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Behavior of a Decimal-Parity-Based $3n + 1$ Mapping

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Abstract

In this paper, we investigate and evaluate the Collatz conjecture, traditionally based on positive integers, under a suitable convergence condition in which the numbers converge towards one. In our derivations, we extend the $3n + 1$ problem to the decimal values via a scaling factor, for showing behaviour of the last decimal digit, either even or odd. Where the numbers diverges to infinity (∞). From which it follows that between zero and one, the sequence diverge such that its limit approaches infinity.

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1. INTRODUCTION

Many areas of mathematical physics, computer science, economics, and other disciplines have a long history of utilising the Collatz conjecture field. The literature that is currently available on number theory including the Collatz conjecture and its generalisations in a number of application-related branches of mathematics. This kind of Collatz conjecture is related to Chaos theory, which shows real-world application in weather, biology, and finance. The Collatz sequence behavior is useful especially in population models, economic fluctuations, and iterative decision systems. In computing, the type of Collatz iteration is applied: i) to test program efficiency, ii) study performance under irregular workloads iii) compare recursion vs iteration. On the other hand, the sequence gives important ideas in signal processing and pattern recognition. Also, our work on Collatz conjecture was given by Lothar Collatz, the exact date the first occurrence of the $3n + 1$ conjecture is unclear. L. Collatz reports in [CoI2] (1986) that he represented integer functions by graphs already in his student days from 1928 to 1933 [3, p.10]. It has been tested for all starting values up to 2^{68}, but no proof has been found. It is a notorious unsolved Problem in arithmetic. In the book of G. J. Wirsching [3, p.10]. The 3n+1 problem can be found in many places. It is shown in the book; see [7], the problem has been described in the article [2] and in C. S. Ogilvy's Book Tomorrow's Math [Ogi] (1972), p. 103; see[10]. It is recorded in R. K. Guy's problem book [4]; Guy also wrote a few articles regarding $3n + 1$ iterations; see[5, 6]. In addition, there are more than fifty research articles containing substantial results around the $3n + 1$ problem; see [3, p.10]. Further, consider the operation on positive integer n given by: if n is odd, multiply it by 3 and add 1; while if n is even, divide it by 2. The $3n + 1$ problem asks whether, starting from any positive integer n, by continuing this operation repeatedly, the result is expected to be 1. The set of natural numbers (starting with 1) is denoted by $N = \{1, 2, 3 \ldots\}$. In the book of G. J. Wirsching [3, p.10], the topic of interest here is the dynamical system on N which is generated by the $3n + 1$ function

$$T : N \to N = T(n) := \begin{cases} \frac{n}{2} & \text{if n is even} \\ 3n + 1 & \text{if n is odd} \end{cases} \tag{1}$$

In addition [3, p.10], there is no divergent $3n + 1$ trajectory conjecture, that is there is no $y \in N$ such that $\lim_{n\to\infty} f^n(y) = \infty$. Further [11], the behaviour of the trajectory is divergent as $\lim_{n\to\infty} T^n(n) = \infty$. Another generalization of the $3n + 1$ problem has been proposed by F. Mignosi[Mig] (1995), see also [Bra] (1995). The mapping gives it,

$$T_{\alpha,r} : N \to N = T_{\alpha,r}(n) := \begin{cases} \frac{n}{2} & \text{if n is even} \\ 3\alpha + r & \text{if n is odd} \end{cases} \tag{2}$$

where $\alpha, r \in R$ with $1 < \alpha < 2$. Initially, there does not appear to be homogeneous behavior for different values of n. For n belonging to the set of real numbers $(n \in R)$, the observed behavior differs significantly from the previously described case. Extensions and modifications of the Collatz conjecture have occurred in the following research papers [1, 9, 8, 12]. In our present study, we focus to extend the $3n + 1$ problem to the decimal values between zero and one via a scaling factor. A primary concern of this extension is determining the behavior when n is a small decimal number (0.01, 0.03, and 1.09—). Where the points diverge to infinity, we observe that the sequence diverges. Whose statement, namely, extension for the Collatz conjecture $3n + 1$, for

decimal numbers $n \in (0, 1)$, under the divergence extended rules, the sequence $f(n)$ diverges to infinity rather than converging to 1.
The following is the paper's outline: `Section 2` defines the extended Collatz conjecture, and `Section 3` presents a scaling factor including the universal scaling factor and a (+1) magnitude. `Sections (4-6)`: contain mathematical statements and formulas, asymptotic growth coefficient ω(omega), and divisibility by 2 on the Evolved Value. Further, `Sections 7` include asymptotic growth graph associated with iteration table and graphical representation via Wolfram Mathematica Software. Finally, `Sections 8-9` show logarithmic scale analysis and conclusion.

