International Journal of Applied Engineering Research ISSN 0973-4562 Volume 10, Number 13 (2015) pp 33424-33429

Sparse Representation Based Image Compression Using Discrete Rajan Transform

Kethepalli Mallikarjuna[1]

Associate Professor, Dept. of ECE,
RGMCET, Nandyal, Kurnool, A.P, India
malli.rgmcet@gmail.com

Dr.Kodati Satya Prasad[2]**, Dr.M.V.Subramanyam**[3]

[2]*Professor, Dept. of ECE, JNTUK, Kakinada,*
[3]*Principal, SREC, Nandyal, A.P, India*
[2]*Prasad_kodati@yahoo.co.in*
[3]*mvsraj@yahoo.com*

Abstract
For the purpose of image coding and compression, many algorithms are available in the existing literature. As an alternative to these traditional approaches, in this paper, we have used Discrete Rajan Transform for sparsification and image compression. On simulation, it was observed that Discrete Rajan Transform yielded higher quality image than the other candidate transforms used, namely Discrete Cosine Transform and Discrete Wavelet Transform. While the Discrete Cosine Transform gives better compression ratio, the image quality degrades because of the artifacts that result from the transform. Discrete Rajan Transform is effective in introducing sparsity in images and thereby improving compressibility, the compromise being a loss of data. The Peak Signal to Noise Ratio is better in comparison with the other candidate transforms used in this research.

Keywords— Discrete Rajan Transform, Image coding, Image quality, Image storage, PSNR

I. INTRODUCTION

With the growth of science and technology, there has been a growth in the field of communication networks. This has led to an increase in the requirement of digital images and multimedia information transmission [1]. Images contain large amounts of redundant information and in order to improve transmission process, compression techniques are being used. Compression is essential to ease transmission of images and other multimedia information [2-4]. In the past few decades, the cost involved in storage has been reduced. However, on the consideration of a large amount of data storage and transmission requirements, the pace of development is still far from our expectations [5]. A number of techniques have been developed in an effort to achieve maximum information content with minimum storage requirements for various data. Alternative to lossless data compression, lossy compression involves accepting information loss as a trade-off to achieve greater compression ratios. These lossy techniques find their greatest utility in the signal processing areas of digitized image and audio. The Joint Photographic Experts Group (JPEG) standardized the concept of transforming image to the frequency domain in order to filter out noisy components in an effort to achieve greater compression. Although computational speeds have been a bottleneck in data compression, current computing technology has shifted the focus away from speed and towards storage capacity limitations. The memory cost involved in the archiving of photographs and documents has become a motive behind the development of efficient information reduction of such data. As the technology in image processing systems advances, such as greater resolutions and color definitions, the information stored in image files has also increased dramatically [6]. And many image compression coding techniques have been developed based on transformation or decomposition algorithms and so on. Algorithms based on transformation techniques could get the good compression effect. However, all need massive calculation and are time-consuming [7].

By the image coding technique, high compression ratio and low distortion characteristics can be achieved. JPEG is one such international standard that has wide applications in the network transmission, wireless communications, medical, multimedia etc [8]. For the purpose of data compression, different calculation methods can be used. These methods are broadly classified into two categories based on the distortions in the input image - lossless compression and lossy compression. Lossless compression, as the name implies, is the technique that involves no loss of data during the compression process and the pre-compressed signal can be restored. In contrary to the Lossless compression technique, Lossy compression technique involves loss of data [9]. However, it has to be realized that images contain complex sets of data and there is no existing transform in the literature that can optimally and efficiently represent the content in the image. For example, consider an image with oscillatory texture, Fourier transform performs better in terms of providing a sparse representation effectively. Similarly, the Discrete Wavelet Transform (DWT) gives better performance if the image has isolated singular textures [10]. This fundamental concept led to the development of many algorithms. There are many algorithms which are standard and widely used in the field of image compression. The JPEG2000 standard is the most popular among these techniques and it is based on the basic fact that the images have a certain sparsity associated with them when they are represented by wavelets [10-11]. As an alternative to these traditional approaches, the use of Discrete Rajan Transform (DRT) for sparsification and image compression has been explored.

