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# Fixed Point Results For Generalized Nonexpansive Mappings in Banach Spaces

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**Abstract**

In this work, we introduce a $(\eta, \theta)$-condition for self-mappings on a nonempty closed convex subset of a Banach space. This condition extends several known nonexpansive-type mappings, including Suzuki's condition (C). We establish convergence results for iterative sequences associated with the mapping and obtain fixed point results under suitable assumptions. An example is provided to illustrate the applicability of the proposed condition. The results improve and extend several known results in the existing literature.

**Keywords:** Fixed point, nonexpansive mapping, condition(C)

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## 1. INTRODUCTION AND PRELIMINARIES

Let $\mathcal{B}$ be a uniformly convex Banach space and $\mathcal{M}$ a nonempty closed convex subset of $\mathcal{B}$. Let $\mathcal{G} : \mathcal{M} \to \mathcal{M}$ be a self-mapping. An element $q \in \mathcal{M}$ is called a fixed point of $\mathcal{G}$ if $\mathcal{G}q = q$, and the set of all fixed points of $\mathcal{G}$ is denoted by $F(\mathcal{G})$.

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Fixed-point theory plays an important role in the study of nonlinear problems in modern analysis and applied sciences. The Banach contraction principle [9], established in 1922, guarantees the existence and uniqueness of fixed points for contraction mappings in complete metric spaces, together with a constructive approximation via the Picard iterative process. Despite its simplicity and effectiveness, the Picard iteration is not applicable to broader classes of nonlinear mappings.

Among such mappings, nonexpansive mappings constitute an important and extensively studied class. A self-mapping $\mathcal{G} : \mathcal{M} \to \mathcal{M}$ is said to be nonexpansive if $\|\mathcal{G}\varkappa - \mathcal{G}\nu\| \leq \|\varkappa - \nu\|, \forall \varkappa, \nu \in \mathcal{M}$. In 1965, Browder [10], Gohde [19], and Kirk [20] established fixed-point results for nonexpansive mappings in Banach spaces. Subsequently, various generalizations of nonexpansive mappings have been introduced and investigated. In 2008, Suzuki [35] introduced condition (C), given by $\frac{1}{2}\|\varkappa - \mathcal{G}\varkappa\| \leq \|\varkappa - \nu\| \Rightarrow \|\mathcal{G}\varkappa - \mathcal{G}\nu\| \leq \|\varkappa - \nu\|, \forall \varkappa, \nu \in \mathcal{M}$. It is stronger than quasi-nonexpansiveness and weaker than nonexpansiveness, and established corresponding convergence and fixed-point results. In 2011, Falset et al. [17] introduced new classes of generalized nonexpansive mappings and investigated their fixed-point existence and asymptotic behavior. In the same year, Aoyama and Kohsaka [5] proposed the class of $\alpha$-nonexpansive mappings in normed spaces, demonstrating that this class properly extends the class of nonexpansive mappings. Later, Pant and Shukla [33] introduced generalized $\alpha$-nonexpansive mappings, which further generalize condition (C)-type mappings, and established several convergence results.

In recent years, the study of generalized nonexpansive mappings and their associated fixed-point results has attracted considerable attention due to their wide applicability in nonlinear functional analysis (see, e.g., [1], [2], [3], [4], [6], [11], [12], [13], [14], [15], [16], [17], [21], [22], [23], [24], [26], [27], [28], [29], [31], [32], [34], [35], [36], [38]).

In this paper, we introduce a $(\eta, \theta)$-type nonexpansive condition for self-mappings defined on a nonempty closed convex subset of a Banach space. This condition extends Suzuki's condition (C) and provides a more flexible framework for the analysis of fixed-point problems. The results presented in this paper establish existence and convergence theorems that extend several previously known results in the literature. The rest of the paper is organized as follows. Section 1 presents the necessary preliminaries and definitions. The main results are established in Section 2, followed by illustrative examples and concluding remarks. At the end, Section 3 provides the conclusions.

Next, we will recall some definitions that will be used in the main results.

**Definition 1.1.** [34] Let $\mathcal{M}$ be a nonempty subset of a Banach space $\mathcal{B}$. A mapping $\mathcal{G} : \mathcal{M} \to \mathcal{B}$ is said to be nonexpansive if $\|\mathcal{G}\varkappa - \mathcal{G}\nu\| \leq \|\varkappa - \nu\|$ for all $\varkappa, \nu \in \mathcal{M}$.

**Definition 1.2.** [34] Let $\mathcal{M}$ be a nonempty subset of a Banach space $\mathcal{B}$. A mapping $\mathcal{G} : \mathcal{M} \to \mathcal{B}$ is said to be quasi nonexpansive if $\|\mathcal{G}\varkappa - p\| \leq \|\varkappa - p\|$ for all $\varkappa, \nu \in \mathcal{M}$ and $p \in F(\mathcal{G})$ where $F(\mathcal{G})$ denotes the set of all fixed points of $\mathcal{G}$.

**Definition 1.3.** [34] Let $\mathcal{M}$ be a nonempty subset of a Banach space $\mathcal{B}$. A mapping $\mathcal{G} : \mathcal{M} \to \mathcal{B}$ is said to satisfy the condition (C) if $\frac{1}{2}\|\varkappa - \mathcal{G}\varkappa\| \leq \|\varkappa - \nu\| \Rightarrow \|\mathcal{G}\varkappa - \mathcal{G}\nu\| \leq \|\varkappa - \nu\|$ for all $\varkappa, \nu \in \mathcal{M}$.

**Definition 1.4.** [17] Let $\mathcal{M}$ be a nonempty subset of a Banach space $\mathcal{B}$, and $\lambda \in (0, 1)$. A mapping $\mathcal{G} : \mathcal{M} \to B$ is said to satisfy the condition $(C_\lambda)$ on $\mathcal{M}$ if $\lambda\|\varkappa - \mathcal{G}\varkappa\| \leq \|\varkappa - \nu\| \Rightarrow \|\mathcal{G}\varkappa - \mathcal{G}\nu\| \leq \|\varkappa - \nu\|$ for all $\varkappa, \nu \in \mathcal{M}$.

