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A Novel Approach: Soft Topology

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Abstract

In this paper, we construct a soft topology on the universe set X by assigning a topological structure to X and investigate some interesting results in this context.

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Key Words: Soft set, Soft topology, Soft open set, Soft closed set, Soft closure, $s-$open set, $s-$closed set, Soft compactness, soft connectedness.

Introduction

In 1999, *Molodtsov* [7] introduced the soft set theory as a general mathematical tool for dealing with uncertainty or vagueness. There are many theories like Fuzzy set theory and Rough set theory to tackle the problem of imperfect knowledge. Soft set theory is still a better approach to deal with problems of uncertainty. *Molodtsov* recognized the importance of the role of parameters and introduced the theory of Soft sets. He has shown several applications of this theory in many fields like economics, engineering, medical sciences, etc. Later, this theory became a very good source of research for many mathematicians and computer scientists of recent years because of its wide range of applicability. The development in the fields of soft set theory and its application has been taking place in a rapid pace.

The notion of topological space for soft sets was first formulated by *Shabir* and *Naz* [10]. Later, many authors ([2] to [5]) have studied various properties of soft topological spaces. There are mainly two concepts in the theory of soft topology. One notion is to define a topology on the universe set and the second one is to make each soft set as a topological space. *Mahanta* and *Das* [6] introduced semi-soft open sets, semi-soft closed sets, soft semi-continuity and related concepts by following the first notion. *Cagman*, *Karatas* and *Enginoglu* [1] introduced the second notion and defined soft open sets, soft neighborhood, soft closure and related concepts.

In this present work, we introduce a soft topology on the universe set X and some soft topological concepts like soft open sets, soft closed sets, soft closure, soft compactness and soft connectedness in a different approach by assigning a topological structure to X.

1. Preliminaries

In this section, we present some basic definitions which are needed in further study of this paper. Let U be an initial universe set and E_U (or simply E) be a collection of all possible parameters with respect to U, where parameters are the characteristics or properties of objects in U. Let $P(U)$ be the collection of all subsets of U.

1.1 Definition: A pair (F,A) is called a *soft set* over U, if $A \subset E$ and $F: A \to P(U)$. We write F_A for (F,A).

1.2 Definition: Let F_A and G_B be soft sets over a common universe set U and $A, B \subset E$. Then we say that

(a) F_A is a *soft subset* of G_B, denoted by $F_A \subset G_B$, if (i) $A \subset B$ and (ii) $F(e) \subset G(e)\ \forall\, e \in A$.

(b) F_A equals G_B, denoted by $F_A = G_B$, if $F_A \subset G_B$ and $G_B \subset F_A$.

1.3 Definition: A soft set F_A over U is called a *null soft set*, denoted by Φ, if $e \in A$, $F(e) = \phi$.

1.4 Definition: A soft set F_A over U is called an *absolute soft set*, denoted by A, if $e \in A,\ F(e) = U$.

1.5 Definition: The *union* of two soft sets F_A and G_B over a common universe U is the soft set H_C, where $C = A \bigcup B,$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e) & if \quad e \in A - B \\ G(e) & if \quad e \in B - A \\ F(e) \bigcup G(e) & if \quad e \in A \bigcap B \end{cases}$$

We write $F_A \bigcup G_B = H_C$.

1.6 Definition: The *intersection* of two soft sets F_A and G_B over a common universe U is the soft set H_C, where $C = A \bigcap B,$ and for all $e \in C$, $H(e) = F(e) \bigcap G(e)$. We write $F_A \bigcap G_B = H_C$.

1.7 Definition: For a soft set F_A over U, the relative *complement* of F_A is denoted by F_A^c and is defined by $F_A^c = F_A^1$, where $F^1 : A \to P(U)$ is a mapping given by

$F^1(e) = U - F(e)$ for all $e \in A$.

2. Soft Topology and Related Concepts

In this section, we present the definition of soft topology on a universe set and some related concepts. Henceforth, let X be an initial universe set and E be the fixed non-empty set of parameters with respect to X unless otherwise mentioned.

2.1 Definition: A collection T of subsets of X is said to be a *topology* on X if it satisfies the following properties.

(i) $\phi \in T$

(ii) $X \in T$

(iii)if $\{G_\alpha : \alpha \in \Delta\}$ is any collection of sets in T, then $\bigcup_{\alpha \in \Delta} G_\alpha \in T$

(iv)the intersection of any finite collection of sets in T is a set in T.

The pair (X, T) is called a topological space.

2.2 Definition: Let τ be the collection of soft sets over X. Then τ is called a *soft topology* on X, if τ satisfies the following properties.

(i) Φ, X belong to τ.

(ii) The union of any number of soft sets in τ belongs to τ.

(iii) The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X.

2.3 Definition: Let F_E be a soft set over X and $x \in X$. We say that $x \in F_E$ if $x \in F(e)\ \forall\, e \in E$. We say that $x \notin F_E$ if $x \notin F(e)$ for some $e \in E$.

