

Decision Feedback Signal Processing based on Constant Modulus Errors for Wireless Sensor Networks

Namyong Kim

*Division of Electronic Information and Communications Eng.
Kangwon National University, Samcheok, 245-711, KOREA
E-mail: namyong@kangwon.ac.kr*

Abstract

In this paper, a recursive estimation of the Euclidean distance (ED) of constant modulus errors and its application to blind algorithms based on decision feedback (DF) structures is proposed for signal processing in severe wireless channel conditions. The proposed DF algorithm based on recursive ED yields the results that the number of multiplications increases linearly with the sample size N while that of the block processing method does squarely. The results of fast convergence performance and low complexity show that the proposed DF algorithm is an efficient signal processing method for wireless sensor networks.

Keywords: Constant modulus error, Euclidean distance, Decision feedback, Computational complexity

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1 Introduction

Wireless sensor network has been an important area of research in recent years [1][2]. The environments of indoor wireless sensor networks include impulsive noise and the networks require blind signal processing without training sequences [3][4]. A blind signal processing method based on constant modulus error (CME) and Euclidean distance (ED) has been developed for robustness to impulsive noise [5]. One of the drawbacks of the ED-CME algorithm is its heavy computational burden. For reducing its computational complexity, a recursive approach to gradient estimation has been proposed [6]. As the wireless channel conditions of indoor sensor networks becomes

worse, a decision feedback (DF) approach to the ED-CME is proposed in this paper to compensate the severe channel distortions, impulsive noise and computational problems.

2 Recursive Estimation of ED-CME cost function

The CME $e_{CME,k}$ at time k is defined as in (1) with the filter output power $|y_k|^2$ and constant modulus R_2 predefined as $R_2 = E[|d_i|^4]/E[|d_i|^2]^2$ with symbol set \mathcal{A} [7].

$$e_{CME,k} = |y_k|^2 - R_2 \quad (1)$$

The ED between the distribution function of CME $f_E(e_{CME})$ and Dirac-delta function $\delta(e_{CME})$ is known as robust against impulsive noise [5].

$$ED = \int [f_E(e) - \delta(e)]^2 de = \int f_E^2(e) de - 2 \int f_E(e) \delta(e) de + \int \delta^2(e) de \quad (2)$$

where the $f_E(e_{CME})$ can be estimated by kernel density estimation method with

Gaussian kernel $G_\sigma(e) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-\frac{e^2}{2\sigma^2})$ and kernel size σ [8].

$$f_E(e_{CME}) = \frac{1}{N} \sum_{i=k-N+1}^k G_\sigma(e_{CME} - e_{CME,i}) \quad (3)$$

Substituting (3) into (2) and discarding the irreducible term $\int \delta^2(e) de$, we have

$$ED = \frac{1}{N^2} \sum_{i=k-N+1}^k \sum_{j=k-N+1}^k G_{\sigma\sqrt{2}}(|y_i|^2 - |y_j|^2) - \frac{2}{N} \sum_{i=k-N+1}^k G_\sigma(-(|y_i|^2 - R_2)) \quad (4)$$

The cost function ED can be used to check the state of the convergence of employed algorithms. The problem is that ED estimation is computationally burdensome due to double summation operations. In this section, a recursive approach to the estimation ED is proposed in order to reduce its computational burden.

We consider two time regions of ED ; the initial state $1 \leq k < N$ when only k CME samples can be utilized and the steady state $k \geq N$ when N samples can fully used. The ED_k^I for the initial state at time k is rewritten as

$$ED_k^I = YY_k^I - YR_k^I = \frac{1}{k^2} \sum_{i=1}^k \sum_{j=1}^k G_{\sigma\sqrt{2}}(|y_i|^2 - |y_j|^2) - \frac{2}{k} \sum_{i=1}^k G_\sigma(-(|y_i|^2 - R_2)) \quad (5)$$

At time $k+1$,

$$YY'_{k+1} = \frac{k^2}{(k+1)^2} \frac{1}{k^2} \sum_{i=1}^{k+1} \sum_{j=1}^{k+1} G_{\sigma\sqrt{2}} (|y_i|^2 - |y_j|^2) \quad (6)$$

$$YR'_{k+1} = \frac{k}{(k+1)} \frac{2}{k} \sum_{i=1}^{k+1} G_{\sigma} (-|y_i|^2 - R_2) \quad (7)$$

In the first place, we separate YY'_k from YY'_{k+1} as the following steps.