**Definition 1.5.** [17] If $\mathcal{M}$ is a closed convex and bounded subset of $\mathcal{B}$, and a self-mapping $\mathcal{G}$ on $\mathcal{M}$ is nonexpansive, then there exist a convergent sequence $\{\varkappa_n\}$ in $\mathcal{M}$ such that $\|\varkappa_n - \mathcal{G}\varkappa_n\| \to 0$. Such a sequence is called almost fixed point sequence for $\mathcal{G}$.

**Definition 1.6.** [30] A Banach space $\mathcal{B}$ said to satisfy Opial's condition if for any sequence $\{t_n\} \in \mathcal{B}, \{t_n\} \rightharpoonup a$, implies that

$$\liminf_{n\to\infty} \|t_n - a\| < \liminf_{n\to\infty} \|t_n - b\|, \tag{1}$$

for all $b \in \mathcal{B}\ with\ a \neq b.$

**Definition 1.7.** [17] Let $\mathcal{M}$ be a nonempty subset of a Banach space $\mathcal{B}$ and $\{\varkappa_n\}$ be a bounded sequence in $\mathcal{B}$. Then

1. An asymptotic radius of $\{\varkappa_n\}$ at $\varkappa$ is defined by

$$r(\varkappa, \{\varkappa_n\}) = \limsup_{n\to\infty} \|\varkappa_n - \varkappa\|;$$

2. An asymptotic radius of $\{\varkappa_n\}$ relative to $\mathcal{M}$ is defined by

$$r(\mathcal{M}, \{\varkappa_n\}) = \inf r\{(u, \{\varkappa_n\}) : u \in \mathcal{M}\};$$

3. An asymptotic center of $\{\varkappa_n\}$ with respect to $\mathcal{M}$ is defined by

$$A(\mathcal{M}, \{\varkappa_n\}) = r(\varkappa, \{\varkappa_n\}) = r(\mathcal{M}, \{u_n\}).$$

We note that $A(\mathcal{M}, \{\varkappa_n\})$ is nonempty and convex whenever $\mathcal{M}$ is weakly compact, if $\mathcal{B}$ is uniformly convex Banach space, then $A(\mathcal{M}, \{\varkappa_n\})$ has exactly one point.

## 2. MAIN RESULTS

In this section, we define the following class of mappings and demonstrate some results for fixed-point theorems, compare the proposed $(\eta, \theta)$-condition with existing conditions such as Suzuki's condition (C).

**Definition 2.1.** Let $\mathcal{M}$ be a nonempty subset of a Banach space $\mathcal{B}$. Let $\eta \in [0,1]$ satisfy $0 \leq \theta < \frac{1}{4}$ and $4\theta \leq \eta \leq 1$. A mapping $\mathcal{G} : \mathcal{M} \to \mathcal{M}$ is said to satisfy the $(\eta, \theta)$-condition if for all $\varkappa, \nu \in \mathcal{M}$,

$$\eta\|\varkappa - \mathcal{G}\varkappa\| \leq \|\varkappa - \nu\| + \theta\|\nu - \mathcal{G}\nu\|$$

implies

$$\|\mathcal{G}\varkappa - \mathcal{G}\nu\| \leq \frac{1-\eta+\theta}{1-\theta}\|\varkappa - \nu\| + \frac{\theta}{1-\theta}[\|\varkappa - \mathcal{G}\nu\| + \|\nu - \mathcal{G}\varkappa\|]$$

**Remark 2.1.** The class of mappings satisfying the $(\eta, \theta)$-condition properly extends the class of nonexpansive mappings. In particular, for $\eta = \theta = 0$, the above condition reduces to the classical nonexpansive condition. Moreover, Suzuki's condition (C) is also included as a special case under suitable choices of parameters. However, the converse does not hold in general.

**Example 2.1.** Let $\mathcal{G} : [0,3] \to \mathbb{R}$ be defined by

$$\mathcal{G}(\varkappa) = \begin{cases} 0, & \text{if } \varkappa \neq 3 \\ 2, & \text{if } \varkappa = 3. \end{cases}$$

Now, for $\varkappa = 1.8,\ \nu = 3,\ \frac{1}{2}\|\mathcal{G}\varkappa - \varkappa\| = 0.9 < 1.2$. But $\|\mathcal{G}\varkappa - \mathcal{G}\nu\| = 2 > 1.2 = \|\varkappa - \nu\|$. So, the condition (C) is not satisfied.

For $\varkappa \neq 3, \nu \neq 3$, obviously $\mathcal{G}$ satisfies $(\eta, \theta)-$ condition for $\eta = 0.8$ and $\theta = 0.2$.

For $\varkappa \neq 3,\ \nu = 3$ and $\eta = 0.8, \theta = 0.2$, we have

$$\|\mathcal{G}\varkappa - \mathcal{G}\nu\| = 2,$$

and

$$\begin{aligned}
&\left(\frac{(1-\eta)+\theta}{1-\theta}\right)\|\varkappa - \nu\| + \left(\frac{\theta}{1-\theta}\right)[\|\varkappa - \mathcal{G}\nu\| + \|\nu - \mathcal{G}\varkappa\|] \\
&= \frac{1}{2}\|\varkappa - 3\| + \frac{1}{4}[\|\varkappa - 2\| + 3] \\
&> 2 = \|\mathcal{G}\varkappa - \mathcal{G}\nu\|.
\end{aligned}$$

For $\varkappa = 3,\ \nu \neq 3$ and $\eta = 0.8$, $\theta = 0.2$, we have

$$\begin{aligned}
&\left(\frac{(1-\eta)+\theta}{1-\theta}\right)\|\varkappa-\nu\| + \left(\frac{\theta}{1-\theta}\right)[\|\varkappa-\mathcal{G}\nu\| + \|\nu-\mathcal{G}\varkappa\|] \\
&= \frac{1}{2}\|\nu-3\| + \frac{1}{4}[3+\|\nu-2\|] \\
&> 2 = \|\mathcal{G}\varkappa-\mathcal{G}\nu\|.
\end{aligned}$$

For $\varkappa = 3, \nu = 3$, obviously $\mathcal{G}$ satisfies $(\eta,\theta)-$condition for $\eta = 0.8$ and $\theta = 0.2$.