2.4 Definition: Let (X, τ, E) be a soft topological space over X. Then

(i) The members of τ are called *soft open* sets in (X, τ, E).

(ii) A soft set F_E over X is said to be a *soft closed* set in (X, τ, E), if $F_E^c \in \tau$.

(iii) A soft set F_E is said to be a *soft neighborhood* of a point $x \in X$ if $x \in F_E$ and F_E is soft open in (X, τ, E).

(iv) The *soft interior* of a soft set F_E is the union of all soft open subsets of F_E. The soft interior of F_E is denoted by $Int_s(F_E)$.

(v) The *soft closure* of F_E is the intersection of all soft closed super sets of F_E. The soft closure of F_E is denoted by $cl_s(F_E)$.

(vi) A point $x \in X$ is said to be a *soft limit point* of a soft set F_E if every soft neighborhood of x contains a point of F_E different from x.

3. A New Approach : Soft Topological Concepts

In this section, we adapt a new approach and introduce soft topological concepts in a different way by assigning a topological structure to the universe set X .

3.1 Definition: Let (X,T) be a topological space. We say that a soft set F_E is $s-$*open* if $F(e)\in T$ for all $e\in E$. We denote the collection of all $s-$open sets with the symbol δ_T .

3.2 Proposition: δ_T forms a soft topology over X .

Proof: Since T is a topology on X ,$\phi\in T$ and $X\in T$. Hence $\Phi\in\delta_T$ and $X\in\delta_T$

Let $\{(F_\alpha)_E:\alpha\in\Delta\}$ be any collection of $s-$open sets in δ_T .

Then $(F_\alpha)_E(e)\in T$ for all $e\in E$ and for every $\alpha\in\Delta$

$\Rightarrow \bigcup_{\alpha\in\Delta}(F_\alpha)_E(e)\in T$ for all $e\in E$

$\Rightarrow \bigcup_{\alpha\in\Delta}(F_\alpha)_E\in\delta_T$.

Let $F_E\in\delta_T$ and $G_E\in\delta_T$. Then $F(e)\in T$ and $G(e)\in T$ for all $e\in E$

$\Rightarrow \quad F(e)\bigcap G(e)\in T$ for all $e\in E$

$\Rightarrow \quad F_E\bigcap G_E\in\delta_T$.

This shows that δ_T is a soft topology over X and hence (X,δ_T,E) forms a soft topological space.

3.3 Definition: Let (X,T) be a topological space. We say that a soft set F_E is $s-$*closed* if $(F(e))^c = X-F(e)\in T$ for all $e\in E$.

3.4 Proposition: In a soft topological space (X,δ_T,E),

(a) F_E is $s-$open $\Leftrightarrow$ F_E is soft open over X .

(b) F_E is $s-$closed $\Leftrightarrow$ F_E is soft closed over X .

(c) F_E is $s-$open $\Leftrightarrow$ F_E^c is $s-$closed.

3.5 Example: Let $T=\{\phi,X\}$. Then T is the indiscrete topology on X . Fix $e_0\in E$. If $F:E\to P(X)$ is defined by

$$F(e)=\begin{cases}\phi & if \quad e=e_0\\ X & if \quad e\neq e_0\end{cases}$$

then this soft set F_E is $s-$open and $F_E\in\delta_T$. From this example, it is clear that δ_T is not simply soft indiscrete eventhough T is indiscrete.

3.6 Example: If $T=P(X)$, the discrete topology on X then the soft topology δ_T corresponding to T becomes the collection of all soft subsets over X . This soft topology δ_T is called the soft discrete topology over X .

3.7 Definition: The *soft closure* of a soft set F_E in $\left(X,\delta_T,E\right)$ is the intersection of all $s-$closed super sets of F_E. The soft closure of F_E is the smallest $s-$closed set containing F_E.

3.8 Definition: The *soft interior* of a soft set F_E in $\left(X,\delta_T,E\right)$ is the union of all $s-$open subsets of F_E. The soft interior of F_E is the largest $s-$open set contained in F_E.

4. Soft Compactness and Soft Connectedness

In this section, we present a few interesting results on soft compactness and soft connectedness.

4.1 Definition: A soft topological space (X,τ,E) is said to be *soft compact* if every soft open cover of X contains a finite sub cover.

4.2 Proposition: (X,T) is compact $\Leftrightarrow$ $\left(X,\delta_T,E\right)$ is soft compact.

Proof: Suppose that (X,T) is compact. Let $\left\{(F_\alpha)_E : \alpha\in\Delta\right\}$ be a collection of $s-$open sets over X such that $X=\bigcup\limits_{\alpha\in\Delta}(F_\alpha)_E$

$\Rightarrow X=\bigcup\limits_{\alpha\in\Delta} F_\alpha(e)$ for any $e\in E$

Thus $\left\{F_\alpha(e):\alpha\in\Delta\right\}$ is an open cover of X. Since (X,T) is compact, there exist finitely many indices $\alpha_1,\alpha_2,\alpha_3,\ldots\alpha_n$ in Δ such that $X=F_{\alpha_1}(e)\bigcup F_{\alpha_2}(e)\bigcup\ldots\bigcup F_{\alpha_n}(e)$ for every $e\in E$.