II. DISCRETE RAJAN TRANSFORM (DRT)

Rajan Transform (RT) is a variant of Hadamard Transform which was initially developed for the purpose of pattern recognition. RT has a homomorphic nature and it is permutation invariant. It is because of this property that RT has potential applications in pattern recognition algorithms like thinning, edge detection, contour detection, and detection of curves and lines in images and isolation of certain points in images [12]. Unlike Rajan Transform, the generalized Discrete Rajan Transform (DRT) has been moulded to exhibit isomorphism when the auxiliary phasor information associated with the spectrum is known apriori. The principle of Discrete Rajan Transformation can be explained as follows:
Consider an N-dimensional signal vector x such that where N = 2^n; here n represents the number of stages required for obtaining DRT transformed signal.
In each stage, a unique operating matrix R_k of dimension [N/2^(k-1)] X [N/2^(k-1)] is required to obtain the intermediate or final DRT spectrum. The general expression for R_k is as follows:

$$R_k = \begin{bmatrix} I_p & I_p \\ -e_k^1.I_p & e_k^1.I_p \end{bmatrix} \quad (1)$$

Where I_p is an identity matrix of order p, and the order is obtained as

$$p_k = \frac{N}{2^k}; k \in \{1, 2, ..., n\} \quad (2)$$

and e_k^1 is the auxiliary phasor information that indicates the inherent relationship between each of the equilibrium segments of the signal spectrum at the k^{th} stage. For example, if the input signal at an arbitrary stage is $\bar{s} = a, b, c, d, e, f, g, h$,
then the auxiliary information e_k is expected to contain inherent phasor relationship between **a** and **e**.

$$e_k^i = \{\begin{matrix} -1, & x_k^i(p_k+1) < x_k^i(1) \\ 1, & \text{otherwise} \end{matrix} \quad (3)$$

Where i = {1, 2,……, 2^{k-1}}.
The output signal sequence in each stage is given by

$$Y_k = R_k X_k = \left[y_k^1, y_k^2 \dots y_k^i \right] \quad (4)$$

Here Y_k has $2^k p_k$ elements in the k^{th} stage.
When k =1, X1=x, the original input sequence, and for k > 1, 2^k-1 equilibrium segments exist for each stage.
The elements of the vector $\mathbf{e_k}$ correspond to the auxiliary information of the respective equilibrium segments in k stages. It is defined by the following equation:

$$e_k = \begin{bmatrix} e_k^1 & e_k^2 & \cdots & e_k^i \end{bmatrix} \quad (5)$$

For example, if S_k = {a,b,c,d,e,f,g,h} and

$$s_{k+1} = \begin{bmatrix} a & e \\ b & f \\ c & g \\ d & h \end{bmatrix}$$

then, e_k^1 and e_k^2 are the auxiliary phasor information that correspond to a and c & e and g respectively. The auxiliary phasor values in the vector e_k are obtained from the equation (3) mentioned above.
In order to construct the common operator matrix R_k in all the stages, auxiliary phasor information e_k^1 from e_k is used. Also for k>1, the transformed output from one stage is reshaped into equilibrium segments iteratively and it is given by the expression:

$$X_{k+1} = \begin{bmatrix} \bar{x}_{k+1}^1 & \mu_k^1.\bar{x}_{k+1}^2 & \cdots & \mu_k^{i-1}.\bar{x}_{k+1}^i \end{bmatrix} \quad (6)$$

Where

$$\mu_k = \begin{bmatrix} \mu_k^1 & \mu_k^2 & \cdots & \mu_k^{i-1} \end{bmatrix} \quad (7)$$