**Example 2.2.** Let $\mathcal{G} : [0,2] \to \mathbb{R}$ be defined by

$$\mathcal{G}(\varkappa) = \begin{cases} (1+\varkappa)\sin(\frac{\pi\varkappa}{4}), & \text{if } \varkappa \in [0,2) \\ 1, & \text{if } \varkappa = 2. \end{cases}$$

Now for $\varkappa = \frac{1}{2}$, $\nu = 0$, $\frac{1}{2}\|\mathcal{G}\varkappa - \varkappa\| = 0.037 < 0.5 = \|\varkappa-\nu\|$. But $\|\mathcal{G}\varkappa-\mathcal{G}\nu\| = 0.574 > 0.5 = \|\varkappa-\nu\|$. So, the condition (C) is not satisfied.

**Case 1.** If $\varkappa \neq 2, \nu \neq 2$, it is evident that $\mathcal{G}$ satisfies $(\eta,\theta)-$condition for $\eta = 0.8$ and $\theta = 0.2$.

**Case 2.** If $\varkappa = 0, \nu = 2$, for $\eta = 0.8$ and $\theta = 0.2$. Then $\|\mathcal{G}\varkappa-\mathcal{G}\nu\| = 1$ and

$$\begin{aligned}
&\left(\frac{1-\eta+\theta}{1-\theta}\right)\|\varkappa-\nu\| + \left(\frac{\theta}{1-\theta}\right)\|\varkappa-\mathcal{G}\varkappa\| \\
&\left(\frac{1-\eta+\theta}{1-\theta}\right)\|\varkappa-\nu\| + \left(\frac{\theta}{1-\theta}\right)[\|\varkappa-\mathcal{G}\nu\| + \|\nu-\mathcal{G}\varkappa\|] \\
&> 1 = \|\mathcal{G}\varkappa-\mathcal{G}\nu\|.
\end{aligned}$$

Hence $\mathcal{G}$ satisfy $(\eta,\theta)-$condition.

**Case 3.** If $\varkappa = 2, \nu = 1$, for $\eta = 0.8$ and $\theta = 0.2$. Then $\|\mathcal{G}\varkappa-\mathcal{G}\nu\| = 0.41$ and

$$\begin{aligned}
&\left(\frac{1-\eta+\theta}{1-\theta}\right)\|\varkappa-\nu\| + \left(\frac{\theta}{1-\theta}\right)[\|\varkappa-\mathcal{G}\nu\| + \|\nu-\mathcal{G}\varkappa\|] \\
&> 0.41 = \|\mathcal{G}\varkappa-\mathcal{G}\nu\|.
\end{aligned}$$

Hence $\mathcal{G}$ satisfy $(\eta,\theta)-$condition.

**Case 4.** If $\varkappa = 2, \nu = 2$, obviously $\mathcal{G}$ satisfy $(\eta,\theta)-$condition for $\eta = 0.8$ and $\theta = 0.2$.

The following lemma shows that $\mathcal{G}$ satisfying the $(\eta, \theta)-$condition is also quasi-nonexpansive.

**Lemma 2.1.** *Let $\mathcal{M}$ be a nonempty closed convex subset of Banach space $\mathcal{B}$, let $\mathcal{G} : \mathcal{M} \to \mathcal{M}$ satisfy the $(\eta, \theta)-$condition. If $q$ is a fixed point of $\mathcal{G}$ on $\mathcal{M}$, then for all $\varkappa \in \mathcal{M}$, the following inequality holds $\|q - \mathcal{G}\varkappa\| \leq \|q - \varkappa\|$.*

*Proof.* Since $q$ is a fixed point, we have $\mathcal{G}q = q$. Thus,

$$\eta\|q - \mathcal{G}q\| = 0 \leq \|q - \varkappa\| + \theta\|\varkappa - \mathcal{G}\varkappa\|.$$

Using the $(\eta, \theta)-$condition, we obtain

$$\begin{aligned} \|\mathcal{G}q - \mathcal{G}\varkappa\| &\leq \left(\frac{1 - \eta + \theta)}{1 - \theta}\right)\|q - \varkappa\| + \left(\frac{\theta}{1 - \theta}\right)(\|\varkappa - \mathcal{G}q\| + \|q - \mathcal{G}\varkappa\|) \\ &= \left(\frac{1 - \eta + \theta)}{1 - \theta}\right)\|q - \varkappa\| + \left(\frac{\theta}{1 - \theta}\right)(\|\varkappa - q\| + \|q - \mathcal{G}\varkappa\|) \end{aligned}$$

$$\|q - \mathcal{G}\varkappa\| \leq \left(\frac{1 - \eta + 2\theta}{1 - 2\theta}\right)\|q - \varkappa\| \leq \|q - \varkappa\|.$$

Since $\mathcal{G}q = q$, this reduces to Since $4\theta \leq \eta$, it follows that

$$\frac{1 - \eta + 2\theta}{1 - 2\theta} \leq 1.$$

Hence,

$$\|q - \mathcal{G}\varkappa\| \leq \|q - \varkappa\|.$$

This shows that $\mathcal{G}$ is quasi-nonexpansive.
However, the converse is not true; Lemma 2.1 does not hold in general. □

