International Journal of Applied Engineering Research
ISSN 0973-4562 Volume 10, Number 11 (2015) pp. 29445-29455

http://www.ripublication.com

State Estimation Based on Probabilistic Contaminated Exponential Square Method

***Gomathi Venugopal, Uma Velayutham**
College of Engineering, Anna University,
Chennai, Tamil Nadu, India

Abstract

This paper presents a detailed study of the probability density function of different algorithms used to solve a power system state estimation problem. The widely used state estimator is the Weighted Least Square (WLS) estimator. Maximum Exponential Square (MES) uses an exponential objective which when maximized solves the non-linear estimation problem in more effective manner. Based on the study, a new solution method using Probabilistic Contaminated Exponential Square (PCES) objective is proposed and its performance is evaluated based on Normalized Error and Computation time.

Keywords: State Estimation**,** WLS, MES, Conventional, Probabilistic Contaminated Exponential Square**.**

Introduction

Power system is a network of electrical components used to supply, transmit and use electric power. Making sure that the voltage, frequency and power in the network will be within permissible limits is one of the greatest challenges in power system engineering. This challenge is overcome by power system state estimation. Power system state estimation is the act of estimating the state of the network from the redundant telemetry measurements [1]. Most of the state estimation (SE) algorithms used are based on Weighted Least Square (WLS) approach.

Conventional WLS state estimator is based on Supervisory Control and Data Acquisition (SCADA) measurements [2]. The conventional state estimator provides the estimates of the power system states, i.e., bus voltages and angles, on the basis of measurements obtained from SCADA system. Usually, a WLS estimator is used to find the estimates of the states [3]. After the advent of Global Positioning System (GPS) synchronized measurements obtained by (Phasor Measurement Units) PMUs, effective techniques are required to incorporate the extremely accurate PMU measurements into State Estimation in order to enhance the performance of WLS

State Estimation. Different applications of PMU measurements that contain a GPS time stamp are discussed in [4]. Optimal placement of PMUs in power system enhances the state estimation problem [5]. Various algorithms that find the minimal set of PMU placement for power system state estimation are discussed in [6-10]. Different ways to incorporate the voltage and current measurements by PMUs, along with conventional measurements in State Estimation are discussed in [11-20]. Three possible ways in which the current measurements by the PMUs can be directly incorporated in a state estimator:

- Inclusion of current phasor in polar form,
- Inclusion of current phasor in rectangular form and
- Inclusion of pseudo voltage measurements.

Pseudo voltage measurements are calculated with the help of current phasor measurement and known line parameters. More accurate estimates can be obtained by including highly accurate PMU measurements.

Recently an alternative SE algorithm based on Maximum Exponential Square (MES) has emerged to solve power system state estimation problems. MES problem is modelled as a maximisation of an exponential square objective of residuals [21]. To obtain a better understanding, the probability density functions of the existing state estimation methods are analysed in this paper. This paper also proposes a new method to solve the state estimation problem based on Probabilistic Contaminated Exponential Square. The proposed technique is tested on IEEE 14-bus system and its estimates are compared with those obtained through other state estimation techniques. This paper is organised as follows: Section 2 gives the Gaussian probability density function of the available state estimation methods. Section 3 proposes the modelling of Probabilistic Contaminated Exponential Square method. Section 4 gives the results and performance analysis, which is followed by conclusion in Section 5.

Gaussian Probability Density Function

This section gives the (Gaussian) probability density function of the existing state estimation methods. The probability density function describes the probability distribution function curve of the objective function. The Gaussian probability density function *f(z)* of Conventional state estimation methods namely conventional WLS [2] and conventional MES are shown in Fig 1.