And

$$\mu_k^{i-1} = e_k^1 \times e_k^i \quad \text{for k>1} \quad (8)$$

Also

$$\mathrm{X}_{k+1} = \begin{bmatrix} y_k^i(1) & y_k^i(2) & \cdots & y_k^i(p_k) \\ y_k^i(p_k+1) & y_k^i(p_k+2) & \cdots & y_k^i(2p_k) \\ \vdots & \vdots & \vdots & \vdots \\ y_k^i(2^{k-1}p_k+1) & \cdots & \cdots & y_k^i(2^k p_k) \end{bmatrix}^T =$$

$$\begin{bmatrix} \overline{X}_{k+1}^1 & \overline{X}_{k+1}^2 & \cdots & \overline{X}_{k+1}^i \end{bmatrix} \quad (9)$$

The equation X_{k+1} represents the general expression for splitting of the signal spectrum into equilibrium segments and this process is continued till n stages to obtain the final DRT transformed output.

The first sample value in the final output of DRT is called Cumulative Point Index (CPI) and it represents the cumulative energy present in the input signal.

The Inverse Discrete Rajan Transform (IDRT) can be used to retrieve the input signal in the presence of the auxiliary phasor information e_k^1 and μ_k. Assuming that these values are known apriori, the DRT operator matrix R_k is obtained. The expression used to retrieve back the input signal in each stage is

$$\tilde{X}_m = \frac{1}{2}\ R_m Y_m \ = \begin{bmatrix} x_m^1 & x_m^2 & \cdots & x_m^i \end{bmatrix}^T \qquad (10)$$

Where m = {k, k-1, ….., 1}.

In the forward process, the sequence was split into equilibrium segments whereas, in the inverse DRT process, they are recombined so as to retrieve the input sequence iteratively.

When m=k, Y_m=Y_k (the final DRT transformed output) and for m < k,

$$Y_{m-1} = \begin{bmatrix} \overline{Y}_m(1) & \mu_k^1.\overline{Y}_m(2) & \cdots & \mu_k^{i-1}.\overline{Y}_m(i) \end{bmatrix} \qquad (11)$$

And

$$\overline{Y}_{m-1} = \begin{bmatrix} x_m^i(1) & x_m^i(2p_k+1) & \cdots & x_m^i(2^{k-1}p_k+1) \\ x_m^i(2) & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ x_m^i(2p_k) & x_m^i(2^2 p_k) & \cdots & x_m^i(2^k p_k) \end{bmatrix} \qquad (12)$$

After the final stage in the inverse process, the length of the output is the same as the input and it can be observed that $x = \tilde{X}$.

III. Sparsity Using Discrete Rajan Transform

With the help of DRT algorithm, both lossless and lossy sparse representation can be obtained. For several input signals, the transformed signal output obtained from DRT was observed. It was found that almost 23% of it was zero coefficients and many of the remaining coefficients were nearly zero. Thus, there is a possibility of achieving better sparse output by using DRT. However, there would have to be a trade off between data recovery and the energy required for it. Hence, this transform was applied for images to exploit its sparsing nature. Different input grey scale images of dimension 512×512 have been considered. Each of the input images was divided into pixel blocks of size 8 and DRT was applied to it and the degree of sparsity associated with the transformed signal output has been calculated. Also, the scope of achieving the maximum possible sparsity has been analyzed by applying thresholding function, which, however, would result in loss of information in the signal. On the simulation of DRT on Lena image, out of the total 512×512 pixels, 9610 zero valued or (N-K) sparse pixels are obtained. The degree of sparsity obtained by considering just the output transformed signal of DRT is 3.7% or nearly 4%. Similarly, for the "cameraman" image, 103078 number of (N-K) sparse or zero valued coefficients were obtained. The degree of sparsity obtained for the cameraman image is 3.93% or nearly 4%. For the 'pepper' image also, a similar trend is observed. This is perfectly lossless and the original image can be recovered with infinite Peak Signal to Noise Ratio- PSNR (since no errors exist in this case). Depending upon the trade off that can be accepted by the end user, various levels of sparsity can be achieved. However, the sparsest representation possible using DRT is only when the CPI of each of the blocks is considered, i.e., the information present in each of the pixels in each block is represented by only one pixel and thus, the total image can only be represented by fewer number of pixels. In this case, for all the three images under consideration, the total number of zero coefficients possible is (512×512) – (64×64), i.e., 258048 and the maximum achievable degree of sparsity would be 98.44%. However, a considerable amount of information loss can be expected. To visualize the impact of the sparse representations, the image is reconstructed using Inverse DRT process. The PSNR (in dB) for the lossy case mentioned above is found to be 20 and 24 respectively. There is always a scope to improve the PSNR or compress the signal by applying relevant algorithms. The application of DRT in the perspective of sparse representation achievable has been focused. Individual algorithms that help in improvising the PSNR and compressibility can always be employed. There is always a scope to improve the degree of sparsity of the signal obtained by considering only the CPI values. However, the compromise would be the loss of information in the signal.