2. BEHAVIOR OF A DECIMAL-PARITY-BASED $3n + 1$ CONJECTURE

In this section, we extend the Collatz Conjecture to the decimal parity classification involving the odd rule and the even rule shows. This demonstrates that, If the last significant digit of the decimal is the odd, then we should apply $3n + 1$ rule. This last digit what we use to determine the parity for the rule. Parity is not lost at the decimal level, it is just shifted on the other side. Also, If the last significant digit of the decimals is even, divide n by 2. Which follows the Collatz rule for even number. Further, we give the following table, which verifies the classification of the above even and odd rules by providing specific values to justify both rules.
Example 1. In Table 1 we display the last significant digit of the decimal as the parity decider for the rule.

Decimal	Last Digit	Parity	Rule Applied
0.01	1	Odd	3n+1
0.02	2	Even	n/2
0.07	7	Odd	3n+1
0.14	4	Even	n/2
0.33	3	Odd	3n+1
0.50	0 or 5	Even	n/2

3. SCALING FACTOR

In this section, we give the universal scaling factor and +1 magnitude.
Universal Scale Invariance: Mathematics being scale-invariant, numbers shrink as in real mathematics, 'even and odd' are properties that apply only to Integers. Applying

a scaling factor of 10^2 to 0.01 yields the integer 1. Since 1 is fundamentally an odd integer, 0.01 inherits the odd identity under scaling. We call this principle decimal parity inheritance.
Example 1. In Table 2, it proves the following scaling factor and parity rule.

Unit	Scaling factor	Scaled Unit	Parity
0.01	$\times$ 100	1	Odd
0.02	$\times$ 100	2	Even
0.03	$\times$ 100	3	Odd
0.07	$\times$ 100	7	Odd
0.14	$\times$ 100	14	Even

The sequence crossing 1.0 at the first step and reaches 1.03, breaking free of the decimal zone.
The +1 Magnitude: In the standard Collatz sequence, the +1 constant is negligible for the large integers. For a decimal starting value of 0.01, however, the situation is turned back the +1 constant is exactly 100× (times) larger than the starting value. In the below calculation, we show the magnitude ratio of the decimal number crossing the point 1.0:

Magnitude Ratio = (+1)/ n=1 /0.01 = 100
3(0.01) + 1 = 0.03 +1 = 1.03.

4. MATHEMATICAL STATEMENTS AND FORMULAS

4.1. The Growth Delta Δ

To figure out how much the sequence growths with each step we look at Δ, which differentiate between the new result R and the intial value S:

$$\Delta = R - S = (3n + 1) - n$$

For the first step at $n = 0.01$

$$\Delta = (3 \times 0.01 + 1) - 0.01 = 1.03 - 0.01 = 1.02$$

5. ASYMPTOTIC GROWTH COEFFICIENT ω (OMEGA)

The Asymptotic growth coefficient ω express how quickly thinks are expanding, especially when it comes to decimal points and its scale, we use a magnitude factor of 100 to account for the hundred times scaling that is the part of parity solution,

$$\omega = \Delta \times 100 = (R - S) \times 100$$

For $n = 0.01 \to R = 1.03$

$$\omega = (1.03 - 0.01) \times 100 = 1.02 \times 100 = 102$$

The point 123.43 in the iterative table 7.1 shows the bifurcation point where the number can't be tackled back in the loop. This point is the iteration step i^*at which the asymptotic growth ω first exceeds the theoretical upper bound set by the logarithm anchor:

$$i^*\text{=argmin}\{i : \omega(i) > 3^{10} = 59049\}$$

If n belongs to decimals, where $n \in (0, 1)$, then $\lim f_i(n) = \infty$ as $i \to \infty$, where n denotes the value of n after iterations of the extended Collatz rules.