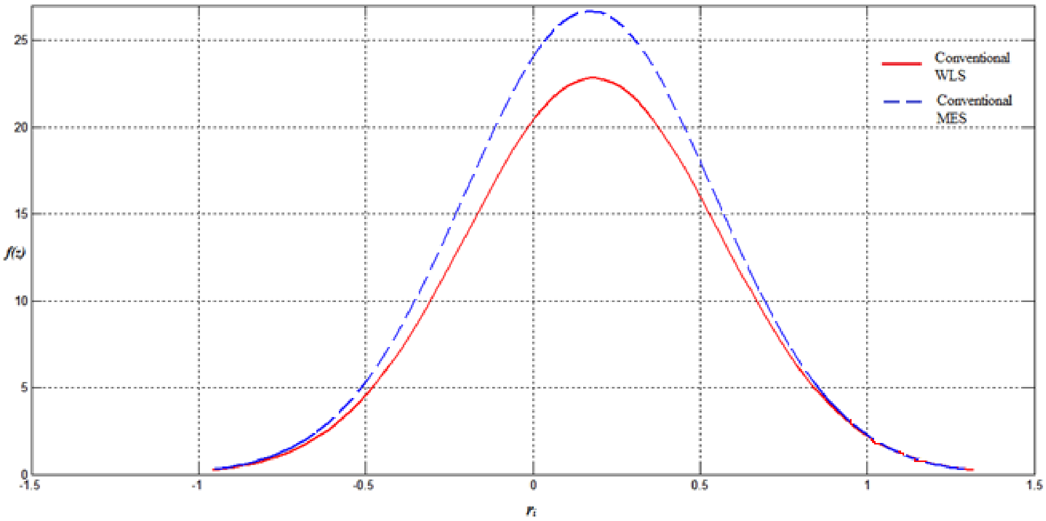

Figure 1: Probability distribution curves of Conventional WLS and MES Estimators

The plot of Gaussian probability density function with respect to the residual is called the Distribution Curve. The distribution curves obtained for conventional WLS and conventional MES have same mean but different variances. The ranges covered by the conventional WLS and MES distribution curves along the x-axis considering the residuals are almost equal. The distribution curves are bell-shaped for all methods yet the areas covered by them are different. The area covered by these distribution curves are said to be the search spaces and are different for various methods. The distribution curve of the MES objective has a peak higher than that of the WLS and also a larger variance. Hence this objective with an exponential function is said to occupy a greater search space than that of WLS. With the greatest search space, MES accurately estimates the state variable when compared to conventional WLS. Therefore it is concluded that the objective function with an exponential distribution is more effective.

Estimator Based on Probabilistic Contaminated Exponential Square

Theoretical Background

Power System State Estimation problem is to solve a over-determined set of non-linear equations [5] in the form

$$z = h(x) + e \tag{1}$$

x is the state vector of the system. z, h and e are $m \times 1$ actual measurement vector, non-linear function of calculated measurement vector and measurement error vector respectively.

The probabilistic contaminated exponential square is a type of contaminated normal distribution which is a mixture of normal distributions [22]. A contaminated normal distribution is a type of mixture distribution for which observed values can come from one of multiple normal distributions. A common type of contaminated normal distribution is a composite of two normal distributions with the same mean, but with different variances, such that only a minority of the values come from the distribution with the larger variance. If the proportion of values from the distribution with the larger variance is small enough, the contaminated normal distribution may look like a normal distribution with outliers.

When considering only one Gaussian Kernal few outliers may exist and in order to formulate the objective function covering those outliers, two Gaussian Kernals are used in the proposed Probabilistic Contaminated Exponential Square method. During an instant, the variable follows any one of the distribution and hence a probability term is introduced. Let W be contaminated normal distributed, then

$$W = zI_{1-\epsilon} + \sigma_2 z(1 - I_{1-\epsilon}) \tag{2}$$

where z is normally distributed and $I_{1-\epsilon}$ is a discrete random variable defined by:

$$I_{1-\epsilon} = \begin{cases} 1 & \text{with the probability } 1-\epsilon \\ 0 & \text{with the probability } \epsilon \end{cases} \tag{3}$$

Assuming z and $I_{1-\epsilon}$ are independent.

The interdependence of z and $I_{1-\epsilon}$ imply that the cumulative distribution function of W is

$$F_W(\omega) = P(W \le \omega)$$

$$= P[W \le \omega | I_{1-\epsilon} = 1]P[I_{1-\epsilon} = 1] + P[W \le \omega | I_{1-\epsilon} = 0]P[I_{1-\epsilon} = 0] \tag{4}$$

When $I_{1-\epsilon} = 1$, $W = z$ and

When $I_{1-\epsilon} = 0$ $W = \sigma_2 z$

$$F_W(\omega) = P[zI_{1-\epsilon} \le \omega](1-\epsilon) + P[\sigma_2 z \le \omega]\epsilon$$

$$= P[z \le \omega / I_{1-\epsilon}](1-\epsilon) + P[z \le W/\sigma_2]\epsilon$$

$$F_W(\omega) = \phi(\omega)(1-\epsilon) + \phi(\omega/\sigma_2)\epsilon \tag{5}$$

