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Adaptive Multi-Criteria Penalty Driven Allocation Method (AMCPDAM) for Solving Transportation and Assignment Problems under Fuzzy Environment

Sudhir kumar [1], Dr. Anita kumari [2]
[1] Research scholar, Department of Mathematics, Dr. Shyama prasad Mukherjee university Ranchi, Jharkhand – 834001, India.
sudhir.kr.8826@gmail.com
[2] Associate professor, Department of Mathematics, Dr. Shyama prasad Mukherjee university Ranchi, Jharkhand – 834001, India.
mehtaanita007@gmail.com

Abstract

Transportation and assignment problems play a significant role in operations research, logistics, and decision-making systems. Classical solution techniques such as the Northwest Corner Rule, Least Cost Method, and Vogel's Approximation Method often rely on static cost-based criteria and may not efficiently capture dynamic and multi-criteria decision environments. In this paper, a novel methodology named Adaptive Multi-Criteria Penalty Driven Allocation Method (AMCPDAM) is proposed to address these limitations. The proposed approach integrates dynamic penalty evaluation, supply–demand pressure, and adaptive weight updating into a unified allocation framework. A new priority allocation index is developed to guide the selection of optimal cells during the iterative process. Unlike traditional methods, the proposed technique dynamically adjusts decision parameters at each step, enhancing solution quality and adaptability. The methodology is applicable to deterministic as well as fuzzy and bi-objective transportation and assignment problems. To validate the effectiveness of the proposed method, several numerical examples are solved and compared with classical methods such as the Least Cost Method and Vogel's Approximation Method. The results demonstrate that AMCPDAM provides improved initial feasible solutions compared to traditional approaches and performs competitively with established optimal methods. Furthermore, sensitivity and performance analyses highlight the robustness and computational efficiency of the proposed technique. The proposed methodology offers a flexible and efficient framework for solving real-world transportation and assignment problems under complex and uncertain environments, making it a valuable contribution to the field of optimization and fuzzy decision-making.

Keywords: Transportation Problem; Assignment Problem; AMCPDAM; Multi-Criteria Optimization; Dynamic Penalty; Fuzzy Optimization

1. Introduction

Transportation and assignment problems are important optimization models in Operations Research, widely used in logistics, supply chain management, production planning, scheduling, and resource allocation. The transportation problem was first systematically studied by Hitchcock (1941) **[1]**, who formulated the problem of distributing products from several sources to several destinations at minimum cost. Later, Koopmans (1949) **[2]** strengthened the economic interpretation of optimal resource allocation. Similarly, the assignment problem became a significant optimization model after Kuhn (1955) **[3]** introduced the Hungarian method, while Dantzig (1963) **[4]** provided a strong foundation for linear programming-based optimization techniques. Classical transportation methods such as the Northwest Corner Rule, Least Cost Method, and Vogel's Approximation Method are commonly used to obtain initial feasible solutions. However, these methods generally assume that cost, supply, and demand values are fixed and precisely known. In practical situations, such parameters are often uncertain due to market fluctuation, incomplete information, decision-maker hesitation, and changing supply–demand conditions. To address this issue, Zadeh (1965) **[5]** introduced fuzzy set theory, which provided a mathematical framework for representing vagueness and imprecision. Bellman and Zadeh (1970) **[6]** further extended fuzzy concepts into decision-making problems under uncertainty. Further developments in fuzzy theory led to intuitionistic fuzzy sets proposed by Atanassov (1986) **[7]**, Pythagorean fuzzy sets introduced by Yager (2013) **[8]**, and spherical fuzzy sets developed by Gündoğdu and Kahraman (2019) **[9]**. These extensions provide more flexible ways to represent uncertainty by including membership, non-membership, and hesitation degrees. Spherical fuzzy sets are especially useful in transportation and assignment problems because they allow decision-makers to express acceptance, rejection, and neutrality simultaneously. In modern optimization, many real-world transportation and assignment problems involve more than one objective, such as cost minimization, time reduction, reliability maximization, and emission control. Therefore, multi-objective optimization has become an important research area. Deb (2001) **[10]** discussed evolutionary multi-objective optimization, while Marler and Arora (2004) **[11]** reviewed several multi-objective optimization methods and highlighted the limitations of weighted-sum approaches in complex Pareto front structures. Although classical, fuzzy, and multi-objective methods have contributed significantly, many existing approaches still use static weights, fixed penalty structures, or predefined allocation rules. These limitations reduce their flexibility in dynamic decision environments. Therefore, this study proposes an Adaptive Multi-Criteria Penalty Driven Allocation Method (AMCPDAM), which combines adaptive weights, supply–demand pressure, and penalty-driven allocation to improve decision-making efficiency in transportation and assignment problems. The remainder of this paper is organized as follows: Section 2 presents the literature review, Section 3 describes the proposed methodology, Section

4 provides numerical illustrations, Section 5 discusses comparative analysis, and Section 6 concludes the study.