$$\begin{aligned} YY'_{k+1} &= \frac{k^2}{(k+1)^2} \frac{1}{k^2} \sum_{i=1}^{k+1} \left(\sum_{j=1}^k G_{\sigma\sqrt{2}} (|y_i|^2 - |y_j|^2) + G_{\sigma\sqrt{2}} (|y_i|^2 - |y_{k+1}|^2) \right) \\ &= \frac{k^2}{(k+1)^2} YY'_k + \frac{2}{(k+1)^2} \sum_{i=1}^k G_{\sigma\sqrt{2}} (|y_i|^2 - |y_{k+1}|^2) + \frac{1}{(k+1)^2} G_{\sigma\sqrt{2}} (0) \end{aligned} \quad (8)$$

Similarly, YR'_k can also be separated from YR'_{k+1} as

$$YR'_{k+1} = \frac{k}{(k+1)} YR'_k + \frac{2}{(k+1)} G_{\sigma} (-|y_{k+1}|^2 - R_2) \quad (9)$$

The equation (8) and (9) indicate that the initial state P'_k can be recursively estimated. Then the steady state ED_k^S becomes.

$$ED_k^S = YY_k^S - YR_k^S = \frac{1}{N^2} \sum_{i=k-N+1}^k \sum_{j=k-N+1}^k G_{\sigma\sqrt{2}} (|y_i|^2 - |y_j|^2) - \frac{2}{N} \sum_{i=k-N+1}^k G_{\sigma} (-|y_i|^2 - R_2) \quad (10)$$

The YY_{k+1}^S and YR_{k+1}^S at time $k+1$ can be expressed in (11) and (12).

$$\begin{aligned} YY_{k+1}^S &= YY_k^S + \frac{2}{N^2} \sum_{j=k-N+1}^k G_{\sigma\sqrt{2}} (|y_{k+1}|^2 - |y_j|^2) \\ &\quad - \frac{2}{N^2} \sum_{j=k-N+1}^k G_{\sigma\sqrt{2}} (|y_{k-N+1}|^2 - |y_j|^2) + \frac{2}{N^2} G_{\sigma\sqrt{2}} (0) - \frac{2}{N^2} G_{\sigma\sqrt{2}} (|y_{k+1}|^2 - |y_{k-N+1}|^2) \end{aligned} \quad (11)$$

$$YR_{k+1}^S - \frac{2}{N} G_{\sigma} (-|y_{k-N+1}|^2 - R_2) = YR_k^S + \frac{2}{N} G_{\sigma} (-|y_{k+1}|^2 - R_2) \quad (12)$$

In short, the initial state ED_k^I of the cost function (4) can be recursively estimated by (8) and (9), and the steady state ED_k^S can be done by (11) and (12).

3 DF algorithm based on the recursive ED

The input vector $\mathbf{X}_k = [x_k, x_{k-1}, \dots, x_{k-P+1}]^T$ is for the feed-forward (FF) filter and the decision symbol vector $\hat{\mathbf{D}}_{k-1} = [\hat{d}_{k-1}, \hat{d}_{k-2}, \dots, \hat{d}_{k-Q-2}]^T$ is for the feedback (FB) filter where \hat{d}_k is a decided symbol through a decision device. With the FF weight vector $\mathbf{W}_k^F = [w_{k,0}^F, w_{k,1}^F, w_{k,2}^F, \dots, w_{k,P-1}^F]^T$ and the FB weight vector $\mathbf{W}_k^B = [w_{k,0}^B, w_{k,1}^B, w_{k,2}^B, \dots, w_{k,Q-1}^B]^T$, the output y_k becomes (13)[9].

$$y_k = \mathbf{X}_k^T \mathbf{W}_k^F + \hat{\mathbf{D}}_{k-1}^T \mathbf{W}_k^B \quad (13)$$

With the convergence parameter μ , \mathbf{W}_k^F and \mathbf{W}_k^B are updated as

$$\mathbf{W}_{k+1}^F = \mathbf{W}_k^F + \mu \cdot \nabla_k^F \quad (14)$$

$$\mathbf{W}_{k+1}^B = \mathbf{W}_k^B + \mu \cdot \nabla_k^B \quad (15)$$

Unlike the approach in our previous work described in [10], the gradients ∇_k^F and ∇_k^B will be obtained in this paper by directly taking derivatives of the proposed recursive ED obtained in the previous section.