The probability density function is:

$$f_W(\omega) = \phi(\omega)(1-\epsilon) + \phi(\omega/\sigma_2)\frac{\epsilon}{\sigma_2} \tag{6}$$

The density function ϕ considering the variable to have an exponential distribution is as follows:

$$f_x(x) = \sum_{i=1}^{m}\left[\frac{1-\epsilon}{m\sqrt{2\pi}\sigma_1}\exp\left(\frac{-1}{2}\left(\frac{z-h(x)}{\sigma_1}\right)^2\right) + \frac{\epsilon}{m\sqrt{2\pi}\sigma_2}\exp\left(\frac{-1}{2}\left(\frac{z-h(x)}{\sigma_2}\right)^2\right)\right] \tag{7}$$

Proposed Formulation

The objective function of the probabilistic contaminated exponential square estimator is formulated as the probabilistic sum of the weighted exponentials of residuals with different variances and differentiated using a term defining probability.

$$\max_x J(x) = \sum_{i=1}^{m} \left[w_i(1-\epsilon) exp\left(\frac{-1}{2}\left(\frac{z-h(x)}{\sigma_1}\right)^2\right) + \frac{w_i\epsilon}{\sigma_2} exp\left(\frac{-1}{2}\left(\frac{z-h(x)}{\sigma_2}\right)^2\right)\right] \quad (8)$$

σ_1 and σ_2 are two different standard deviations. w_i is the weight of the i^{th} measurement. The state vector consists of the set of bus voltages and phase angles.

The Probabilistic Contaminated Exponential Square estimator is formulated as a maximisation problem. A term defining probability is included in the objective so as to include only a minority of the values from the distribution with larger variance

The term contaminated comes into existence with mixing up of two different normal distribution curves having different standard deviations σ_1 and σ_2. By proper selection of σ_1 and σ_2, two different normal distribution can be brought together to fit into the objective function and better estimates could be obtained. An exponential square objective function is considered as its distribution curve covers a larger distribution than that covered by the squares of measurement error.

Solution Method

Redefining the objective function in Quadratic Form and converting the non-linear set of equations into linear set of equations using Maclaurin's Series neglecting the higher order terms, we get the following iterative formula

$$Q_1\Delta x + Q_2\Delta x + (1-\in)H^T Y_1\big(z-h(x)\big) + \in H^T Y_2\big(z-h(x)\big) = 0 \quad (9)$$

$$\Delta x = (Q_1+Q_2)^{-1}\{-[(1-\in)H^T Y_1\big(z-h(x)\big) + \in H^T Y_2\big(z-h(x)\big)]\} \quad (10)$$

Here just like WLS and MES $\partial H/\partial x$ is ignored and an approximate Hessian Matrix is considered.

$$\frac{\partial^2 J}{\partial x^2} = -\left\{H^T Y_1(x)\left[I - diag\left\{\frac{z-h(x)}{\sigma_1}\right\}^2\right]H + H^T Y_2(x)\left[I - diag\left\{\frac{z-h(x)}{\sigma_2}\right\}^2\right]H\right\} = -(Q_1+Q_2) \quad (11)$$

where $Y_j(x)$ is a diagonal matrix with diagonal element

$$Y_{j_{ii}}(x) = w_i exp\left(-\left(\frac{z-h(x)}{\sqrt{2}\,\sigma_j}\right)^2\right) \quad (12)$$

The proposed distribution is assumed to possess zero expectation and unit variance. Then,

$$\sigma_2 = \frac{1}{\epsilon} - \frac{(1-\epsilon)}{\epsilon}\left({\sigma_1}^2 + \frac{\mu_1}{\epsilon}\right) \quad (13)$$

In probabilistic contaminated exponential square method, Δx^k is solved iteratively and the state vector is updated during every iteration k ($x^{k+1} = x^k + \Delta x^k$) until all the elements of Δx becomes less than a pre-specified convergence limit. The steps followed to solve a State Estimation problem based on Probabilistic Contaminated Exponential Square Method are illustrated in Fig. 2.