**Example 2.3.** Let $\mathcal{G} : [0, 3] \to [0, 3]$ be defined by

$$\mathcal{G}(\varkappa) = \begin{cases} 0, & \text{if } \varkappa \neq 3, \\ 2, & \text{if } \varkappa = 3. \end{cases}$$

Clearly, $\mathcal{G}(0) = 0$, so $0$ is a fixed point. We show that $\mathcal{G}$ does not satisfy the $(\eta, \theta)$-condition. Take $\varkappa = 3$ and $\nu = 2$. Then

$$\mathcal{G}(3) = 2, \quad \mathcal{G}(2) = 0.$$

First, check the condition:

$$\eta\|\varkappa - \mathcal{G}\varkappa\| = \eta|3 - 2| = \eta,$$

and

$$\|\varkappa - \nu\| + \theta\|\nu - \mathcal{G}\nu\| = |3 - 2| + \theta|2 - 0| = 1 + 2\theta.$$

Thus,

$$\eta \leq 1 + 2\theta,$$

which holds for all admissible $\eta, \theta$. Now,

$$\|\mathcal{G}\varkappa - \mathcal{G}\nu\| = |2 - 0| = 2.$$

On the other hand,

$$\begin{aligned}
&\left(\frac{1-\eta+\theta}{1-\theta}\right)\|\varkappa - \nu\| + \left(\frac{\theta}{1-\theta}\right)\|\varkappa - \mathcal{G}\varkappa\| \\
&= \left(\frac{1-\eta+\theta}{1-\theta}\right)(1) + \left(\frac{\theta}{1-\theta}\right)(1). \\
&= \frac{1-\eta+2\theta}{1-\theta}.
\end{aligned}$$

Since $4\theta \leq \eta$, we get

$$1 - \eta + 2\theta \leq 1 - 2\theta,$$

hence

$$\frac{1-\eta+2\theta}{1-\theta} < 2.$$

Therefore,

$$\|\mathcal{G}\varkappa - \mathcal{G}\nu\| = 2 > \frac{1-\eta+2\theta}{1-\theta}.$$

Hence, $\mathcal{G}$ does not satisfy the $(\eta, \theta)$-condition.

The general strategy is inspired by existing approaches in the literature (see Patir et al. [31]). However, the current proof is developed independently and involves significant modifications due to the generalized $(\eta, \theta)$-framework. Below are some fundamental properties of mappings that satisfy the $(\eta, \theta)-$condition.

**Proposition 2.1.** *Let $\mathcal{M}$ be a nonempty subset of Banach space $\mathcal{B}$. Let $\mathcal{G} : \mathcal{M} \to \mathcal{B}$ satisfy the $(\eta, \theta)-$condition on $\mathcal{M}$. Then, for all $\varkappa, \nu \in \mathcal{M}$ and for $m \in [0, 1]$,*

(i) $\|\mathcal{G}\varkappa - \mathcal{G}^2\varkappa\| \leq \|\varkappa - \mathcal{G}\varkappa\|$, if $\theta = \eta = 0$,

(ii) at least one of the following ((a) and (b)) holds:
(a) $\frac{m}{2}\|\varkappa - \mathcal{G}\varkappa\| \leq \|\varkappa - \nu\|$
(b) $\frac{m}{2}\|\varkappa - \mathcal{G}^2\varkappa\| \leq \|\mathcal{G}\varkappa - \nu\|$.
The condition (a) implies $\|\mathcal{G}\varkappa - \mathcal{G}\nu\| \leq \left(\frac{(1-\frac{m}{2})+\theta}{1-\theta}\right)\|\varkappa - \nu\| + \left(\frac{\theta}{1-\theta}\right)\|\varkappa - \mathcal{G}\varkappa\|$
The condition (b) implies $\|\mathcal{G}^2\varkappa - \mathcal{G}\nu\| \leq \left(\frac{(1-\frac{m}{2})+\theta}{1-\theta}\right)\|\varkappa - \nu\| + \left(\frac{\theta}{1-\theta}\right)\|\mathcal{G}\varkappa - \mathcal{G}^2\varkappa\|$.

(iii) $\|\nu - \mathcal{G}\nu\| \leq \big(\frac{(3-m)+\theta}{1-\theta}\big)\|\mathcal{G}\varkappa - \varkappa\| + \big(\frac{(1-\frac{m}{2})+\theta}{1-\theta}\big)\|\varkappa - \nu\|$
$+\big(\frac{\theta}{1-\theta}\big)\big[2\|\mathcal{G}\varkappa - \varkappa\| + \|\nu - \mathcal{G}\varkappa\| + \|\varkappa - \mathcal{G}\nu\| + 2\|\mathcal{G}\varkappa - \mathcal{G}^2\varkappa\|\big].$

*Proof.* (i) We have, for all $\varkappa \in M$

$$\|\varkappa - \mathcal{G}\varkappa\| \leq \|\varkappa - \mathcal{G}\varkappa\| + \theta\|\mathcal{G}\varkappa - \mathcal{G}^2\varkappa\|.$$

So, by the $(\eta, \theta)-$condition, replacing by $\nu$ by $\mathcal{G}\varkappa$

$$\begin{aligned}
\|\mathcal{G}\varkappa - \mathcal{G}^2\varkappa\| &\leq \left(\frac{1-\eta+\theta}{1-\theta}\right)\|\varkappa - \mathcal{G}\varkappa\| + \left(\frac{\theta}{1-\theta}\right)\|\varkappa - \mathcal{G}^2\varkappa\| \\
&\leq \left(\frac{1-\eta+\theta}{1-\theta}\right)\|\varkappa - \mathcal{G}\varkappa\| + \left(\frac{\theta}{1-\theta}\right)\Big[\|\varkappa - \mathcal{G}\varkappa\| \\
&\quad + \|\mathcal{G}\varkappa - \mathcal{G}^2\varkappa\|\Big] \\
&\leq \left(\frac{1-\eta+2\theta}{1-2\theta}\right)\|\varkappa - \mathcal{G}\varkappa\| \\
\Rightarrow \|\mathcal{G}\varkappa - \mathcal{G}^2\varkappa\| &\leq \left(\frac{1-\eta+2\theta}{1-2\theta}\right)\|\varkappa - \mathcal{G}\varkappa\| \leq \|\varkappa - \mathcal{G}\varkappa\|.
\end{aligned}$$

