Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 21, Number 4 (2025), pp. 1067-1078

http://www.ripublication.com/gjpam.htm

Coefficient Estimates for a New Subclass of Bi-Univalent Functions Defined by Differential Operators

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Abstract

This article proposes and studies a new operator based subclass of bi-univalent functions associated with the Sălăgean and Al–Oboudi differential operators. We obtain explicit upper bounds for the initial Taylor–Maclaurin coefficients. In addition, we establish an estimate for the Fekete–Szegö functional and derive a bound for the second Hankel determinant. Several consequences and special cases are included to relate our findings to earlier results in the literature.

Keywords : Analytic functions; bi-univalent functions; Sălăgean operator; Al–Oboudi operator; Fekete–Szegö inequality; Hankel determinant

2020 Mathematics Subject Classification. 30C45, 30C50.

1. INTRODUCTION

Let $\mathcal{A}$ denote the family of analytic functions in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$ of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n. \tag{1.1}$$

A function $f \in \mathcal{A}$ is called *univalent* in $\mathbb{U}$ if it is injective there, and the set of all such functions is denoted by $\mathcal{S}$.

Two fundamental subclasses of $\mathcal{S}$ are the families of starlike and convex functions of order $\varphi \in [0, 1)$, written as $\mathcal{S}^*(\varphi)$ and $\mathcal{C}(\varphi)$, respectively, and defined by

$$\mathcal{S}^*(\varphi) = \left\{ f \in \mathcal{S} : \Re\left(\frac{zf'(z)}{f(z)} \right) > \varphi,\ z \in \mathbb{U} \right\}$$

and

$$\mathcal{C}(\varphi) = \left\{ f \in \mathcal{S} : \Re\left(1 + \frac{zf''(z)}{f'(z)} \right) > \varphi,\ z \in \mathbb{U} \right\}.$$

When $\varphi = 0$, these reduce to the classical classes $\mathcal{S}^*$ and $\mathcal{C}$.

For $f \in \mathcal{S}$, the inverse mapping f^{-1} exists at least in a disk $D_{r_0} = \{\omega \in \mathbb{C} : |\omega| < r_0\}$ with $r_0(f) \geq \frac{1}{4}$, and it satisfies $f^{-1}(f(z)) = z$ together with $f(f^{-1}(\omega)) = \omega$. Moreover, f^{-1} admits the expansion

$$f^{-1}(\omega) = \omega - a_2\omega^2 + (2a_2^2 - a_3)\omega^3 - (5a_2^3 - 5a_2a_3 + a_4)\omega^4 + \cdots . \tag{1.2}$$

A function f is said to be *bi-univalent* in $\mathbb{U}$ if both f and f^{-1} are univalent in $\mathbb{U}$; the corresponding class is denoted by Σ. The coefficient problem for Σ is substantially more delicate than for $\mathcal{S}$, and in many situations only partial bounds are known.

The study of coefficient estimates for bi-univalent functions goes back to Lewin [1] and was further developed by Brannan and Taha [2]. After the work of Srivastava *et al.* [3], there has been renewed activity, and numerous subclasses of Σ have been proposed using operator methods and subordination ideas; see, for example, [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 15, 17, 18, 19, 20, 21].

In recent investigations, differential operators such as the Sălăgean operator and its generalizations have proved useful for constructing subclasses and obtaining coefficient bounds; see [17, 19, 16]. Guided by this viewpoint, we define a new operator-driven subclass of Σ and derive explicit estimates for the initial Taylor–Maclaurin coefficients.

Two further quantities frequently used in modern coefficient problems are the second Hankel determinant and the Fekete–Szegö functional. For $r \in \mathbb{N}$, Noonan and Thomas [25] introduced the Hankel determinant

$$\mathcal{H}_r(s) = \begin{vmatrix} b_s & b_{s+1} & \cdots & b_{s+r-1} \\ b_{s+1} & b_{s+2} & \cdots & b_{s+r} \\ \vdots & \vdots & \ddots & \vdots \\ b_{s+r-1} & b_{s+r} & \cdots & b_{s+2r-2} \end{vmatrix},$$

which has been used to capture structural information about analytic (and meromorphic) functions; see [26, 27, 28]. The systematic study of Hankel determinants for univalent functions was initiated by Pommerenke [29, 30], and many refinements for various subclasses were later obtained; see [33, 35, 34, 31, 32].