$\Rightarrow X=\left(F_{\alpha_1}\right)_E\bigcup\left(F_{\alpha_2}\right)_E\bigcup\ldots\bigcup\left(F_{\alpha_n}\right)_E$

Thus every soft open cover of X has a finite subcover.

Hence $\left(X,\delta_T,E\right)$ is soft compact. Conversely, suppose that $\left(X,\delta_T,E\right)$ is soft compact. Let $\left\{G_\alpha:\alpha\in\Delta\right\}$ be an open cover of X. Then $X=\bigcup\limits_{\alpha\in\Delta}G_\alpha$.

Define $F_\alpha : E\to P(X)$ such that $F_\alpha(e)=G_\alpha \quad \forall\ e\in E$.

Then $X=\bigcup\limits_{\alpha\in\Delta}(F_\alpha)_E$. Since $\left(X,\delta_T,E\right)$ is soft compact, there exist finitely many indices $\alpha_1,\alpha_2,\alpha_3,\ldots\alpha_n$ in Δ such that $X=\left(F_{\alpha_1}\right)_E\bigcup\left(F_{\alpha_2}\right)_E\bigcup\ldots\bigcup\left(F_{\alpha_n}\right)_E$

$\Rightarrow X=G_{\alpha_1}\bigcup G_{\alpha_2}\bigcup\ldots\bigcup G_{\alpha_n}$

Thus every open cover contains a finite sub cover. Hence (X,T) is compact.

4.3 Definition: A soft topological space (X,τ,E) is said to be *soft disconnected* if there exist two non null soft open sets F_E and G_E such that $X=F_E\bigcup G_E$ and $F_E\bigcap G_E=\Phi$. A soft topological space (X,τ,E) is said to be *soft connected* if it is not soft disconnected.

4.4 Proposition: (X,T) is disconnected $\Rightarrow$ (X,δ_T,E) is soft disconnected.

Proof: Suppose that (X,T) is disconnected. Then there exist two non-empty open sets V and W such that $X = V \bigcup W$ and $V \bigcap W = \phi$.
Define $F : E \to P(X)$ and $G : E \to P(X)$ such that $F(e) = V$ and $G(e) = W$ for all $e \in E$. Then F_E and G_E are two non null soft open sets in (X,δ_T,E) such that $X = F_E \bigcup G_E$ and $F_E \bigcap G_E = \Phi$. Hence (X,δ_T,E) is soft disconnected.

4.5 Proposition: (X,δ_T,E) is soft connected $\Rightarrow$ (X,T) is connected.

4.6 Remark: The converse of the above Proposition-(4.5) is not true. It is not necessary that (X,δ_T,E) is soft connected eventhough (X,T) is connected. It can be seen from the following example.

4.7 Example: Let $X = \{a,b,c\}$ and $T = \{\phi, X, \{a\}\}$. Then (X,T) is connected.
Define $F : E \to P(X)$ and $G : E \to P(X)$ such that

$$F(e) = \begin{cases} \phi & if \quad e = e_0 \\ X & if \quad e \neq e_0 \end{cases} \text{ and } G(e) = \begin{cases} X & if \quad e = e_0 \\ \phi & if \quad e \neq e_0 \end{cases}$$

Where $e_0 \in E$ is a fixed parameter. Clearly F_E and G_E are two non null soft open sets in (X,δ_T,E) such that $X = F_E \bigcup G_E$ and $F_E \bigcap G_E = \Phi$.

Hence (X,δ_T,E) is soft disconnected.

5. Soft T_1 and T_2 Spaces

5.1 Definition: A $T_1 - space$ is a topological space in which, given any pair of distinct points, each has a neighborhood which does not contain other.

5.2 Definition: A topological space (X,T) is said to be a $T_2 - space$ if for each pair of distinct points x and y in X there exist two neighborhoods V and W of x and y respectively such that $V \bigcap W = \phi$.

5.3 Definition: We say that a soft set $F : E \to P(X)$ is a *soft single point set* if $F(e) = \{x\}$ for all $e \in E$ and for some $x \in X$.

5.4 Definition: We say that a soft topological space (X,τ,E) is a *soft* $T_1 - space$ if each soft single point set is soft closed in (X,τ,E).

5.5 Proposition: (X,T) is T_1- space $\Leftrightarrow$ (X,δ_T,E) is a soft T_1-space.

5.6 Definition: We say that a soft topological space (X,τ,E) is a *soft* $T_2 - space$ if each pair of distinct points x and y in X there exist two soft neighborhoods F_E and G_E of x and y respectively such that $F_E \cap G_E = \Phi$.

5.7 Proposition: (X,T) is T_2 −space $\Leftrightarrow$ $\left(X,\delta_T,E\right)$ is a soft T_2 −space.

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