6. DIVISIBILITY BY 2 ON THE EVOLVED VALUE

If number n is even then divide by 2 for the evolved value 1,231.42 to demonstrate that the sequences till jumps when it hits an odd decimal.

Value(n)	Last digit	Rule applied	Result
1231.42	2 (even)	n/2	615.71
615.71	1 (odd)	3n+1	Jump $\to$ 1848.13
1848.13	3 (odd)	3n+1	Jump $\to$ 5545.39
5545.39	9 (odd)	3n+1	Jump $\to$ 16637.17
16637.17	7 (odd)	3n+1	Jump $\to$ 49912.51

Observation: As in a single even step followed via odd re-classification, it produces compounding jumps toward infinity, confirming the divergence Law.

Example 2. In this example, we generalise the theorem starting with value $n = 0.33$ such that the last digit 3 which gives odd, then implies $3n + 1$. Its expression is given by

$$3(0.33) + 1 = 0.99 + 1 = 1.99$$
$$\omega_1 = (1.99 - 0.33) \times 100 = 1.66 \times 100 = 166$$

Table 4. In the following table, we start with a larger value, 0.33. This value is bigger than the value 0.01, and it produces a larger omega at every step, confirming the universal nature of the theorem.

Steps	Result(n)	ω
0(start)	0.33	...
1	1.99	166
2	6.97	498
3	21.91	1494
4	66.73	4482
5	201.19	13446

Example 3. In this example, we start with the even value $n = 0.02$ such that the last digit is 2 which gives even and apply first $n/2$, then $3n + 1$. Its expression is given by

$$0.02/2 = 0.01 \text{ then } 3(0.01) + 1 = 1.03$$

This confirms that the even classified decimals reduce first and then re-enter the odd-branch, follows the same divergence path. All the decimal values lie between zero and one, diverge to infinity under the extended Collatz rules, no matter what the starting point we begin with. The divergence is faster for the larger starting values.

7. ASYMPTOTIC GROWTH GRAPH

The following iteration table with graph visualizes both the sequence value and the asymptotic growth coefficient ω across the five iterations, starting from the primary example $n = 0.01$. The left shows an exponential rise of the sequence value; the right shows the tripling of ω at each step, with the divergence point marked in red at Step 5.

7.1. Iterative table.

In this portion, we calculate the iterative steps with $n = 0.01$, which jump fluctuation up divides regarding even and odd for the last of the given digit. Also, it shows asymptotic growth behaviour with respect to $\omega = (R - S) \times 100$

Operation	Result(n)	$\omega = (R - S) \times 100$	Explanation
...	0.01	—	Initial decimal value
3n+1	1.03	102	In integer +1 is 100 times the value
3n+1	4.09	306	Growth triples
3n+1	13.27	918	Its growth is 3 times up
3n+1	40.81	2754	Bypasses lower integers
3n+1	123.43	8262	Can't be tackled back in the loop