IV. Image Compression Using DRT

For the purpose of improving sparse representation, a special case of DRT can be used in wherein the auxiliary phasor information in each stage is considered as 1. Considering the lossy case, even higher degree of sparsity can be obtained using (i) Cumulative Point Index (CPI, the first spectral component) values and (ii) CPI along with the mid-spectral component of DRT transformed output at different block lengths. But, in this case, a compromise has to be made in information loss as in the signal [13]. Discrete Rajan Transform (DRT) has potential applications in the image processing and compression. In this paper, only the application of DRT to obtain a sparse representation of images is considered. Standard images have been considered and to make the DRT processing easier, the image has been divided into the blocks of size 8. DRT is applied to it and the special case of DRT, as mentioned earlier, is considered

The block diagram for image compression using sparsifying transform- DRT is shown in the Fig.1. The Fig.2 describes the sequence of steps used in the algorithm for image compression using DRT. A similar algorithm is used for the case of DCT and DWT. The results, thus, obtained have been presented in the following section. In Fig.3, column (a) describes the original images, column (b) describes the reconstructed images using Discrete Wavelet Transform (DWT), column(c) describes the reconstructed images using Discrete Cosine Transform (DCT), and column (d) describes the reconstructed images using Discrete Rajan Transform (DRT). It is observed that the image reconstruction using DRT is better than that using other candidate transforms.