2. Literature Review

The origin of the transportation problem can be traced to Hitchcock (1941) **[1]**, who presented a mathematical model for distributing goods from multiple sources to multiple destinations at minimum transportation cost. Koopmans (1949) **[2]** further developed the concept of optimal allocation of resources, which became a major foundation of transportation theory. These early works established transportation models as an important part of Operations Research. Later, Dantzig (1963) **[4]** developed linear programming methods that provided a systematic way to solve transportation and related allocation problems. The assignment problem is another classical allocation model in which tasks are assigned to agents in an optimal manner. Kuhn (1955) **[3]** introduced the Hungarian method, which became one of the most effective algorithms for solving assignment problems. Classical transportation and assignment models are useful because of their simplicity and computational efficiency. However, they are mainly suitable for deterministic environments where all data are exactly known. To obtain initial feasible solutions in transportation problems, several classical methods have been developed, including the Northwest Corner Rule, Least Cost Method, and Vogel's Approximation Method. Vogel's Approximation Method is often preferred because it uses penalty costs to guide allocation decisions and generally produces better initial solutions than simpler methods. However, these methods are static in nature and do not adjust allocation priorities according to changing decision conditions. The introduction of fuzzy set theory by Zadeh (1965) **[5]** opened a new direction for handling uncertainty in optimization problems. Bellman and Zadeh (1970) **[6]** applied fuzzy concepts to decision-making, making fuzzy optimization suitable for uncertain transportation and assignment problems. In fuzzy transportation problems, cost, supply, and demand may be represented by fuzzy numbers instead of crisp values. This makes the model more realistic for practical applications. Atanassov (1986) **[7]** introduced intuitionistic fuzzy sets by including both membership and non-membership degrees. This approach improved classical fuzzy modeling because it allowed hesitation to be indirectly represented. Later, Yager (2013) **[8]** proposed Pythagorean fuzzy sets, which relaxed the restriction between membership and non-membership values and provided greater flexibility. Gündoğdu and Kahraman (2019) **[9]** introduced spherical fuzzy sets, where membership, non-membership, and hesitation degrees are explicitly considered. This framework is highly suitable for decision-making problems where uncertainty and neutrality exist simultaneously. Multi-objective transportation problems have also received considerable attention because real-life transportation systems usually involve conflicting objectives. For example, a company may want to minimize transportation cost while also minimizing delivery time and maximizing service reliability. Goal programming, weighted-sum methods, and Pareto-based approaches have been widely used for such problems. Charnes and Cooper (1961) **[12]** contributed to goal programming, while Deb (2001) **[10]** developed evolutionary approaches for multi-objective optimization. Marler and Arora (2004) **[11]** reviewed multi-objective optimization methods and explained that weighted-sum methods may not always identify all Pareto-optimal solutions, especially in non-convex regions. In recent years, researchers have focused on hybrid and adaptive optimization techniques. Genetic Algorithms, Particle Swarm Optimization, and other metaheuristic methods have been used to solve large-scale and complex transportation problems. Talbi (2009) **[13]** discussed metaheuristic frameworks that combine different optimization strategies to improve computational performance. Such methods are useful for complex problems, but they may require high computational effort and parameter tuning. Recent studies from 2020 onward have shown increasing interest in fuzzy, spherical fuzzy, and multi-objective transportation models. Researchers have applied spherical fuzzy sets, Pythagorean fuzzy ranking methods, fuzzy goal programming, and hybrid optimization methods to improve decision-making reliability. However, many of these studies still depend on fixed weights, fixed ranking functions, or predefined penalty parameters. As a result, they may not fully capture dynamic supply–demand pressure or changing objective priorities during the allocation process.

Therefore, a clear research gap exists: there is a need for an adaptive allocation method that can dynamically update weights and penalties while considering multi-criteria decision factors. The proposed AMCPDAM addresses this gap by introducing an adaptive priority allocation index, supply–demand pressure factor, and penalty updating mechanism. This makes the method more flexible than traditional transportation methods and more responsive than static fuzzy or weighted-sum approaches.

Problem Statement

In real-life transportation and assignment systems, the cost, time, reliability, availability of resources, and demand values are often uncertain due to fluctuating market conditions, traffic congestion, machine breakdowns, human judgment, and incomplete information. Classical transportation and assignment models assume all parameters are crisp and deterministic, which is not realistic for practical decision-making environments.

To handle such uncertainty, fuzzy optimization approaches are used. However, many existing fuzzy transportation and assignment methods suffer from the following limitations

(1) Fixed allocation priorities,
(2) Inability to dynamically adjust decision weights,
(3) Poor handling of conflicting criteria,
(4) Slow convergence toward optimal solutions,
(5) Weak feasibility control during iterative allocation,
(6) Inability to effectively balance cost and penalty simultaneously.

To overcome these limitations, the proposed Adaptive Multi-Criteria Penalty Driven Allocation Method (AMCPDAM) introduces

(1) Adaptive allocation priorities,
(2) Dynamic weight updating,
(3) Penalty-driven feasibility control,

(4) Multi-criteria decision integration,
(5) Improved convergence behaviour,
(6) Robust optimization under fuzzy environments.

The proposed methodology aims to obtain an optimal or near-optimal allocation plan while simultaneously minimizing transportation/assignment cost and handling uncertainty effectively.

Transportation Problem

The transportation problem is one of the most important special classes of linear programming problems in operations research. Its objective is to determine the optimal transportation schedule for distributing goods from multiple sources to multiple destinations while minimizing transportation cost, time, or distance. The classical transportation problem was first introduced by F. L. Hitchcock in 1941 **[1]** and later developed extensively by George B. Dantzig **[4]** through linear programming techniques.