Another classical quantity is the Fekete–Szegö functional $|b_3 - \mu b_2^2|$ $(\mu \in \mathbb{R})$ introduced by Fekete and Szegö [36]; related sharp estimates can be found, for instance, in [37, 38]. More recently, both the Fekete–Szegö problem and Hankel determinant bounds have also been treated for bi-univalent subclasses involving operators and q-calculus; see [42, 43, 44, 45, 46, 47, 48].

Motivated by these developments, we obtain new coefficient bounds together with estimates for the Fekete–Szegö functional and the second Hankel determinant for an operator-defined subclass of bi-univalent functions.

2. DEFINITION OF THE NEW CLASS

Definition 2.1. A function $f \in \Sigma$ is said to belong to the class $\mathcal{BS}_\Sigma(\alpha, m)$ if

$$\Re\left(\frac{D^m f(z)}{z}\right) > \alpha \quad (z \in \mathbb{U}), \tag{2.1}$$

and

$$\Re\left(\frac{D^m f^{-1}(w)}{w}\right) > \alpha \quad (w \in \mathbb{U}), \tag{2.2}$$

where $0 \leq \alpha < 1$ and $m \in \mathbb{N}$.

Classes formulated through operator inequalities of the form (2.1)–(2.2) have been used repeatedly in the recent theory of bi-univalent functions; see, for instance, [12, 17, 19].

There exists a function p with $\Re p(z) > 0$ such that

$$p(z) = 1 + c_1 z + c_2 z^2 + c_3 z^3 + \cdots . \tag{2.3}$$

Similarly there exists q with $\Re q(w) > 0$ such that

$$q(w) = 1 + d_1 w + d_2 w^2 + d_3 w^3 + \cdots . \tag{2.4}$$

3. COEFFICIENT ESTIMATES

Theorem 3.1. *Let $f \in \mathcal{BS}_\Sigma(\alpha, m)$, where $0 \leq \alpha < 1$ and $m \in \mathbb{N}$. Then the initial coefficients satisfy*

$$|a_2| \leq \frac{2(1-\alpha)}{2^m}, \tag{3.1}$$

$$|a_3| \leq \frac{2(1-\alpha)}{3^m}, \tag{3.2}$$

$$|a_4| \leq \frac{2(1-\alpha)}{4^m}. \tag{3.3}$$

Proof. Since $f \in \mathcal{BS}_\Sigma(\alpha, m)$, by Definition 2.1 we have

$$\Re\left(\frac{D^m f(z)}{z}\right) > \alpha \qquad (z \in \mathbb{U}) \tag{3.4}$$

and

$$\Re\left(\frac{D^m g(w)}{w}\right) > \alpha \qquad (w \in \mathbb{U}), \tag{3.5}$$

where $g = f^{-1}$.

Hence there exist analytic functions p and q with $\Re p(z) > 0$ and $\Re q(w) > 0$ in $\mathbb{U}$ such that

$$\frac{D^m f(z)}{z} = \alpha + (1-\alpha)p(z), \qquad p(z) = 1 + c_1 z + c_2 z^2 + c_3 z^3 + \cdots, \tag{3.6}$$

and

$$\frac{D^m g(w)}{w} = \alpha + (1-\alpha)q(w), \qquad q(w) = 1 + d_1 w + d_2 w^2 + d_3 w^3 + \cdots. \tag{3.7}$$

Using the series expansion $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ and the Sălăgean operator (Definition **??**), we obtain

$$D^m f(z) = z + \sum_{n=2}^{\infty} n^m a_n z^n = z + 2^m a_2 z^2 + 3^m a_3 z^3 + 4^m a_4 z^4 + \cdots. \tag{3.8}$$

Dividing (3.8) by z yields

$$\frac{D^m f(z)}{z} = 1 + 2^m a_2 z + 3^m a_3 z^2 + 4^m a_4 z^3 + \cdots. \tag{3.9}$$

On the other hand, expanding the right-hand side of (3.6) gives

$$\begin{aligned} \alpha + (1-\alpha)p(z) &= \alpha + (1-\alpha)\big(1 + c_1 z + c_2 z^2 + c_3 z^3 + \cdots\big) \\ &= 1 + (1-\alpha)c_1 z + (1-\alpha)c_2 z^2 + (1-\alpha)c_3 z^3 + \cdots. \end{aligned} \tag{3.10}$$

Since (3.9) and (3.10) represent the same analytic function, equating coefficients of z, z^2, and z^3 gives

$$2^m a_2 = (1-\alpha)c_1. \tag{3.11}$$

$$3^m a_3 = (1-\alpha)c_2. \tag{3.12}$$

$$4^m a_4 = (1-\alpha)c_3. \tag{3.13}$$

Taking absolute values in (3.11)–(3.13) and applying Lemma **??**, we obtain the desired bounds.