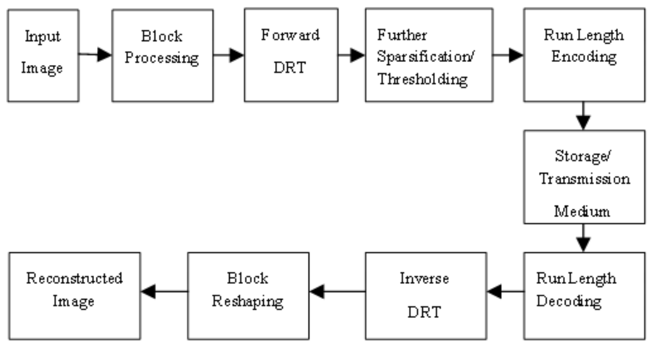


Fig 1. Block Diagram of Image Compression using DRT

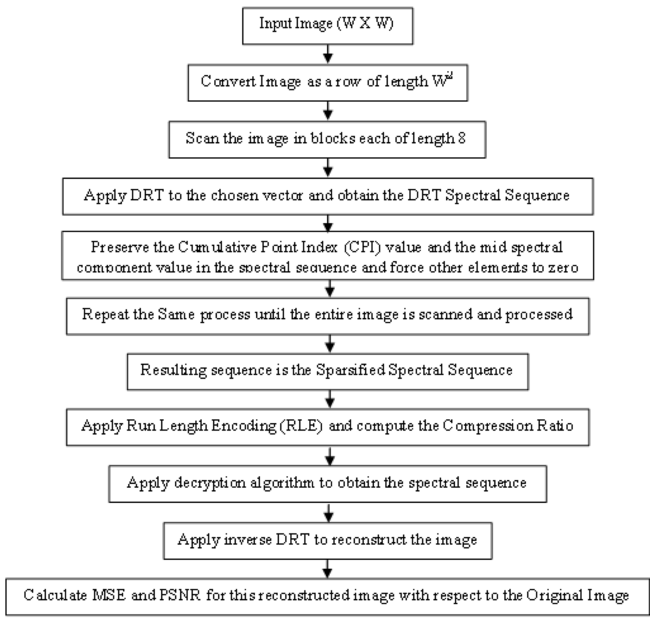


Fig. 2. DRT Compression Algorithm

V. SIMULATION RESULTS

For the purpose of simulation, different input grey scale images (Lena, Cameraman and Peppers, etc.) of dimension 512×512 have been considered. Each of the input images was divided into 1×8-pixel blocks and DRT was applied to it and the degree of sparsity associated with the transformed signal output has been calculated. The scope of achieving the maximum possible sparsity has also been analyzed by applying thresholding function, which, however, would result in the loss of information in the signal. On the simulation of DRT on Lena image, out of the total 512×512 pixels, 9610 zero valued or (N-K) sparse pixels are obtained.

The degree of sparsity obtained by considering just the output transformed signal of DRT is 3.7% or nearly 4%. Similarly, for the ‘cameraman’ image, 103078 number of (N-K) sparse or zero valued coefficients were obtained. The degree of sparsity obtained for the cameraman image is 3.93% or nearly 4%. For the ‘pepper’ image also, a similar trend is observed. This is perfectly lossless and the original image can be recovered with infinite Peak Signal to Noise Ratio PSNR (since no errors exist). Further sparse representation of the transformed signal can be obtained by applying thresholding function. After thresholding (threshold T=0) was considered for the Lena image, 134568 samples could be approximated to zero. The degree of sparsity that could be obtained is 51.3%. Similarly, for the cameraman image, the number of samples that could be approximated to zero is 180273 and the degree of sparsity achieved is 68.7% and for the pepper image, the number of zero coefficients obtained is 131959, and the achieved degree of sparsity being nearly 51%. However, this would lead to loss of information in the image and thus, can be classed as lossy. The transformed output from DRT was sparsified as mentioned above and RLE (Run Length Encoding) encryption technique was applied. This step was introduced so as to obtain more PSNR. To have a visual perception of the sparsity and the amount of information loss, the decryption algorithm was applied and reconstruction using IDRT has been done. For the purpose of comparison, Discrete Cosine Transform (DCT) and Discrete Wavelet Transform (DWT) have been considered alongside DRT. This is depicted in Fig.3.

The results of the experiment were analyzed on the basis of two parameters – Peak Signal to Noise ratio (PSNR) and compression ratio. PSNR for a set of 7 images has been analyzed and it was found that DRT gives better image quality in comparison with other candidate transforms. The PSNR observed, in this case, has been tabulated below:

TABLE I. **PSNR IN DECIBELS**

S.No	*Grayscale Images*	*DWT*	*DCT*	*DRT*
1	Lena	6.9023	27.0072	30.7203
2	Boat	6.5912	24.2723	27.9927
3	Zelda	9.4019	30.2277	34.2956
4	Baboon	6.7218	19.9528	21.8813
5	Cameraman	4.2623	21.1088	23.8764
6	Barbara	7.1399	22.7127	25.4919
7	Peppers	7.8607	25.4598	28.9709

It can be observed that PSNR is better in the case of DRT in comparison to DCT and DWT. The other parameter that was considered for the analysis is compression ratio. The compression ratio obtained in percentage (%) is tabulated below

