = 1] of combination of variables is selected, and the computation of repeats is done. In the new method, for (9, 0), it is  $50/7 \approx 7$ , for (9, 1), it is  $50/7 \approx 7$ , for (9, 2), it is  $50/6 \approx 8$ , for (9, 3), it is  $50/7 \approx 7$ , for (9, 4), it is  $50/14 \approx 4$ , for (9, 5), it is  $50/8 \approx 6$ , and for (9, 6), it is  $50/8 \approx 6$ .

In this range, the new method is less than the complexity of the Grover's method, and then, in for (9, 4), the probability is the maximum value 28%.

5-10. 9 Variables, Repeat Qubits from 0 to 6, and  $RAM = [1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1]$

There are  $RAM = [1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1]$ ,  $t = 5$ , and  $k = 31$ . And then, [(the logical formula) = 1] of combination of variables is selected, and the computation of repeats is done. In the new method, for (9, 0), it is  $50/10 = 5$ , for (9, 1), it is  $50/10 = 5$ , for (9, 2), it is  $50/3 \approx 17$ , for (9, 3), it is  $50/11 \approx 5$ , for (9, 4), it is  $50/14 \approx 4$ , for (9, 5), it is  $50/9 \approx 6$ , and for (9, 6), it is  $50/10 = 5$ .

In this range, the new method is less than the complexity of the Grover's method, and then, in for (9, 4), the probability is the maximum value 28%.

5-11. 9 Variables, Repeat Qubits from 0 to 6, and  $RAM = [1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1]$

There are  $RAM = [1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1]$ ,  $t = 5$ , and  $k = 25$ . And then, [(the logical formula) = 1] of combination of variables is selected, and the computation of repeats is done. In the new method, for (9, 0), it is  $50/6 \approx 8$ , for (9, 1), it is  $50/5 = 10$ , for (9, 2), it is  $50/9 \approx 6$ , for (9, 3), it is  $50/4 \approx 13$ , for (9, 4), it is  $50/9 \approx 6$ , for (9, 5), it is  $50/5 = 10$ , and for (9, 6), it is  $50/7 \approx 7$ .

In this range, the new method is less than the complexity of the Grover's method, and then, in for (9, 2), and (9, 4), the probability is the maximum value 18%.

5-12. 9 Variables, Repeat Qubits from 0 to 6, and  $RAM = [1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1]$

There are  $RAM = [1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1]$ ,  $t = 5$ , and  $k = 19$ . And then, [(the logical formula) = 1] of combination of variables is selected, and the computation of repeats is done. In the new method, for (9, 0), it is  $50/9 \approx 6$ , for (9, 1), it is  $50/8 \approx 6$ , for (9, 2), it is  $50/12 \approx 4$ , for (9, 3), it is  $50/9 \approx 6$ , for (9, 4), it is  $50/5 = 10$ , for (9, 5), it is  $50/15 \approx 3$ , and for (9, 6), it is  $50/10 = 5$ .

In this range, the new method is less than the complexity of the Grover's method, and then, in for (9, 5), the probability is the maximum value 30%.

5-13. 9 Variables, Repeat Qubits from 0 to 6, and RAM = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

There are RAM = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1],  $t = 4$ , and  $k = 13$ . And then, [(the logical formula) = 1] of combination of variables is selected, and the computation of repeats is done. In the new method, for (9, 0), it is  $50/3 \approx 17$ , for (9, 1), it is  $50/7 \approx 7$ , for (9, 2), it is  $50/4 \approx 13$ , for (9, 3), it is  $50/8 \approx 6$ , for (9, 4), it is  $50/8 \approx 6$ , for (9, 5), it is  $50/5 = 10$ , and for (9, 6), it is  $50/7 \approx 7$ .

In this range, the new method is less than the complexity of the Grover's method, and then, in for (9, 3), and (9, 4), the probability is the maximum value 16%.

### Summary

The quantum algorithm for the 3-SAT problem of 9 variables by the quantum Fourier transform with the repeat qubits, and the weight changes on the QCEngine, and its example are reported.

The complexity of this method is several times, and then, in for (9, 5), and RAM = [1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1], the probability is the maximum value 30%.

I will apply this method for other problems.

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