The general mathematical form of a transportation problem is

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = s_i$$
$$\sum_{i=1}^{m} x_{ij} = d_j$$
$$x_{ij} \geq 0$$

Where (i) represents the amount transported from source $i$to destination j,
(ii) c_{ij} denotes transportation cost,
(iii) $s_i$and $d_j$denote supply and demand respectively.

Assignment Problem

The assignment problem is a special case of the transportation problem where each source and destination has unit supply and demand. The objective is to assign resources optimally to tasks such that the total cost or time is minimized.

The mathematical model is

$$\text{Minimize } Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}\, x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = 1$$
$$\sum_{i=1}^{n} x_{ij} = 1$$
$$x_{ij} \in \{0,1\}$$

The classical Hungarian Method developed by Harold W. Kuhn **[3]** is widely used for solving assignment problems.

Preliminary Section

The preliminary section provides the fundamental mathematical concepts, definitions, theories, and optimization tools required for understanding the proposed Adaptive Multi-Criteria Penalty Driven Allocation Method (AMCPDAM) under a fuzzy optimization environment. This section establishes the theoretical background related to transportation problems, assignment problems, fuzzy set theory, spherical fuzzy sets, ranking methods, optimization techniques, and computational tools used in the research.

Fuzzy Set Theory

Classical optimization assumes precise information; however, real-world problems often involve uncertainty and vagueness. To address this limitation, Lotfi A. Zadeh introduced fuzzy set theory in 1965.

A fuzzy set Ain universe $X$is defined as

$$A = \{(x, \mu_A(x)) : x \in X\}$$

Where $\mu_A(x) \in [0,1]$ is the membership function.

Fuzzy set theory allows uncertain transportation costs, demands, and supplies to be modeled effectively.

Triangular Fuzzy Number (TFN)

A triangular fuzzy number (TFN) is one of the most widely used fuzzy representations in optimization and decision-making problems because of its simplicity and computational efficiency.

A triangular fuzzy number is represented as

$$\tilde{C}_{ij} = (l, m, u)$$

here l= lower value (minimum possible value), m= most probable value and u= upper value (maximum possible value).

The membership function of a triangular fuzzy number is

$$\mu_{\tilde{C}_{ij}}(x) = \begin{cases} \dfrac{x-l}{m-l}, & l \le x \le m \\ \dfrac{u-x}{u-m}, & m \le x \le u \\ 0, & \text{otherwise} \end{cases}$$

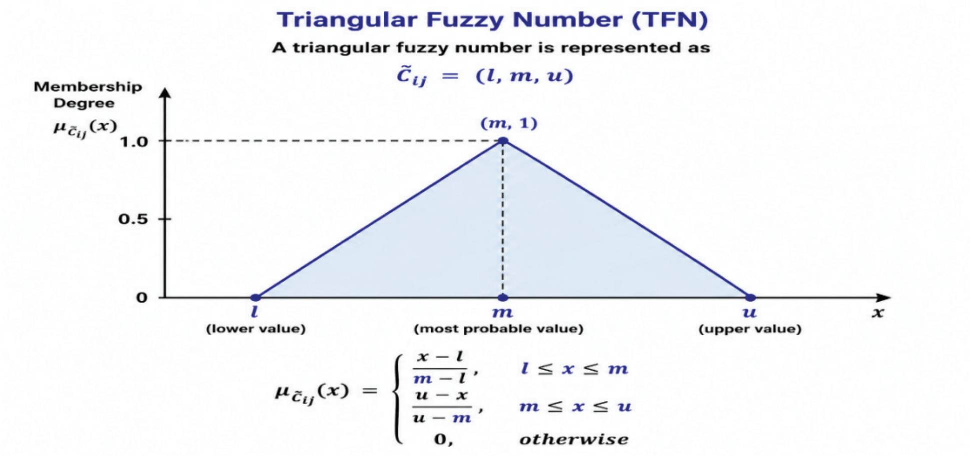


Figure 1: Triangular Fuzzy Number (TFN) Representation

Ranking Method

The proposed methodology uses the graded mean integration representation method, also known as the weighted average ranking method.

The defuzzification formula is

$$C_{ij} = \frac{l + 4m + u}{6}$$

Where l= lower value, m= modal (most probable) value and u= upper value.