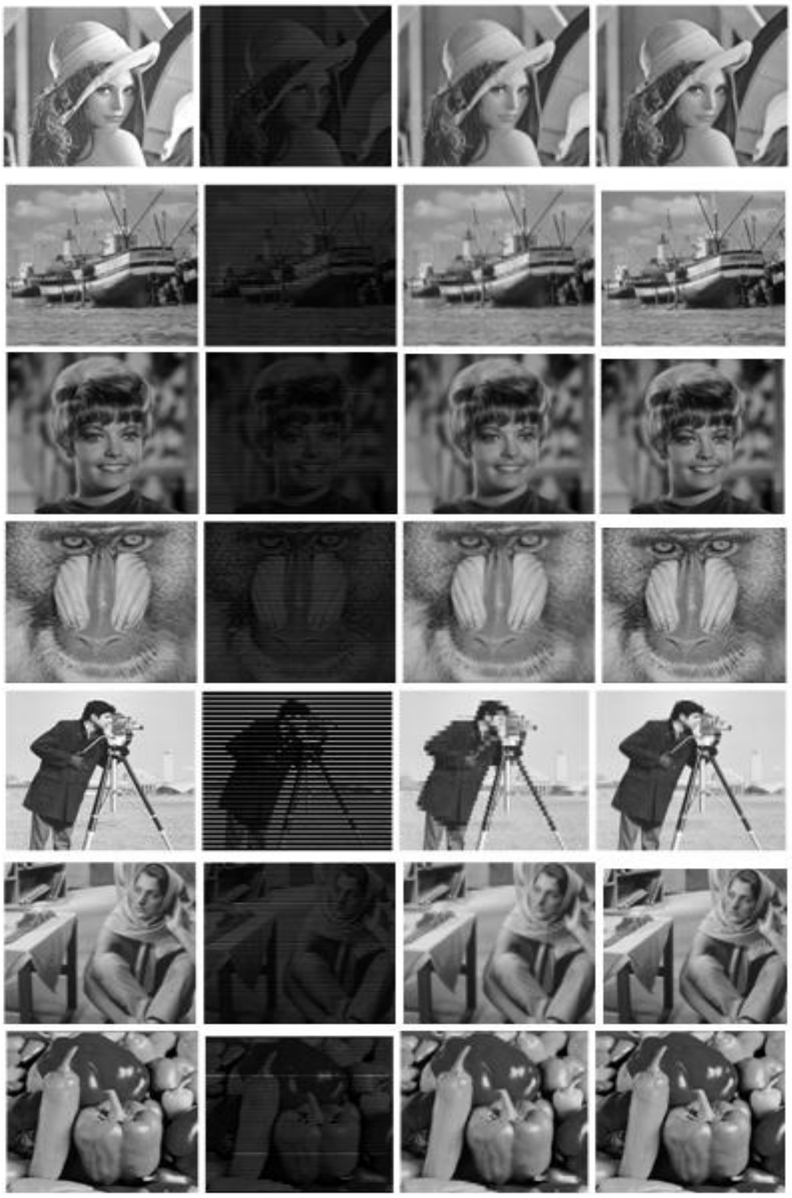
International Journal of Applied Engineering Research ISSN 0973-4562 Volume 10, Number 13 (2015) pp 33424-33429

TABLE II. **COMPRESSION RATIO(%)**

Sl. No	*Grayscale Images*	*DWT*	*DCT*	*DRT*
1	Lena	48.2468	48.9304	49.3568
2	Boat	49.0181	49.3683	49.6521
3	Zelda	48.3963	48.8831	49.5102
4	Baboon	49.5232	49.7063	49.7986
5	Cameraman	48.0988	48.9868	49.1974
6	Barbara	48.7633	49.2279	49.5750
7	Peppers	48.7129	49.2165	49.4339

The compression provided by DRT is almost equal to that of DCT. Though the compression provided by DWT is better, but its PSNR is very poor. Thus, DRT is better in reconstructing the images with good quality.