The proof is complete. □

Corollary 3.2. *If we take $m = 0$ in Theorem 3.1 then it reduce to the known estimates for the classical bi-univalent class Σ. Consequently, our results recover those established earlier in [1, 2, 6].*

Remark 3.3. Bounds of the form (3.11)–(3.13) are consistent with many operator-defined subclasses of Σ; compare with [5, 12, 17, 19].

4. SECOND HANKEL DETERMINANT

Theorem 4.1. *Let $f \in \mathcal{BS}_\Sigma(\alpha, m)$. Then*

$$|a_2a_4 - a_3^2| \leq \frac{4(1-\alpha)^2}{2^m 4^m} + \frac{4(1-\alpha)^2}{3^{2m}}. \tag{4.1}$$

Proof. From the definition of the class $\mathcal{BS}_\Sigma(\alpha, m)$, there exist functions p and q with positive real part such that

$$\frac{D^m f(z)}{z} = \alpha + (1-\alpha)p(z), \quad \frac{D^m f^{-1}(w)}{w} = \alpha + (1-\alpha)q(w), \tag{4.2}$$

where

$$p(z) = 1 + c_1z + c_2z^2 + c_3z^3 + \cdots, \quad q(w) = 1 + d_1w + d_2w^2 + d_3w^3 + \cdots. \tag{4.3}$$

Equating coefficients of like powers of z, we obtain

$$2^m a_2 = (1-\alpha)c_1, \tag{4.4}$$

$$3^m a_3 = (1-\alpha)c_2, \tag{4.5}$$

$$4^m a_4 = (1-\alpha)c_3. \tag{4.6}$$

Using these relations, we write

$$a_2a_4 - a_3^2 = \frac{(1-\alpha)^2}{2^m 4^m}c_1c_3 - \frac{(1-\alpha)^2}{3^{2m}}c_2^2. \tag{4.7}$$

Taking absolute values and applying the triangle inequality yields

$$|a_2a_4 - a_3^2| \leq \frac{(1-\alpha)^2}{2^m 4^m}|c_1||c_3| + \frac{(1-\alpha)^2}{3^{2m}}|c_2|^2. \tag{4.8}$$

Since $\Re p(z) > 0$, Lemma 2.1 gives

$$|c_n| \leq 2 \quad (n \geq 1).$$

Therefore,

$$|a_2a_4 - a_3^2| \leq \frac{4(1-\alpha)^2}{2^m 4^m} + \frac{4(1-\alpha)^2}{3^{2m}}. \tag{4.9}$$

This completes the proof. □

Corollary 4.2. *If $m = 0$, then the bound in the above Theorem it reduces to the corresponding second Hankel determinant estimate for bi-starlike functions of order α(see [39, 40, 41, 43]).*

5. FEKETE–SZEGÖ FUNCTIONAL

We now consider the classical Fekete–Szegö functional

$$\Phi_\mu(f) := a_3 - \mu a_2^2, \qquad \mu \in \mathbb{R}, \tag{5.1}$$

which has become standard in contemporary bi-univalent studies; see, for example, [16, 20, 21].

Theorem 5.1. *Let $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ belong to the class $\mathcal{BS}_\Sigma(\alpha, m)$. Then for $\mu \in \mathbb{R}$,*

$$|a_3 - \mu a_2^2| \leq \frac{2(1-\alpha)}{3^m} \max\left\{1, \left|1 - \frac{2\mu(1-\alpha)}{2^m}\right|\right\}. \tag{5.2}$$