Firstly, taking the derivative of the recursive ED_k^I , that is, (8) and (9), with respect to \mathbf{W}^F we have FF gradient in the initial state $\nabla_k^{F,I} = \frac{\partial YY_k^I}{\partial \mathbf{W}^F} - \frac{\partial YR_k^I}{\partial \mathbf{W}^F}$ as

$$\frac{\partial YY_{k+1}^I}{\partial \mathbf{W}^F} = \frac{k^2}{(k+1)^2} \frac{\partial YY_k^I}{\partial \mathbf{W}^F} + \frac{2}{(k+1)^2} \sum_{i=1}^k \frac{\partial}{\partial \mathbf{W}^F} G_{\sigma\sqrt{2}}(|y_i|^2 - |y_{k+1}|^2) \quad (16)$$

$$\frac{\partial YR_{k+1}^I}{\partial \mathbf{W}^F} = \frac{k}{(k+1)} \frac{\partial YR_k^I}{\partial \mathbf{W}^F} + \frac{2}{(k+1)} \frac{\partial}{\partial \mathbf{W}^F} G_{\sigma}(-|y_{k+1}|^2 - R_2) \quad (17)$$

Inserting (13) into (16) and (17) leads to

$$\frac{\partial YY_{k+1}^I}{\partial \mathbf{W}^F} = \frac{k^2}{(k+1)^2} \frac{\partial YY_{k+1}^I}{\partial \mathbf{W}^F} + \frac{2}{\sigma^2(k+1)^2} \sum_{i=1}^k (|y_i|^2 - |y_{k+1}|^2) \cdot G_{\sigma\sqrt{2}}(|y_i|^2 - |y_{k+1}|^2) (y_{k+1} \mathbf{X}_{k+1} - y_i \mathbf{X}_i) \quad (18)$$

$$\frac{\partial YR_{k+1}^I}{\partial \mathbf{W}^F} = \frac{k}{(k+1)} \frac{\partial YR_k^I}{\partial \mathbf{W}^F} - \frac{4}{\sigma^2(k+1)} G_\sigma(|y_{k+1}|^2 - R_2) \cdot (R_2 - |y_{k+1}|^2) \cdot y_{k+1} \cdot \mathbf{X}_{k+1} \quad (19)$$

Similarly, we have FB gradient in the initial state $\nabla_k^{B,I} = \frac{\partial YY_k^I}{\partial \mathbf{W}^B} - \frac{\partial YR_k^I}{\partial \mathbf{W}^B}$ as

$$\frac{\partial YY_{k+1}^I}{\partial \mathbf{W}^B} = \frac{k^2}{(k+1)^2} \frac{\partial YY_k^I}{\partial \mathbf{W}^B} + \frac{2}{\sigma^2(k+1)^2} \sum_{i=1}^k (|y_i|^2 - |y_{k+1}|^2) \cdot G_{\sigma\sqrt{2}}(|y_i|^2 - |y_{k+1}|^2) (y_{k+1} \hat{\mathbf{D}}_k - y_i \hat{\mathbf{D}}_{i-1}) \quad (20)$$

$$\frac{\partial YR_{k+1}^I}{\partial \mathbf{W}^B} = \frac{k}{(k+1)} \frac{\partial YR_k^I}{\partial \mathbf{W}^B} - \frac{4}{\sigma^2(k+1)} G_\sigma(|y_{k+1}|^2 - R_2) \cdot (R_2 - |y_{k+1}|^2) \cdot y_{k+1} \quad (21)$$

For the steady state, $\nabla_k^{F,S} = \frac{\partial YY_k^S}{\partial \mathbf{W}^F} - \frac{\partial YR_k^S}{\partial \mathbf{W}^F}$ and $\nabla_k^{B,S} = \frac{\partial YY_k^S}{\partial \mathbf{W}^B} - \frac{\partial YR_k^S}{\partial \mathbf{W}^B}$ are

$$\begin{aligned} \frac{\partial YY_{k+1}^S}{\partial \mathbf{W}^F} &= \frac{\partial YY_k^S}{\partial \mathbf{W}^F} + \frac{2}{\sigma^2 N^2} \sum_{i=k-N+1}^k (|y_i|^2 - |y_{k+1}|^2) \cdot G_{\sigma\sqrt{2}}(|y_i|^2 - |y_{k+1}|^2) (y_{k+1} \mathbf{X}_{k+1} - y_i \\ &- \frac{2}{\sigma^2 N^2} \sum_{i=k-N+1}^k (|y_i|^2 - |y_{k-N+1}|^2) \cdot G_{\sigma\sqrt{2}}(|y_i|^2 - |y_{k-N+1}|^2) (y_{k-N+1} \mathbf{X}_{k-N+1} - y_i \mathbf{X}_i) \quad (22) \end{aligned}$$