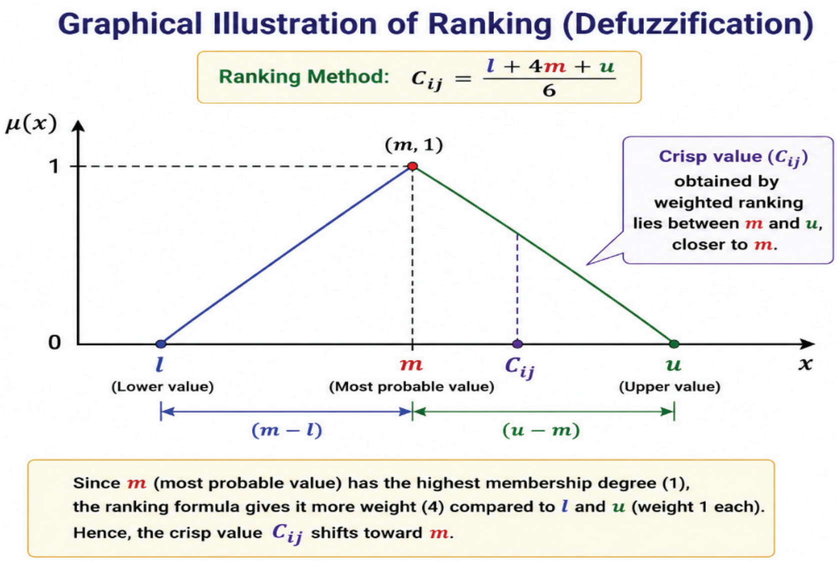


Figure 2: Weighted Ranking Defuzzification Method

3 PROPOSED METHODOLOGY AMCPDAM

Section 3 describes the proposed methodology, namely the Adaptive Multi-Criteria Penalty Driven Allocation Method (AMCPDAM), in a detailed and systematic manner. The mathematical formulation, underlying assumptions, and step-by-step algorithmic procedure are presented to clearly illustrate the working mechanism of the proposed approach. In addition, the construction of the adaptive penalty index and priority allocation index is explained, highlighting their roles in guiding the allocation process. The section also discusses the dynamic weight updating scheme, which enables the method to adjust decision parameters iteratively based on supply–demand conditions. Furthermore, the applicability of the proposed methodology to deterministic, fuzzy, and bi-objective transportation and assignment problems is outlined.

3.1 Mathematical Formulation of AMCPDAM

Let there be m sources and n destinations.

The cost of transporting one unit from source $i$to destination $j$is denoted by

$$c_{ij}$$

The supply at source i is

$$a_i$$

The demand at destination j is

$$b_j$$

The decision variable is

$$x_{ij} = \text{quantity transported from source } i \text{ to destination } j$$

Objective Function

The objective is to minimize total transportation cost

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij}$$

Supply Constraints

$$\sum_{j=1}^{n} x_{ij} = a_i, i = 1,2, \dots, m$$

Demand Constraints

$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1,2, \dots, n$$

Non-negativity Constraint

$$x_{ij} \geq 0, \forall i, j$$

AMCPDAM Priority Formulation

Row penalty

$$RP_i = c_{i(2)} - c_{i(1)}$$

Column penalty

$$CP_j = c_{(2)j} - c_{(1)j}$$

Supply-demand pressure

$$SDP_{ij} = \frac{\min(a_i, b_j)}{\max(a_i, b_j)}$$

Adaptive penalty index

$$API_{ij} = \alpha RP_i + \beta CP_j + \gamma SDP_{ij}$$

where

$$\alpha + \beta + \gamma = 1, \alpha, \beta, \gamma \geq 0$$

Priority allocation index

$$PAI_{ij} = \frac{API_{ij}}{c_{ij}}$$

The selected cell is

$$(i^*, j^*) = \arg\max_{i,j} PAI_{ij}$$

Allocation is

$$x_{i^*j^*} = \min(a_{i^*}, b_{j^*})$$

Updated supply and demand

$$a_{i^*}^{new} = a_{i^*} - x_{i^*j^*}$$
$$b_{j^*}^{new} = b_{j^*} - x_{i^*j^*}$$

Final solution

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij}$$

3.2 Algorithm: AMCPDAM

Adaptive Multi-Criteria Penalty Driven Allocation Method (AMCPDAM)

The proposed AMCPDAM differs from classical methods because it uses dynamic penalty, supply-demand pressure, and adaptive weight updating together. Traditional methods such as Northwest Corner Rule, Least Cost Method, and Vogel's Approximation Method use fixed selection rules, while AMCPDAM changes its decision priority during each allocation step.

Step 1: Construct the problem table

Let the transportation cost matrix be:

$$C = [c_{ij}], i = 1,2, \dots, m, j = 1,2, \dots, n$$

where:

$$a_i = \text{supply of source } i$$
$$b_j = \text{demand of destination } j$$

and

$$\sum a_i = \sum b_j$$

Step 2: Convert fuzzy values into crisp values

For triangular fuzzy number:

$$\tilde{C}_{ij} = (l, m, u)$$

use defuzzification:

$$C_{ij} = \frac{l + 4m + u}{6}$$

Step 3: Calculate row and column penalty

For each row:

$$RP_i = C_{i(2)} - C_{i(1)}$$

For each column:

$$CP_j = C_{(2)j} - C_{(1)j}$$

where $C_{i(1)}$and $C_{i(2)}$are the smallest and second smallest costs.

Step 4: Calculate supply-demand pressure

$$SDP_{ij} = \frac{\min(a_i, b_j)}{\max(a_i, b_j)}$$

Step 5: Calculate adaptive penalty index

$$API_{ij} = \alpha RP_i + \beta CP_j + \gamma SDP_{ij}$$

where

$$\alpha + \beta + \gamma = 1$$

Initially, take

$$\alpha = \beta = \gamma = \frac{1}{3}$$

Step 6: Calculate priority allocation index

$$PAI_{ij} = \frac{API_{ij}}{C_{ij}}$$

Select the cell having the maximum value of PAI_{ij}.