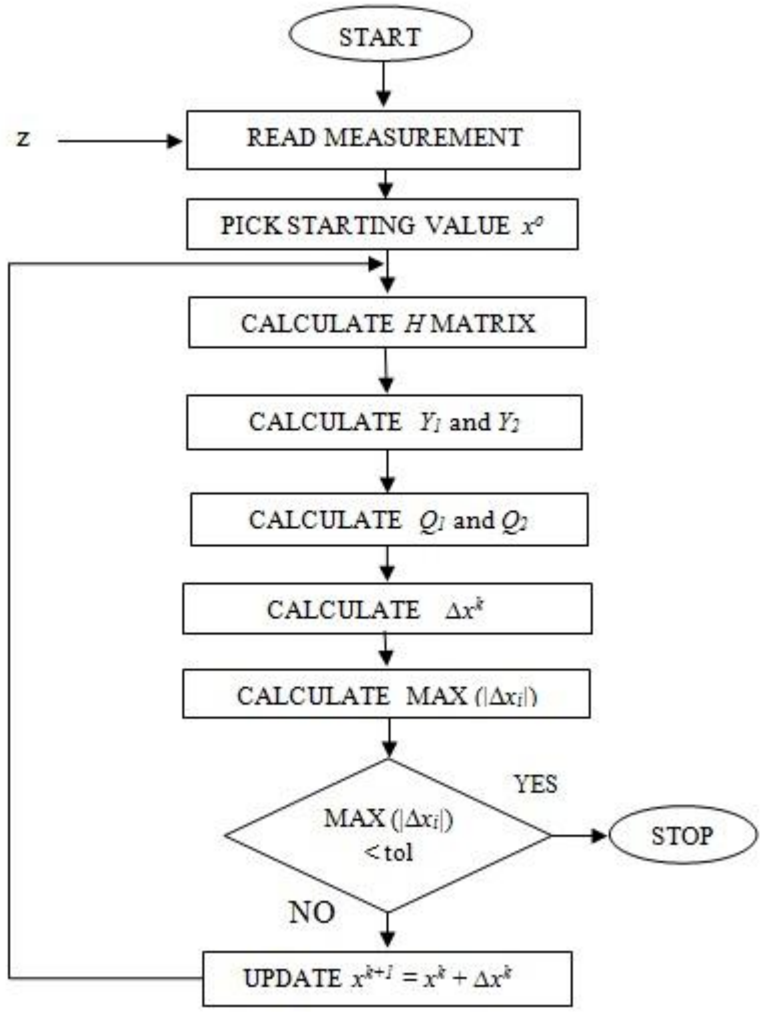

Figure 2: Steps to implement Probabilistic Contaminated Exponential Square Estimator

Results

The state estimation methods are applied on the IEEE 14-bus. The test system was assumed to have conventional measurements [23]. The optimal locations of the PMUs to make the system completely observable are the buses 6 and 9. Bus 1 is the reference bus in the test system. An additional PMU is placed on bus 1 to provide the reference phase angle [4]. The estimates obtained by the proposed method are compared with those of the available methods. The performance of the proposed method is analysed based on different parameters namely Redundancy Factor, Normalized Error and Computation Time.

Comparison of Estimates From Various Methods

The state estimates are obtained using Conventional WLS, Conventional MES and PCES State Estimators. The voltages and phase angles estimated by the proposed Probabilistic Contaminated Exponential Square methods are given in bold in Table 1 and Table 2. The estimates obtained for the proposed method are compared with those obtained from Weighted Least Square and Maximum Exponential Square estimators. The Voltage estimates obtained at the buses using different state estimation methods both different forms of conventional and hybrid methods are given in Table 1 and the phase angles are given in Table 2.

Table 1: Voltage Estimates (in pu) obtained by different State Estimation Criteria

Bus Number	True Values	Conventional WLS	Conventional MES	Probabilistic Contaminated Exponential Square
1	1.06	1.0068	1.0479	**1.0479**
2	1.045	0.9899	1.0316	**1.0316**
3	1.01	0.9518	0.9948	**0.9948**
4	1.019	0.9579	0.9995	**0.9995**
5	1.02	0.9615	1.003	**1.003**
6	1.07	1.0185	1.0572	**1.0572**
7	1.062	0.9919	1.0323	**1.0323**
8	1.09	1.0287	1.0672	**1.0672**
9	1.056	0.9763	1.0171	**1.0171**
10	1.051	0.9758	1.0165	**1.0165**
11	1.057	0.9932	1.033	**1.033**
12	1.055	1.0009	1.0403	**1.0403**
13	1.05	0.994	1.0336	**1.0336**
14	1.036	0.9647	1.0058	**1.0058**

Performance Analysis

The performances of different State Estimation methods are analysed using two factors namely: Nomalized Error [23] and Computation time. The Performance Index reflects the degree of approximation between the estimated value and the true value.