(ii) We assume on the contrary that $\frac{m}{2}\|\varkappa - \mathcal{G}\varkappa\| \geq \|\varkappa - \nu\|$ and $\frac{m}{2}\|\varkappa - \mathcal{G}^2\varkappa\| \geq \|\mathcal{G}\varkappa - \nu\|$ for some $\varkappa, \nu \in \mathcal{M}$.
Now,

$$\begin{aligned}
\|\varkappa - \mathcal{G}\varkappa\| &\leq \|\varkappa - \nu\| + \|\nu - \mathcal{G}\varkappa\| \\
&< \frac{m}{2}\|\varkappa - \mathcal{G}\varkappa\| + \frac{m}{2}\|\mathcal{G}\varkappa - \mathcal{G}^2\varkappa\| \\
&\leq \frac{m}{2}\|\varkappa - \mathcal{G}\varkappa\| + \frac{m}{2}\|\varkappa - \mathcal{G}^2\varkappa\| \ (by(i)) \\
&\leq \|\varkappa - \mathcal{G}\varkappa\| \ (since\ m \leq 1)
\end{aligned}$$

i.e.,

$$\|\varkappa - \mathcal{G}\varkappa\| < \|\varkappa - \mathcal{G}\varkappa\|,$$

which is impossible. So, at least one of (a) and (b) holds.

(iii) We assume that, $\|\varkappa - \mathcal{G}\nu\| \leq \|\varkappa - \mathcal{G}\varkappa\| + \|\mathcal{G}\varkappa - \mathcal{G}\nu\|$. If (ii)(a) holds, we have

$$\begin{aligned}
\|\varkappa - \mathcal{G}\nu\| &\leq \|\varkappa - \mathcal{G}\varkappa\| + \left(\frac{(1-\frac{m}{2})+\theta}{1-\theta}\right)\|\varkappa - \nu\| + \left(\frac{\theta}{1-\theta}\right)\|\varkappa - \mathcal{G}\varkappa\| \\
&\leq \left(\frac{(3-m)+\theta}{1-\theta}\right)\|\varkappa - \mathcal{G}\varkappa\| + \left(\frac{(1-\frac{m}{2})+\theta}{1-\theta}\right)\|\varkappa - \nu\| \\
&\quad + \left(\frac{\theta}{1-\theta}\right)\Big[2\|\varkappa - \mathcal{G}\varkappa\| + \|\varkappa - \mathcal{G}\nu\| + \|\nu - \mathcal{G}\varkappa\|\Big].
\end{aligned}$$

If (ii) (b) holds,

$$\begin{aligned}
\|\varkappa - \mathcal{G}\nu\| &\leq \|\varkappa - \mathcal{G}\varkappa\| + \|\mathcal{G}\varkappa - \mathcal{G}^2\varkappa\| + \|\mathcal{G}^2\varkappa - \mathcal{G}\nu\| \\
&\leq \|\varkappa - \mathcal{G}\varkappa\| + \left(\frac{(1-\frac{m}{2})+\theta}{1-\theta}\right)\|\mathcal{G}\varkappa - \varkappa\| + \left(\frac{\theta}{1-\theta}\right)\Big[\|\mathcal{G}\varkappa - \mathcal{G}^2\varkappa\|\Big] \\
&\quad + \left(\frac{(1-\frac{m}{2})+\theta}{1-\theta}\right)\|\mathcal{G}\varkappa - \nu\| + \left(\frac{\theta}{1-\theta}\right)\Big[\|\mathcal{G}\varkappa - \mathcal{G}^2\nu\|\Big] \\
&\leq \|\varkappa - \mathcal{G}\varkappa\| + \left(\frac{(1-\frac{m}{2})+\theta}{1-\theta}\right)\|\mathcal{G}\varkappa - \varkappa\| + \left(\frac{\theta}{1-\theta}\right)\Big[\|\mathcal{G}\varkappa - \mathcal{G}\varkappa\| \\
&\quad + \|\varkappa - \mathcal{G}^2\varkappa\|\Big] + \left(\frac{(1-\frac{m}{2})+\theta}{1-\theta}\right)\|\mathcal{G}\varkappa - \nu\| \\
&\quad + \left(\frac{\theta}{1-\theta}\right)\Big[\|\mathcal{G}\varkappa - \mathcal{G}^2\nu\| + \|\nu - \mathcal{G}^2\varkappa\|\Big] \\
&\leq \|\varkappa - \mathcal{G}\varkappa\| + \left(\frac{(1-\frac{m}{2})+\theta}{1-\theta}\right)\|\mathcal{G}\varkappa - \varkappa\| + \left(\frac{(1-\frac{m}{2})+\theta}{1-\theta}\right)\|\mathcal{G}\varkappa - \varkappa\| \\
&\quad + \left(\frac{(1-\frac{m}{2})+\theta}{1-\theta}\right)\|\varkappa - \nu\| + \left(\frac{\theta}{1-\theta}\right)\Big[\|\mathcal{G}\varkappa - \mathcal{G}\varkappa\| + \|\varkappa - \mathcal{G}^2\varkappa\| \\
&\quad + \|\mathcal{G}\varkappa - \mathcal{G}\nu\| + \|\nu - \mathcal{G}^2\varkappa\|\Big] \\
&\leq \left(\frac{(3-m)+\theta}{1-\theta}\right)\|\mathcal{G}\varkappa - \varkappa\| + \left(\frac{(1-\frac{m}{2})+\theta}{1-\theta}\right)\|\varkappa - \nu\| \\
&\quad + \left(\frac{\theta}{1-\theta}\right)\Big[2\|\mathcal{G}\varkappa - \varkappa\| + \|\nu - \mathcal{G}\varkappa\| + \|\varkappa - \mathcal{G}\nu\| + 2\|\mathcal{G}\varkappa - \mathcal{G}^2\varkappa\|\Big].
\end{aligned} \tag{2}$$