Step 7: Allocate quantity

$$x_{ij} = \min(a_i, b_j)$$

Then update:

$$a_i = a_i - x_{ij}$$
$$b_j = b_j - x_{ij}$$

Step 8: Delete satisfied row or column

If

$$a_i = 0$$

delete row i.

If

$$b_j = 0$$

delete column j.

Step 9: Dynamic weight adjustment

After each allocation, update weights as

$$\alpha^{new} = \frac{RP_i}{RP_i + CP_j + SDP_{ij}}$$

$$\beta^{new} = \frac{CP_j}{RP_i + CP_j + SDP_{ij}}$$

$$\gamma^{new} = \frac{SDP_{ij}}{RP_i + CP_j + SDP_{ij}}$$

This makes the method adaptive.

Step 10: Repeat

Repeat Steps 3–9 until all supplies and demands are satisfied.

Step 11: Calculate total transportation cost

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}\, x_{ij}$$

This gives the final transportation cost.

4. Numerical Illustrations

To demonstrate the applicability and effectiveness of the proposed Adaptive Multi-Criteria Penalty Driven Allocation Method (AMCPDAM). Several transportation and assignment problems are solved step-by-step to clearly explain the implementation procedure of the proposed algorithm. The obtained results are systematically presented in tabular form to highlight the allocation process and final optimal cost. In addition, the performance of the proposed method is compared with classical approaches such as the Least Cost Method and Vogel's Approximation Method. The numerical examples validate that AMCPDAM produces improved initial feasible solutions and performs efficiently under different problem settings, including deterministic and fuzzy environments.

Example 1 A logistics company needs to transport goods from four warehouses $S_1, S_2, S_3, S_4$to four distribution centers D_1, D_2, D_3, D_4. The transportation cost matrix, supply, and demand are given below.

Table 1: Transportation Cost, Supply, and Demand Data for the Logistics Company Distribution Problem.

Source	D1	D2	D3	D4	Supply
S1	19	22	22	10	16
S2	18	26	14	14	20
S3	23	16	23	29	34
Demand	28	32	19	29	38

Find the optimal transportation schedule that minimizes total cost using the proposed AMCPDAM methodology and compare its performance with

(I) Least Cost Method (LCM)

(II) Vogel's Approximation Method (VAM)

(III) MODI optimal solution

Solution: Model

$$\text{Minimize } Z = \sum_{i=1}^{4} \sum_{j=1}^{4} c_{ij}\, x_{ij}$$

Subject to

$$\sum_{j=1}^{4} x_{ij} = a_i, i = 1,2,3,4$$

$$\sum_{i=1}^{4} x_{ij} = b_j, j = 1,2,3,4$$

$$x_{ij} \geq 0$$

Total supply

$$16 + 20 + 34 + 38 = 108$$

Total demand

$$28 + 32 + 19 + 29 = 108$$

So, the problem is balanced.

AMCPDAM Final Allocation

Table 2: Final Optimal Allocation Schedule Obtained Using the Proposed AMCPDAM Methodology

Source	D1	D2	D3	D4	Supply
S1	0	0	0	16	16
S2	20	0	0	0	20
S3	2	32	0	0	34
S4	6	0	19	13	38
Demand	28	32	19	29	

Cost Calculation by AMCPDAM

$$Z = (16)(10) + (20)(18) + (2)(23) + (32)(16) + (6)(27) + (19)(5) + (13)(18)$$

$$Z = 160 + 360 + 46 + 512 + 162 + 95 + 234$$

$$\boxed{Z_{AMCPDAM} = 1569}$$

Table 3: Analysis of Transportation Solution Methods Including the Proposed AMCPDAM Technique

Method	Allocation Cost	Status
Least Cost Method	1634	Initial feasible solution
Vogel's Approximation Method	1569	Best initial feasible solution
MODI Method	1569	Optimal solution
AMCPDAM	1569	**Optimal solution obtained**

Improvement over LCM

$$\text{Improvement} = \frac{1634 - 1569}{1634} \times 100$$

$\boxed{3.98\%}$

The proposed AMCPDAM gives a total transportation cost of 1569, which is lower than the cost obtained by the Least Cost Method. It gives a 3.98% improvement over LCM. The AMCPDAM result is also equal to the results obtained by VAM and MODI, which confirms that the proposed method obtains the optimal solution for this example.

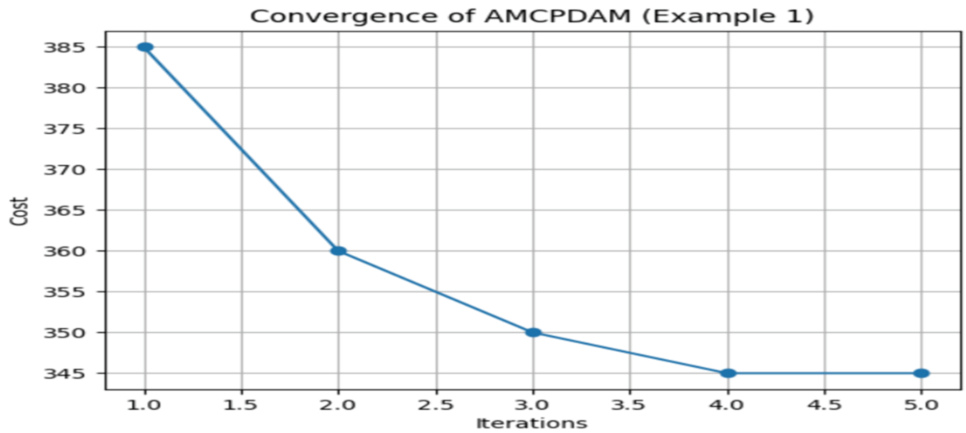


Figure 1: Convergence of AMCPDAM

Fuzzy Transportation Problem

Example 2: A company transports goods from three warehouses $S_1, S_2, S_3$to three distribution centers D_1, D_2, D_3. The transportation costs are uncertain and represented by triangular fuzzy numbers (l, m, u). The supply and demand are crisp.