Measurement Redundancy (k) is the ratio of number of measurements (m) to the number of unknown state variables (n).

$$k = \frac{m}{n} \tag{14}$$

Table 2: Phase Angle Estimates (In Degree) Obtained By Different State Estimation Criteria

Bus Number	True Values	Conventional WLS	Conventional MES	Probabilistic Contaminated Exponential Square
1	0	0	0	**0**
2	-4.98	-5.53	-5.1019	**-5.0995**
3	-12.72	-14.2	-13.0783	**-13.0716**
4	-10.33	-11.4	-10.5112	**-10.5059**
5	-8.78	-9.76	-8.9896	**-8.9851**
6	-14.22	-16.1	-14.8243	**-14.817**
7	-13.37	-14.8	-13.5858	**-13.5789**
8	-13.36	-14.8	-13.5855	**-13.5787**
9	-14.94	-16.5	-15.2108	**-15.2031**
10	-15.1	-16.7	-15.4313	**-15.4237**
11	-14.79	-16.5	-15.2467	**-15.2392**
12	-15.07	-17	-15.6959	**-15.6882**
13	-15.16	-17.1	-15.7308	**-15.7231**
14	-16.04	-17.9	-16.4936	**-16.4854**

Greater the number of measurements higher is the measurement redundancy. This suppresses bad data efficiently. So that the Normalized error is reduced as redundancy level is increased. In practice the range of the redundancy factor k, has been found useful if its value is between 1.5 and 2.8. i.e.,$1.5 \leq k \leq 2.8$.

The Normalized Error is used to analyse the performance of a method. Normalized Error is used to evaluate the accuracy of a method. Normalized error (NE) is evaluated using the following formula.

$$NE = \frac{\|\hat{x} - x_t\|_2}{\|x_t\|_2} \tag{15}$$

where $\hat{x}$ is the estimated value and x_t is the true value.

NE represents the degree of closeness between the estimated and true values. If the NE is small the method is said to be more accurate. Table 3 shows the NE in estimating the voltages and phase angles using each method. From the results it could be observed that the method for which the search space of the distribution curve is large gives more accurate estimates and its NE is the least.

Table 3: Normalized Error obtained by different State Estimation Methods

State Estimation Methods	Redundancy Factor	Normalized Error in estimating the Voltage Magnitudes	Normalized Error in estimating the phase angles
Conventional WLS	1.57	0.003568	0.013412
Conventional MES	1.57	0.000485	0.000863
Probabilistic Contaminated Exponential Square	1.57	0.000485	0.000835

The different state estimation algorithms are simulated for IEEE 14 bus test system in Matlab7.10.0 (R2010a) and their computation time is found and compared. The computation time is the time taken for the iteration to converge. The computation time of various state estimation methods is given in Table 4.

Table 4: Computation Time for Different State Estimation Methods

State Estimation Methods	Computation Time (seconds)
Conventional Weighted Least Square State Estimator	0.5140
Conventional Maximum Exponential Square State Estimator	0.5520
Probabilistic Contaminated Exponential Square State Estimator	0.5540

The time taken for the Probabilistic contaminated Exponential Square method to converge is slightly higher when compared to that of rest of the methods but the results prove that, the estimate obtained by Probabilistic contaminated Exponential Square method is more close to the actual values. Hence it could be said that this method has a very good convergence characteristics.

Conclusion

In this context, this paper presents a comparative analysis of the estimates obtained by different state estimation methods for IEEE 14 bus test system with same Redundancy Factor based on Normalized Error and Computation Time. The distribution curves of already existing conventional state estimation solution methods have been analysed. Based on that, a new state estimation solution method has been proposed. The iterative formula for the proposed objective has been formulated. The Probabilistic Contaminated Exponential Square objective function has a larger search space as it combines two different Gaussian Kernals. The proposed objective function covers the outliers unlike other state estimators and hence yields more accurate estimates. Normalized error for the proposed method is low even for a lower redundancy factor and has a very good convergence compared to all other existing methods.

References

[1] J. Bialek, 'Modified power-to-current measurement transformation for power system state estimation', International Journal of Electrical Power & Energy Systems, Volume 6, Issue 4, Pages 195-202, October 1984.