$$\begin{aligned} \frac{\partial YR_{k+1}^S}{\partial \mathbf{W}^F} &= \frac{\partial YR_k^S}{\partial \mathbf{W}^F} - \frac{4}{\sigma^2 N} G_\sigma(|y_{k+1}|^2 - R_2) \cdot (R_2 - |y_{k+1}|^2) \cdot y_{k+1} \cdot \mathbf{X}_{k+1} \\ &+ \frac{4}{\sigma^2 N} G_\sigma(|y_{k-N+1}|^2 - R_2) \cdot (R_2 - |y_{k-N+1}|^2) \cdot y_{k-N+1} \cdot \mathbf{X}_{k-N+1} \quad (23) \end{aligned}$$

$$\begin{aligned} \frac{\partial YY_{k+1}^S}{\partial \mathbf{W}^B} &= \frac{\partial YY_k^S}{\partial \mathbf{W}^B} + \frac{2}{\sigma^2 N^2} \sum_{i=k-N+1}^k (|y_i|^2 - |y_{k+1}|^2) \cdot G_{\sigma\sqrt{2}}(|y_i|^2 - |y_{k+1}|^2) (y_{k+1} \hat{\mathbf{D}}_k - y_i \hat{\mathbf{D}}_{i-1}) \\ &- \frac{2}{\sigma^2 N^2} \sum_{i=k-N+1}^k (|y_i|^2 - |y_{k-N+1}|^2) \cdot G_{\sigma\sqrt{2}}(|y_i|^2 - |y_{k-N+1}|^2) (y_{k-N+1} \hat{\mathbf{D}}_{k-N} - y_i \hat{\mathbf{D}}_{i-1}) \quad (24) \end{aligned}$$

$$\frac{\partial YR_{k+1}^S}{\partial \mathbf{W}^B} = \frac{\partial YR_k^S}{\partial \mathbf{W}^B} - \frac{4}{\sigma^2 N} G_\sigma(|y_{k+1}|^2 - R_2) \cdot (R_2 - |y_{k+1}|^2) \cdot y_{k+1} \cdot \hat{\mathbf{D}}_k$$

$$+ \frac{4}{\sigma^2 N} G_\sigma (|y_{k-N+1}|^2 - R_2) \cdot (R_2 - |y_{k-N+1}|^2) \cdot y_{k-N+1} \cdot \hat{\mathbf{D}}_{k-N} \quad (25)$$

It is noticeable that the FF and FB gradients for (14) and (15) can be expressed in the recursive form being derived from the recursive ED.

4 Results and Discussion

The transmitted data d_k is binary and sent through the channel $H(z) = 0.407 + 0.815z^{-1} + 0.407z^{-2}$ [11]. The number of weights is $P=7$ and $Q=4$. The impulsive noise is the same as in [4]. The convergence parameter is $\mu=0.06$ for the proposed, and $\mu=0.002$ for least mean square algorithm (DF-LMS) [6]. The sample size and kernel size for kernel estimation are $N=20$ and $\sigma=7$, respectively. All these parameters are chosen to show the lowest MSE values. As shown in Fig. 1, the proposed DF algorithm converges up to -16 dB while the linear one stays at about -8 dB and the supervised DF-LMS algorithm at around -5dB.