□

**Proposition 2.2.** *Let $\mathcal{M}$ be a nonempty, closed, convex, subset of a Banach space $\mathcal{B}$, and let $\mathcal{G} : \mathcal{M} \to \mathcal{M}$ be a mapping satisfying the $(\eta, \theta)$-condition. Let $\{\varkappa_n\}$ be the sequence generated by the Mann iteration [25]*

$$\varkappa_{n+1} = (1-\alpha)\varkappa_n + \alpha\mathcal{G}\varkappa_n, \quad n \geq 0, \tag{3}$$

*where $\alpha \in (0,1)$. Then $\{\varkappa_n\}$ is an almost fixed point sequence of $\mathcal{G}$, that is,*

$$\lim_{n\to\infty} \|\mathcal{G}\varkappa_n - \varkappa_n\| = 0.$$

*Proof.* Since $F(\mathcal{G}) \neq \emptyset$, let $q \in F(\mathcal{G})$. By Lemma 2.1, the mapping $\mathcal{G}$ is quasi-nonexpansive. Hence, From the definition of the Mann iteration, we have

$$\varkappa_{n+1} = (1-\alpha)\varkappa_n + \alpha\mathcal{G}\varkappa_n,$$

and consequently,

$$\varkappa_{n+2} = (1-\alpha)\varkappa_{n+1} + \alpha\mathcal{G}\varkappa_{n+1}.$$

Subtracting $\varkappa_{n+1}$ from both sides yields

$$\varkappa_{n+2} - \varkappa_{n+1} = \alpha(\mathcal{G}\varkappa_{n+1} - \varkappa_{n+1}).$$

Taking norms, we obtain

$$\|\varkappa_{n+2} - \varkappa_{n+1}\| = \alpha\|\mathcal{G}\varkappa_{n+1} - \varkappa_{n+1}\|.$$

Since $\mathcal{M}$ is bounded and $\{\varkappa_n\} \subset \mathcal{M}$, the sequence $\{\varkappa_n\}$ is bounded. Moreover, it is well known that for the Mann iteration (3) associated with a quasi-nonexpansive mapping, we have

$$\|\varkappa_{n+2} - \varkappa_{n+1}\| \to 0 \quad \text{as } n \to \infty.$$

Therefore,

$$\|\mathcal{G}\varkappa_{n+1} - \varkappa_{n+1}\| = \frac{1}{\alpha}\|\varkappa_{n+2} - \varkappa_{n+1}\| \to 0 \quad \text{as } n \to \infty.$$

Hence,

$$\lim_{n\to\infty} \|\mathcal{G}\varkappa_n - \varkappa_n\| = 0,$$

and thus $\{\varkappa_n\}$ is an almost fixed point sequence of $\mathcal{G}$. □

**Theorem 2.1.** *Let $\mathcal{M}$ be a compact subset of a Banach space $\mathcal{B}$. Let $\mathcal{G} : \mathcal{M} \to \mathcal{M}$ be a mapping satisfying the $(\eta, \theta)$-condition with $F(\mathcal{G}) \neq \emptyset$. For $\varkappa_0 \in \mathcal{M}$, let $\{\varkappa_n\}$ be the sequence defined in Proposition 2.2. Then $\{\varkappa_n\}$ converges strongly to a fixed point of $\mathcal{G}$.*

*Proof.* Since $\mathcal{M}$ is compact, there exists a subsequence $\{\varkappa_{n_i}\}$ of $\{\varkappa_n\}$ and $q \in \mathcal{M}$ such that $\{\varkappa_{n_i}\}$ converges $q$.
Now, by proposition 2.1 (ii), for $\eta = \frac{m}{2}, m \in [0,1]$.

$$\begin{aligned} &\eta\|\varkappa_{n_i} - \mathcal{G}\varkappa_{n_i}\| \le \|\varkappa_{n_i} - q\| \\ \Rightarrow\ &\eta\|\varkappa_{n_i} - \mathcal{G}\varkappa_{n_i}\| \le \|\varkappa_{n_i} - q\| + \theta\|q - \mathcal{G}q\|. \end{aligned}$$

So by the $(\eta, \theta)-$condition,

$$\begin{aligned} \|\mathcal{G}\varkappa_{n_i} - \mathcal{G}q\| &\le \left(\frac{(1-\eta)+\theta}{1-\theta}\right)\|\varkappa_{n_i} - q\| + \left(\frac{\theta}{1-\theta}\right)\|\varkappa_{n_i} - \mathcal{G}\varkappa_{n_i}\| \\ &\le \left(\frac{(1-\eta)+\theta}{1-\theta}\right)\|\varkappa_{n_i} - q\| + \left(\frac{\theta}{1-\theta}\right)\Big[\|\varkappa_{n_i} - \mathcal{G}q\| + \|q - \mathcal{G}\varkappa_{n_i}\|\Big]. \end{aligned} \tag{4}$$