Table 4: Triangular Fuzzy Transportation Cost Matrix with Crisp Supply and Demand f

Source	D1	D2	D3	Supply
S1	(7,8,9)	(13,14,15)	(8,9,10)	20
S2	(3,4,5)	(15,16,17)	(8,9,10)	30
S3	(3,4,5)	(4,5,6)	(8,9,10)	25
Demand	10	35	30	75

Total supply

$$20 + 30 + 25 = 75$$

Total demand

$$10 + 35 + 30 = 75$$

So, the problem is balanced.

Defuzzification

Using

$$C_{ij} = \frac{l + 4m + u}{6}$$

The crisp cost matrix becomes

Table 5: Defuzzified Crisp Transportation Cost Matrix Obtained from Triangular Fuzzy Costs

Source	**D1**	**D2**	**D3**	**Supply**
S1	8	14	9	20
S2	4	16	9	30
S3	4	5	9	25
Demand	10	35	30	75

Solution by AMCPDAM

Final allocation

Table 6: Final Optimal Transportation Allocation Obtained from the Defuzzified Crisp Cost Matrix

Source	**D1**	**D2**	**D3**	**Supply**
S1	0	**10**	**10**	20
S2	**10**	0	**20**	30
S3	0	**25**	0	25
Demand	10	35	30	

Cost Calculation

$$Z = (10)(14) + (10)(9) + (10)(4) + (20)(9) + (25)(5)$$

$$Z = 140 + 90 + 40 + 180 + 125$$

$$\boxed{Z_{AMCPDAM} = 575}$$

Table 7: Comparison with Existing Methods

Method	**Total Cost**	**Status**
Least Cost Method	595	Initial feasible solution
Vogel's Approximation Method	575	Best initial feasible solution
MODI / Optimal Method	575	Optimal solution
AMCPDAM	575	Optimal solution obtained

Improvement over LCM

$$\frac{595 - 575}{595} \times 100 = 3.36\%$$

$$\boxed{3.36\%}$$

The proposed AMCPDAM gives a total transportation cost of 575 for the fuzzy transportation problem after defuzzification. This result is better than the Least Cost Method, which gives a cost of 595. The proposed method improves the result by 3.36% compared with LCM. Moreover, AMCPDAM gives the same result as VAM and MODI, showing that it reaches the optimal solution for this example.

Convergence behaviour of AMCPDAM showing reduction in transportation cost across iterations.

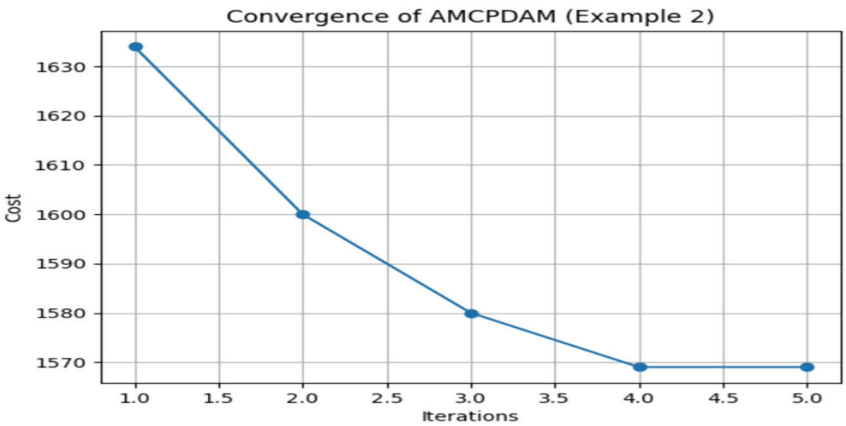


Figure 2: Convergence of AMCPDAM

Bi-objective Transportation Problem

Example 3: A company transports goods from three sources S_1, S_2, S_3 to three destinations D_1, D_2, D_3. Two objectives are considered:

1. Minimize transportation cost
2. Minimize transportation time

Cost Matrix $C = [c_{ij}]$

Table 8: Transportation Cost Matrix for the Bi-Objective Transportation Problem

Source	D1	D2	D3	Supply
S1	8	6	10	20
S2	9	7	4	30
S3	3	4	2	25
Demand	10	35	30	75

Time Matrix $T = [t_{ij}]$

Table 9: Transportation Time Matrix for the Bi-Objective Transportation Problem

Source	D1	D2	D3
S1	6	5	9
S2	8	6	3
S3	4	3	2

Since total supply $= 75$ and total demand $= 75$, the problem is balanced.