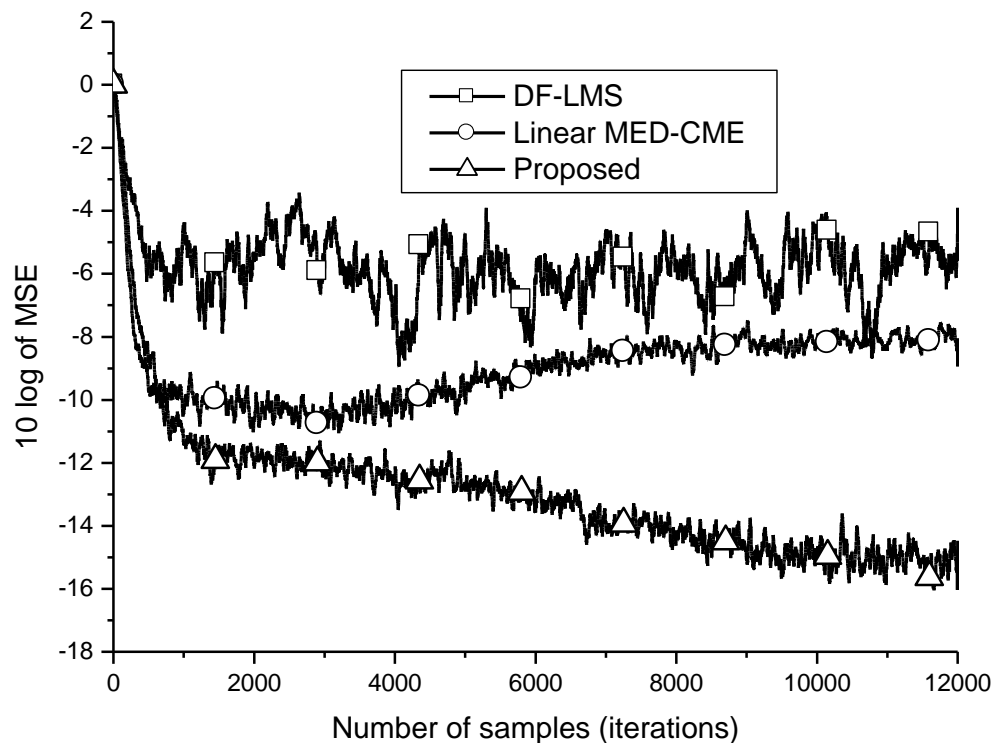


Fig 1. MSE learning performance

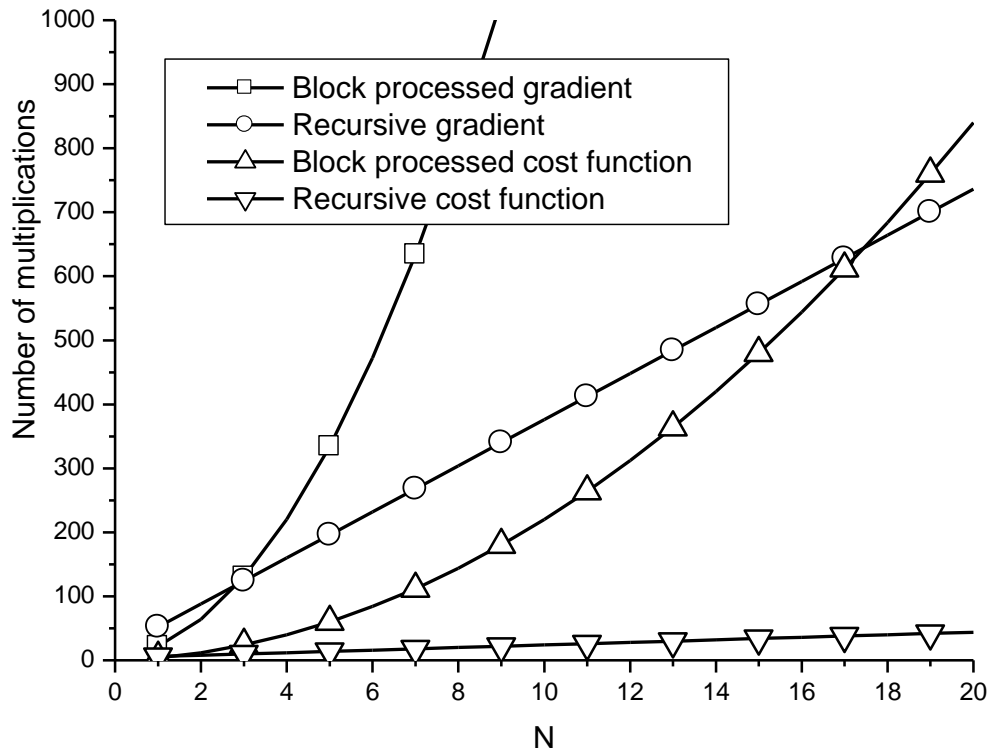


Fig 2. Number of multiplications

In the aspect of computational complexity, the block processing DF method and proposed recursive one are compared. The block processing ED requires $2N^2 + 2N$ multiplications, but the recursive ED does only $2N + 4$. With the number of multiplication operations revealed in [10], the complexity comparison of the two methods is depicted in Fig. 2. Considering the fact that the sample size is required to be large for accurate estimation, we observe that the complexity difference is significant.

5 Conclusion

For blind signal processing in indoor wireless sensor networks, recursive estimation of ED and its gradients based on DF structure is proposed in this paper. The estimation of recursive ED and gradients solves the computational complexity problem having linearly increasing complexity with the sample size. From the results of convergence performance and complexity analysis, it can be concluded that the

proposed DF method is a reliable and efficient signal processing method for sensor data transmission.

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