Again

$$\begin{aligned}\|\varkappa_{n_i}-\mathcal{G}q\| &\le \|\varkappa_{n_i}-\mathcal{G}\varkappa_{n_i}\|+\|\mathcal{G}\varkappa_{n_i}-\mathcal{G}q\|\\ &\le \|\varkappa_{n_i}-\mathcal{G}\varkappa_{n_i}\|+\left(\frac{(1-\eta)+\theta}{1-\theta}\right)\|\varkappa_{n_i}-q\|+\left(\frac{\theta}{1-\theta}\right)\|\varkappa_{n_i}-\mathcal{G}\varkappa_{n_i}\|\\ &\le \|\varkappa_{n_i}-\mathcal{G}\varkappa_{n_i}\|+\left(\frac{(1-\eta)+\theta}{1-\theta}\right)\|\varkappa_{n_i}-q\|+\left(\frac{\theta}{1-\theta}\right)\Big[\|\varkappa_{n_i}-\mathcal{G}q\|\\ &\quad+\|q-\mathcal{G}\varkappa_{n_i}\|\Big]\\ &\le \|\varkappa_{n_i}-\mathcal{G}\varkappa_{n_i}\|+\left(\frac{(1-\eta)+\theta}{1-\theta}\right)\|\varkappa_{n_i}-q\|\\ &\quad+\left(\frac{\theta}{1-\theta}\right)\Big[\|\varkappa_{n_i}-\mathcal{G}q\|+\|q-\varkappa_{n_i}\|+\|\varkappa_{n_i}-\mathcal{G}\varkappa_{n_i}\|\Big].\end{aligned}$$

So, taking $n_i \to \infty$ and using proposition 2.2, we obtain $\|q-\mathcal{G}q\| = 0 \Rightarrow \mathcal{G}q = q$, shows that $q$ is a fixed point for $\mathcal{G}$.

Now, using Mann iteration (3)

$$\begin{aligned}\|\varkappa_{n+1}-q\| &\le \alpha\|\mathcal{G}\varkappa_n-q\|+(1-\alpha)\|\varkappa_n-q\|\\ &\le \|\varkappa_n-q\|+(1-\alpha)\|\varkappa_n-q\|\\ &\le \|\varkappa_n-q\|,\ \forall n\in\mathbb{N}\cup\{0\}.\end{aligned}$$

Thus $\{\|\varkappa_n-q\|\}$ is a non-increasing. Since $\|\varkappa_n-q\|\ge 0$, it follows that the sequence is bounded below and therefore convergent to some real, say $t$. Now

$$\begin{aligned}\|\varkappa_n-\mathcal{G}q\| \le \|\varkappa_n-\mathcal{G}\varkappa_n\|+\left(\frac{1-\eta+\theta}{1-\theta}\right)\|\varkappa_n-q\|\\ +\left(\frac{\theta}{1-\theta}\right)\Big[2\|\varkappa_n-\mathcal{G}q\|+\|\varkappa_n-\mathcal{G}\varkappa_n\|\Big].\end{aligned}$$

Taking limit as $n\to\infty$, we obtain $t \le \left(\frac{1-\eta+\theta}{1-\theta}\right)t+\left(\frac{2\theta}{1-\theta}\right)t \Rightarrow (\eta-4\theta)t\le 0$, which is possible only for $t=0$, since $4\theta\le\eta$. Hence, $\{\varkappa_n\}$ converges strongly to $q$. □

**Theorem 2.2.** *Let $\mathcal{M}$ be a weakly compact and convex subset of a uniformly convex Banach space $\mathcal{B}$. Let $\mathcal{G}$ be a self- mapping on $\mathcal{M}$ satisfying the $(\eta,\theta)-$condition. Then $\mathcal{G}$ has a fixed point.*

*Proof.* Consider the sequence $\{\varkappa_n\}$ in $\mathcal{M}$ as defined in proposition 2.2. Then $\limsup\|\mathcal{G}\varkappa_n-\varkappa_n\| = 0$. As in [17], let $\phi$ be a continuous convex function from $\mathcal{M}$ into $[0,\infty)$ defined by

$$\phi(\varkappa)=\limsup_{n\to\infty}\|\varkappa_n-\varkappa\|.$$

For all $\varkappa \in \mathcal{M}$. Again, since $\mathcal{M}$ is weakly compact and $\phi$ is weakly lower-continuous, there is $q \in \mathcal{M}$ such that

$$\phi(q) = \min\{\phi(\varkappa) : \varkappa \in \mathcal{M}\}.$$

Now, by proposition 2.1 (iii) (for $\eta = \frac{m}{2}$)

$$\begin{aligned}\|\varkappa_n - \mathcal{G}q\| \leq &\left(\frac{3-2\eta+\theta}{1-\theta}\right)\|\varkappa_n - \mathcal{G}\varkappa_n\| + \left(\frac{1-\eta+\theta}{1-\theta}\right)\|\varkappa_n - q\| \\ &+ \left(\frac{\theta}{1-\theta}\right)[2\|\mathcal{G}\varkappa_n - \varkappa_n\| + \|\varkappa_n - \mathcal{G}q\| + \|\varkappa_n - q\| \\ &+ \|\varkappa_n - \mathcal{G}\varkappa_n\| + 2\|\varkappa_n - \mathcal{G}\varkappa_n\|].\end{aligned}$$

So,

$$\begin{aligned}(1-2\theta)\limsup_{n\to\infty}\|\varkappa_n - \mathcal{G}q\| &\leq (1-\eta+2\theta)\limsup_{n\to\infty}\|\varkappa_n - q\| \\ \Rightarrow \limsup_{n\to\infty}\|\varkappa_n - \mathcal{G}q\| &\leq \left(\frac{1-\eta+2\theta}{1-2\theta}\right)\limsup_{n\to\infty}\|\varkappa_n - q\| \\ &\Rightarrow \phi(\mathcal{G}q) \leq \phi(q).\end{aligned}$$

Since $\phi(q)$ is the minimum, $\phi(\mathcal{G}q) = \phi(q)$. Now, if $\mathcal{G}q \neq q$, then as $\phi$ is strictly quasi-convex [17], we have $\phi(q) \leq \phi(\alpha\mathcal{G}q + (1-\alpha)q) < \max\{\phi(q), \phi(\mathcal{G}q)\} = \phi(q)$, which is a contradiction. Hence, $\mathcal{G}q = q$.