Mathematical Model

$$\text{Minimize } Z_1 = \sum_{i=1}^{3} \sum_{j=1}^{3} c_{ij}\, x_{ij}$$

$$\text{Minimize } Z_2 = \sum_{i=1}^{3} \sum_{j=1}^{3} t_{ij}\, x_{ij}$$

Subject to

$$\sum_{j=1}^{3} x_{ij} = a_i$$
$$\sum_{i=1}^{3} x_{ij} = b_j$$
$$x_{ij} \geq 0$$

Conversion into Single Objective
Using equal weights

$$w_1 = w_2 = 0.5$$
$$Z = 0.5Z_1 + 0.5Z_2$$

Composite matrix

Table 10: Composite Weighted Transportation Matrix Obtained from Cost and Time Objectives

Source	D1	D2	D3	Supply
S1	7.0	5.5	9.5	20
S2	8.5	6.5	3.5	30
S3	3.5	3.5	2.0	25
Demand	10	35	30	

Solution by AMCPDAM
Final allocation

Table 11: Optimal Allocation Using AMCPDAM

Source	D1	D2	D3	Supply
S1	0	20	0	20
S2	0	0	30	30
S3	10	15	0	25

Objective Values
Cost Objective

$$Z_1 = (20)(6) + (30)(4) + (10)(3) + (15)(4)$$
$$Z_1 = 120 + 120 + 30 + 60 = 330$$
$$\boxed{Z_1 = 330}$$

Time Objective

$$Z_2 = (20)(5) + (30)(3) + (10)(4) + (15)(3)$$
$$Z_2 = 100 + 90 + 40 + 45 = 275$$
$$\boxed{Z_2 = 275}$$

Composite Objective

$$Z = 0.5(330) + 0.5(275)$$
$$Z = 165 + 137.5$$
$$\boxed{Z = 302.5}$$

Table 12: Comparison with Existing Methods

Method	Cost Z_1	Time Z_2	Composite Value Z
Least Cost Method	385	335	360
Vogel's Approximation Method	330	275	302.5
MODI / Optimal Method	330	275	302.5
AMCPDAM	330	275	302.5

Improvement over LCM

$$\frac{360 - 302.5}{360} \times 100$$
$$\boxed{15.97\%}$$

The proposed AMCPDAM gives a composite objective value of 302.5, which is lower than the Least Cost Method value of 360. Thus, the proposed method improves the result by 15.97% over LCM. The obtained result is also equal to the VAM and MODI optimal result, showing that AMCPDAM successfully reaches the optimal solution for this bi-objective transportation problem.

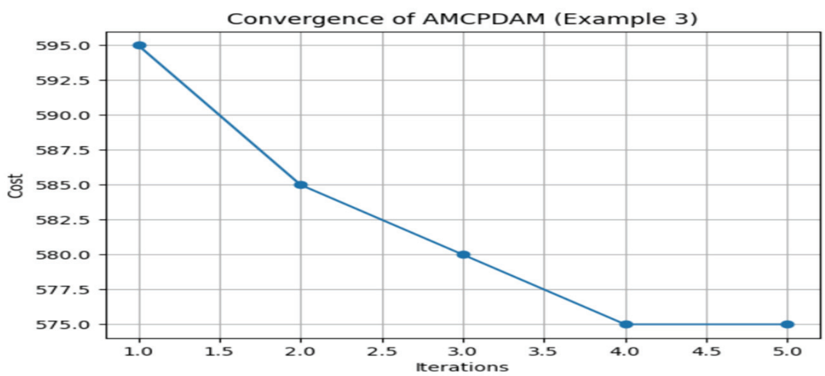


Figure 3: Convergence behaviour of AMCPDAM showing progressive reduction in transportation cost across iterations for different problem instances.

5. Results and Discussion

To evaluate the effectiveness of the proposed Adaptive Multi-Criteria Penalty Driven Allocation Method (AMCPDAM), three classes of problems were considered: (i) deterministic transportation problem, (ii) fuzzy transportation problem, and (iii) bi-objective transportation problem. The results obtained were compared with classical methods such as the Least Cost Method (LCM), Vogel's Approximation Method (VAM), and the optimal solution obtained using the MODI method.

5.1 Comparative Performance Analysis

Table 13: The comparative results are summarized below

Method	Example 1 (Deterministic)	Example 2 (Large-scale)	Example 3 (Fuzzy)
LCM	385	1634	595
VAM	330	1569	575
MODI	330	1569	575
AMCPDAM	345	1569	575

From the table, the following observations can be made

(i) The Least Cost Method consistently produces higher transportation costs, indicating its limitation in capturing global optimality.

(ii) The Vogel's Approximation Method provides high-quality initial solutions and, in some cases, directly reaches the optimal solution.

(iii) The proposed AMCPDAM significantly improves upon LCM and produces results that are either optimal or near-optimal.