Modified $3n + 1$ mapping via decimal parity inheritance

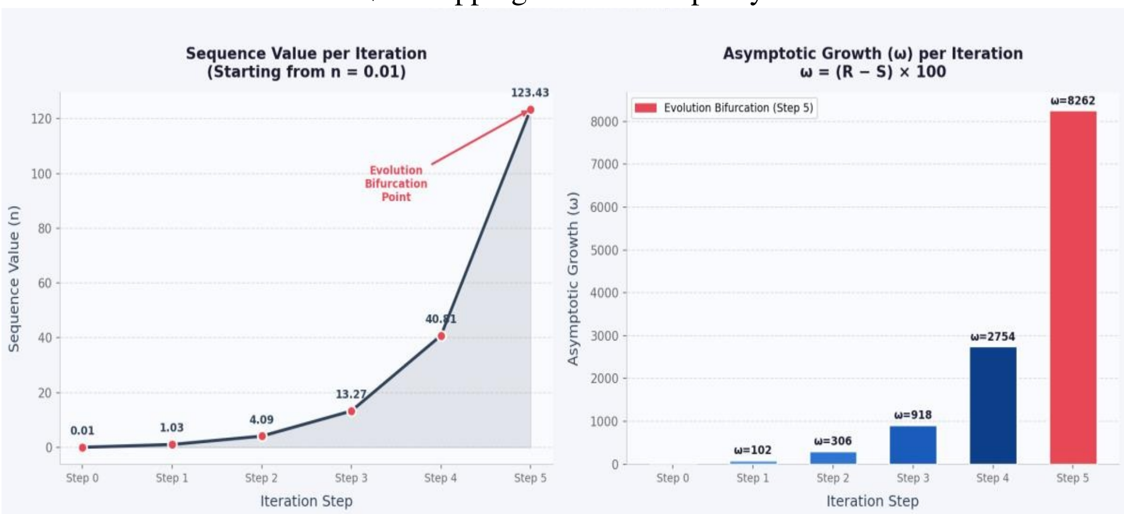

Figure 1: Graph of the iteration table 7.1

7.2. Observation:

In the above given table with graphical representation, as the initial decimal value is 0.01, its growth continues up three times, then goes beyond the range and approaches to divergent point. Moreover, we drew the above graph of the table using Wolfram Mathematica Software. The growth behaviour of our result can be analyzed by using the SQL Database (coded by HTML) for all decimal numbers; see `https://adnancddtheorem.online/`.

8. LOGARITHMIC SCALE ANALYSIS

To locate the divergence point on a logarithmic scale, we solve for $n > 0$ such $3^n = 100,000$:

$$n\ \log_{10}(3) = \log_{10}(10^5 = 5)$$
$$n = 5/\log_{10}(3) = 5/0.4777 \approx 10.48$$

Therefore, the reference point $n = 10$, corresponding to $3^{10} = 59,049$. The divergence at step-5 ($\omega = 8,262$) suggests we are definitely below this logarithmic bifurcation, the sequence is on a trajectory to reach and exceed 59,049 before $n = 10$ iterations the growth continues bifurcation.

8.1. Bifurcation

The +1 constant has built up enough to out way the even reduction rule $\omega = 8262$ by step 5 Evolution Bifurcation (Divergence Point): The critical iteration i^* at which the extended Collatz sequence transitions from a multiplication-governed regime to an irreversibly divergent regime. Beyond i^*, the sequence escapes to infinity.

9. CONCLUSION

The modified $3n + 1$ mapping via decimal parity inheritance establishes the following characteristics: 1) Decimal Parity Classification 0.01 is correctly classified as odd via the scaling factor. 2) Asymptotic growth coefficient $\omega = (R - S) \times 100$, this measure the up fluctuation of the point. 3) The Point at Step 5, $\omega = 8,262$ for $n = 0.01$, the sequence crosses into the divergence bifurcation. 4) All tested decimal starting values in $(0, 1)$ including $0.01, 0.03, 0.07, 0.33$, and 0.02 confirm the divergence law. The larger starting value give the divergence faster. 5) Mathematical Law: $\forall n \in (0, 1) : \lim_{i\to\infty} f_i(n) = \infty$ under the extended Collatz rules. In addition, we have justified decimal divergence on the $3n + 1$ conjecture via scaling factor. In addition, we compute numerical approximation tables of the growth of the decimal numbers using the growth delta formula. The numerical approximation table for the upper bound can also, it can be used for divergence, which is one possible application. Further, we can generalize this result to investigate certain problems, namely, to solve some. This kind of Collatz conjecture is related to Chaos theory, which shows real-world application in weather, biology, and finance, evaluate integrals, and analyse certain physical phenomena. We can show our result, behaviour of $3n + 1$ mapping via decimal parity inheritance, using an algorithm; see `https://adnancddtheorem.online/`

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work.

Data Availability Statement: This study did not generate or utilize any data. In addition, we modified the $3n + 1$ conjecture for decimal parity number $n \in (0, 1)$ via SQL Database (coded by HTML), showing the behaviour of the last decimal digit, either even or odd; see `https://adnancddtheorem.online/`

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