□

**Theorem 2.3.** *(Demiclosedness type). Let $\mathcal{M}$ be a weakly compact and convex subset of a Banach space $\mathcal{B}$ satisfying the Opial condition. Let $\mathcal{G} : \mathcal{M} \to \mathcal{M}$ be a mapping satisfying the $(\eta,\theta)-$condition. If $\{\varkappa_n\}$ is a sequence in $\mathcal{M}$ such that*

*(i)* $\{\varkappa_n\} \rightharpoonup q$ *(weakly),*
*(ii)* $\lim_{n\to\infty}\|\mathcal{G}\varkappa_n - \varkappa_n\| = 0$,

*then $q \in \mathcal{M}, \mathcal{G}q = q$.*

*Proof.* By the proposition 2.1 (ii), for $\eta = \frac{m}{2}, m \in [0,1]$.

$$\eta\|\varkappa_n - \mathcal{G}\varkappa_n\| \leq \|\varkappa_n - q\| \leq \|\varkappa_n - q\| + \theta\|q - \mathcal{G}q\|.$$

So by the $(\eta,\theta)-$condition,

$$\begin{aligned}\|\mathcal{G}\varkappa_n - \mathcal{G}q\| &\leq \left(\frac{1-\eta+\theta}{1-\theta}\right)\|\varkappa_n - q\| + \left(\frac{\theta}{1-\theta}\right)\|\varkappa_n - \mathcal{G}\varkappa_n\| \\ &\leq \left(\frac{1-\eta+\theta}{1-\theta}\right)\|\varkappa_n - q\| + \left(\frac{\theta}{1-\theta}\right)[\|\varkappa_n - \mathcal{G}q\| + \|q - \mathcal{G}\varkappa_n\|].\end{aligned} \tag{5}$$

Now,

$$\begin{aligned}
\|\varkappa_n - \mathcal{G}q\| &\leq \|\varkappa_n - \mathcal{G}\varkappa_n\| + \|\mathcal{G}\varkappa_n - \mathcal{G}q\| \\
&\leq \|\varkappa_n - \mathcal{G}\varkappa_n\| + \left(\frac{1-\eta+\theta}{1-\theta}\right)\|\varkappa_n - q\| \\
&\quad + \left(\frac{\theta}{1-\theta}\right)[\|\varkappa_n - \mathcal{G}q\| + \|q - \varkappa_n\| + \|\varkappa_n - \mathcal{G}\varkappa_n\|] \qquad (by (5)).
\end{aligned}$$

So, taking limit $n \to \infty$ and using (ii), we obtain

$$\begin{aligned}
\|\varkappa_n - \mathcal{G}q\| &\leq \left(\frac{1-\eta+\theta}{1-\theta}\right)\|\varkappa_n - q\| \\
&\leq \|\varkappa_n - q\|.
\end{aligned}$$

So,

$$\liminf_{n\to\infty} \|\varkappa_n - \mathcal{G}q\| \leq \liminf_{n\to\infty} \|\varkappa_n - q\|. \tag{6}$$

Let $\mathcal{G}q \neq q$. Since $\varkappa_n \rightharpoonup q$ (weakly), by the Opial condition, we have

$$\liminf_{n\to\infty} \|\varkappa_n - \mathcal{G}q\| < \liminf_{n\to\infty} \|\varkappa_n - q\|,$$

which is a contradiction (6). So, $\mathcal{G}q = q$. □

**Theorem 2.4.** *Let $\mathcal{M}$ be a weakly compact and convex subset of a uniformly convex Banach space $\mathcal{B}$. Let $\mathcal{G} : \mathcal{M} \to \mathcal{M}$ be a mapping satisfying the $(\eta, \theta)$-condition with $F(\mathcal{G}) \neq \emptyset$. Let $\{\varkappa_n\} \subset \mathcal{M}$ be the sequence defined in Proposition 2.2. Then $\{\varkappa_n\}$ converges weakly to a fixed point of $\mathcal{G}$.*

*Proof.* By Proposition 2.2, $\|\mathcal{G}\varkappa_n - \varkappa_n\| \to 0$ as $n \to \infty$. Since $\mathcal{M}$ is weakly compact, there exists a subsequence $\{\varkappa_{n_i}\}$ of $\{\varkappa_n\}$ and $q \in \mathcal{M}$ such that $\{\varkappa_{n_i}\}$ converges weakly to $q$. Now, by Theorem 2.3, $q$ is a fixed point of $\mathcal{G}$.
We assume that $\{\varkappa_n\}$ does not converges weakly to $q$. Then there is a subsequence $\{\varkappa_{n_k}\}$ of $\{\varkappa_n\}$ and $l \in \mathcal{M}$ such that $\{\varkappa_{n_k}\}$ converges weakly to $l$ and $l \neq q$. Again, $\mathcal{G}l = l$ by Theorem 2.3.
Now,

$$\begin{aligned}
\liminf_{n\to\infty} \|\varkappa_n - q\| &= \liminf_{n_i\to\infty} \|\varkappa_{n_i} - q\| \\
&< \liminf_{n_i\to\infty} \|\varkappa_{n_i} - l\| \\
&= \liminf_{n_k\to\infty} \|\varkappa_{n_k} - l\| \\
&< \liminf_{n_k\to\infty} \|\varkappa_{n_k} - l\| \\
&= \liminf_{n\to\infty} \|\varkappa_n - q\|.
\end{aligned}$$

Which is a contradiction. Hence, $\{\varkappa_n\}$ converges weakly to $q$. □

## 3. CONCLUSION

We have presented a $(\eta, \theta)$- condition for mappings in Banach spaces. This condition extends and unifies several existing nonexpansive-type conditions, including Suzuki's condition (C). We established convergence results for iterative sequences and proved fixed point results under the proposed framework. The proposed condition opens new directions for studying broader classes of generalized nonexpansive mappings.

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