[2] A. Abur and A. Gomez Exposito, Power System state estimation: theory and implementation, New York, NY: Marcel Gekker, 2004.

[3] IEEE Power Engineering Society, IEEE standard for synchrophasors for power systems, IEEE Std C37.118TM-2005.

[4] Phadke, A.G., Thorp, J.S.: 'Synchronized phasor measurements and their applications' (Springer Science and Buisness Media, New York, 2008).

[5] S. Chakrabarti, E. Kyriakides, and D. G. Eliades, 'Placement of synchronized measurements for power system observability', IEEE Trans. Power Delivery vol. 24, no. 1, pp. 12-19, Sep. 2008.

[6] Nuqui, R.F., Phadke, A.G.: 'Phasor measurement unit placement techniques for complete and incomplete observability', IEEE Trans. Power Syst., 2005, 20, (4), pp. 2381-2388.

[7] Gou, B.: 'Generalized integer linear programming formulation for optimal PMU placement', IEEE Trans. Power Syst., 2008, 23, (3), pp. 1099-1104.

[8] Chakrabarti, S., Kyriakides, E.: 'Optimal placement of phasor measurement units for power system observability', IEEE Trans. Power Syst., 2008, 23, (3), pp. 1433-1440.

[9] Gou, B.: 'Optimal placement of PMUs by integer linear programming', IEEE Trans. Power Syst., 2008, 23, (3), pp. 1525-1526.

[10] N. M. Manousakis, G. N. Korres, P. S., Georgilakis: 'Taxonomy of PMU Placement Methodologies', IEEE Transactions on Power Systems, Vol. 27, No. 2, pp. 1070-1077, May 2012.

[11] Nuqui, R.F., Phadke, A.G.: 'Hybrid linear state estimation utilizing synchronized phasor measurements', Proc. IEEE Power Tech. Conf., 2007.

[12] Zhou, M., Centeno, V.A., Thorp, J.S., Phadke, A.G.: 'An alternative for including phasor measurements in state estimators', IEEE Trans. Power Syst., 2006, 21, (4), pp. 1930-1937.

[13] Bi, T.S., Qin, X.H., Yang, Q.X.: 'A novel hybrid state estimator for including synchronized phasor measurements', Electr. Power Syst. Res., 2009, 78, (8), pp. 2452-2458.

[14] Vanfretti, L., Chow, J.H., Sarawgi, S., Ellis, D., Faradanesh, B.: 'A framework for estimation of power systems based on synchronized phasor measurement data' Proc. PES General Meeting, 2009.

[15] Valverde, G., Chakrabarti, S., Kyriakides, E., Terzija, V.: 'A constrained formulation for hybrid state estimation', IEEE Trans. Power Syst., 2011, 26, (3), pp. 1102-1109.

[16] Sodhi, R., Srivastava, S.C., Singh, S.N.: 'Phasor-assisted hybrid state estimator', Electr. Power Compon. Syst., 2010, 38, (5), pp. 533-544.

[17] Chakrabarti, S., Kyriakides, E., Ledwich, G., Ghosh, A.: 'A comparative study of the methods of inclusion of PMU current phasor measurements in

a hybrid state estimator'. Proc. IEEE Power Society General Meeting, 2010.

[18] G. N. Korres, N. M. Manousakis: 'State estimation and bad data processing for systems including PMU and SCADA measurements', Electric Power Systems Research, Vol. 81, pp. 1514-1524.

[19] G. N. Korres, N. M. Manousakis: 'State estimation and observability analysis for phasor measurement unit measured systems', IET Generation, Transmission and Distribution, Vol. 6, No. 9, pp. 902-913, Sep. 2012.

[20] Chakrabarti, S., Kyriakides, E., Ledwich, G., Ghosh, A.: 'Inclusion of PMU current phasor measurements in a power system state estimator', IET Generation, Transmission and Distribution, 2010, 4, (10), pp. 1104-1115.

[21] W. Wu, Y. Guo, B. Zhang, A. Bose and S. Hongbin: 'Robust State Estimation based on Maximum Exponential Square', IET Generation, Transmission and Distribution, 2011, Vol. 5 Iss.11, pp.1165-1172.

[22] Hogg, McKean and Craig: Introduction to Mathematical statistics, 6th ed., Pearson Education, 2004

[23] Fang Chen, Xueshan Han, Zhiyuan Pan and Li Han, 'State estimation model and algorithm including PMU', DRPT, Nanjing, April 2008.