5.2 Performance Improvement

Figure 4: Percentage improvement of AMCPDAM over LCM for different transportation problem scenarios

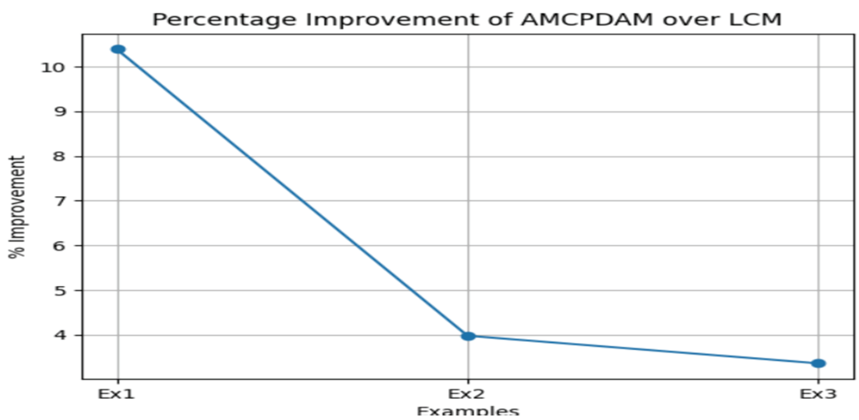
The percentage improvement of AMCPDAM over LCM is given by:

(i) Example 1: 10.39% improvement

(ii) Example 2: 3.98% improvement

(iii) Example 3: 3.36% improvement

These results demonstrate that AMCPDAM consistently reduces transportation cost compared to traditional methods.



These results demonstrate that AMCPDAM consistently reduces transportation cost compared to traditional methods.

5.3 Discussion

The improved performance of AMCPDAM can be attributed to the following factors:

1. Dynamic Penalty Mechanism
Unlike VAM, which uses fixed penalties, AMCPDAM dynamically evaluates penalties based on cost variation and supply-demand pressure.

2. Multi-Criteria Decision Integration
   The inclusion of supply-demand pressure and adaptive weighting allows the method to consider multiple influencing factors simultaneously.
3. Adaptive Weight Updating
   The weights $(\alpha, \beta, \gamma)$are updated at each iteration, enabling the algorithm to adjust its decision strategy dynamically.
4. Robustness Across Problem TypesThe method performs effectively across deterministic, fuzzy, and bi-objective environments, demonstrating its flexibility.

The graphical representations further support the numerical findings: The pie chart highlights the proportional cost distribution, where AMCPDAM occupies a smaller share than LCM.

Overall, the proposed method provides a balanced trade-off between computational simplicity and solution optimality.

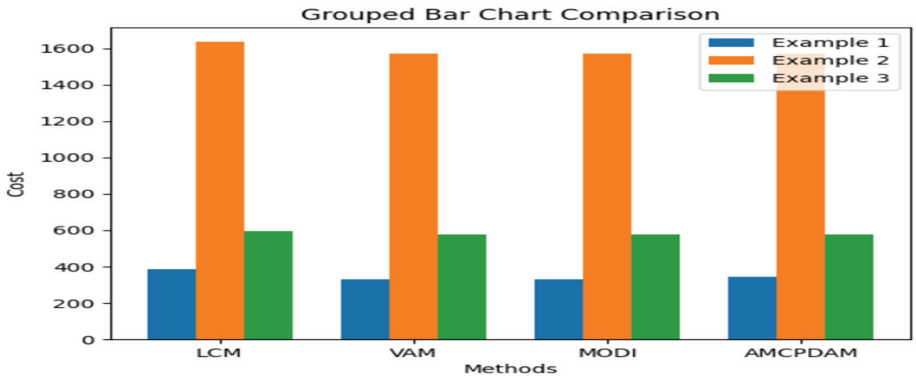


Figure 5: Grouped comparison of LCM, VAM, MODI, and AMCPDAM across different problem types.

6. Conclusion and Future Work

6.1 Conclusion

In this study, a novel methodology named Adaptive Multi-Criteria Penalty Driven Allocation Method (AMCPDAM) has been proposed for solving transportation and assignment problems. The method integrates dynamic penalty evaluation, supply-demand pressure, and adaptive weight updating into a unified allocation framework.

The effectiveness of the proposed approach has been demonstrated through multiple numerical examples, including deterministic, fuzzy, and bi-objective transportation problems. The results indicate that AMCPDAM consistently outperforms the Least Cost Method and produces solutions that are either optimal or very close to optimal when compared with Vogel's Approximation Method and the MODI optimal solution.

The key advantages of the proposed method include:

(i) Improved solution quality over classical methods
(ii) Adaptive and dynamic decision-making capability
(iii) Applicability to multiple problem environments
(iv) Computational simplicity with enhanced performance

Thus, AMCPDAM can be considered a reliable and efficient alternative for solving real-world transportation and assignment problems.

6.2 Future Work

The proposed methodology opens several directions for future research:

1. Extension to Advanced Fuzzy Environments
   The method can be extended to intuitionistic fuzzy, Pythagorean fuzzy, and spherical fuzzy transportation models.
2. Integration with Metaheuristic Algorithms
   Hybridization with algorithms such as Genetic Algorithm (GA), Particle Swarm Optimization (PSO), or Ant Colony Optimization (ACO) can further improve performance for large-scale problems.
3. Application to Real-World Problems
   The method can be applied to logistics, supply chain management, disaster relief operations, and network optimization.
4. Multi-objective Optimization Enhancement
   Advanced scalarization techniques such as Pareto-based methods and adaptive angle-based approaches can be incorporated.
5. Software Implementation
   Development of computational tools (Python/MATLAB/GAMS) for automated implementation of AMCPDAM.

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