Proof. From Definition 3.1, there exist functions p and q with $\Re p(z) > 0$ and $\Re q(w) > 0$ such that

$$\frac{D^m f(z)}{z} = \alpha + (1-\alpha)p(z), \tag{5.3}$$

and

$$\frac{D^m f^{-1}(w)}{w} = \alpha + (1-\alpha)q(w). \tag{5.4}$$

Let

$$p(z) = 1 + c_1 z + c_2 z^2 + \cdots, \quad q(w) = 1 + d_1 w + d_2 w^2 + \cdots. \tag{5.5}$$

Comparing coefficients of like powers of z and w, we obtain

$$2^m a_2 = (1-\alpha)c_1, \tag{5.6}$$

$$3^m a_3 = (1-\alpha)c_2, \tag{5.7}$$

and

$$-2^m a_2 = (1-\alpha)d_1. \tag{5.8}$$

From the above relations, we have

$$a_3 - \mu a_2^2 = \frac{1-\alpha}{3^m}\left[c_2 - \frac{3^m \mu(1-\alpha)}{2^{2m}} c_1^2\right]. \tag{5.9}$$

Applying Lemma 2.1 and the classical estimate

$$|c_2 - \nu c_1^2| \leq 2\max\{1, |2\nu - 1|\},$$

we arrive at

$$|a_3 - \mu a_2^2| \leq \frac{2(1-\alpha)}{3^m} \max\left\{1, \left|1 - \frac{2\mu(1-\alpha)}{2^m}\right|\right\}. \tag{5.10}$$

This completes the proof. □

Corollary 5.2. *(see [36, 38, 43, 20]) For $\mu = 1$, the estimate in the above Theorem yields a bound for $|a_3 - a_2^2|$ for functions belonging to the class $\mathcal{BS}_\Sigma(\alpha, m)$ (see [36, 38, 43, 20]).*

6. EXTENSION USING THE AL–OBOUDI OPERATOR

Operator-based generalizations are widely used in recent bi-univalent studies; see [17, 19].

Definition 6.1. Let $\lambda \geq 0$. The Al–Oboudi differential operator D_λ^m is defined recursively by

$$D_\lambda^0 f(z) = f(z), \quad D_\lambda^m f(z) = (1-\lambda)D_\lambda^{m-1} f(z) + \lambda z (D_\lambda^{m-1} f(z))'. \tag{6.1}$$

If $f \in \mathcal{A}$ is given by (1.1), then

$$D_\lambda^m f(z) = z + \sum_{n=2}^{\infty} [1 + \lambda(n-1)]^m a_n z^n. \tag{6.2}$$

Definition 6.2. A function $f \in \Sigma$ is said to belong to the class $\mathcal{BS}_\Sigma^\lambda(\alpha, m)$ if

$$\Re\left(\frac{D_\lambda^m f(z)}{z}\right) > \alpha \quad (z \in \mathbb{U}), \tag{6.3}$$

and

$$\Re\left(\frac{D_\lambda^m f^{-1}(w)}{w}\right) > \alpha \quad (w \in \mathbb{U}), \tag{6.4}$$

where $0 \leq \alpha < 1$.

Theorem 6.3. *Let $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ belong to the class $\mathcal{BS}_\Sigma^\lambda(\alpha, m)$. Then*

$$|a_2| \leq \frac{2(1-\alpha)}{[1+\lambda]^m}. \tag{6.5}$$

Proof. Proceeding as in Theorem 4.1, there exists a function p with $\Re p(z) > 0$ such that

$$\frac{D_\lambda^m f(z)}{z} = \alpha + (1-\alpha)p(z). \tag{6.6}$$

Comparing coefficients of z, we obtain

$$[1+\lambda]^m a_2 = (1-\alpha)c_1. \tag{6.7}$$

Applying Lemma 2.1 yields

$$|a_2| \leq \frac{2(1-\alpha)}{[1+\lambda]^m}. \tag{6.8}$$

This completes the proof. □

Remark 6.4. For $\lambda = 1$, in the above theorem the Al–Oboudi operator reduces to the Sălăgean operator, and the above results coincide with those obtained in(see[36, 38, 43, 20])

7. CONCLUSION

An operator-defined subclass of bi-univalent functions associated with the Sălăgean operator has been introduced, and explicit bounds have been obtained for the initial coefficients. We also established estimates for the Fekete–Szegö functional and for the second Hankel determinant. Finally, an extension based on the Al–Oboudi operator was discussed. Possible continuations include higher-order Hankel determinants and related subclasses generated by other operator families.

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