DRT is effective in introducing sparsity in images and thereby improving compressibility, the compromise being a loss of data. The PSNR is better in comparison with the other candidate transforms used in this research. Individual algorithms that help in improvising the PSNR and compressibility can always be employed.



(a) (b) (c) (d)

Column(a): Original Images; Column(b):DWT Reconstructed Images; Column(c): DCT Reconstructed Images; Column(d):DRT Reconstructed Images

Fig. 3. Simulation Results

Acknowledgment

I am heartily thankful to my supervisor, Dr.K.Satya Prasad, Professor, Department of ECE, JNTUK, Kakinada, my Co-Supervisor, Dr.M.V.Subramanyam, Principal, SREC, Nandyal, Dr.E.G.Rajan, Director, Pentagram Research Centre, Hyderabad and Ms.G.Prashanthi, MSc, Staffordshire University, UK, whose constant encouragement and guidance during the period of this work enabled me complete it with better perception of the subject.

References

[1] Zhu Gui bin, Cao Chang xiu, Hu Zhong yu, He Shi biao, Sen Bai, 2003, "An image scrambling and encryption algorithm based on affine transformation [J], Journal of Computer-Aided Design and Computer Graphics", Issue 6, pp. 711-715

[2] En-hui Yang, Longji Wang., 2009, "Joint Optimization of Run-Length Coding, Huffman Coding, and Quantization Table With Complete Baseline JPEG Decoder Compatibility[J]", IEEE Transaction on Image Processing, 18(1): pp.63-74

[3] Q Xia,X Li,L Zhou,K M Lam., 2012, " Visual sensitivity-based low-bit-rate image compression algorithm[J]", IEEE Transaction on Image Processing, 6(7): pp.910-918.

[4] Zhiwei Xiong,Xiaoyan Su,Feng Wu., 2010, "Block-Based Image Compression With Parameter-Assistant Inpainting [J]", IEEE Transactions on image processing, 19(6):1651-1657.

[5] M. Zhang, G. Shao and K. Yi., 2004, "T-matrix and Its Applications in Image Processing [J]", Electronics Letters, Vol.40, No.25, pp.1583-1584.

[6] Jackson.D.J, Hannah.S.J., 1993, "Comparative analysis of image compression techniques, System Theory, 1993", Proceedings SSST '93., Twenty-Fifth Southeastern Symposium on , vol., no. 7-9, pp.513,517.

[7] Shengli Chen, Xiaoxin Cheng, Jiapin Xu., 2012, "Research on image compression algorithm based on Rectangle Segmentation and storage with sparse matrix, Fuzzy Systems and Knowledge Discovery (FSKD)", 2012 9th International Conference on, vol., no., pp.1904, 1908, 29-31.

[8] Sen Bai, Changxiu Cao, Longhan Cao., 2001, "Image detail hiding technology based on knight cruise transform[J]", Journal of Image and Graphics, Vol.6, No.11, pp.1096-1100.

[9] Li Zhiqianga, Sun Xiaoxin, Du Changbin, Ding Qun., 2013, "JPEG Algorithm Analysis and Application in Image Compression Encryption of Digital Chaos, Instrumentation, Measurement, Computer, Communication and Control (IMCCC) 2013", Third International Conference on , vol., no., pp.185,189.

[10] Elad Michael: Sparse and Redundant Representations., 2010 , "From Theory to Applications in Signal and Image Processing", USA: Springer ISBN 978-1-4419-7011-4.

[11] Starck J.L, Murtagh F, Fadili J.M., "Sparse Image and Signal Processing: Wavelets, Curvelets, Morphological Diversity, New York: Cambridge University Press". ISBN-13: 978-0521119139 ISBN-10: 0521119138.

[12] Ekambaram Naidu Mandalapu, Rajan E. G., 2009, "Rajan Transform and its Uses in Pattern Recognition", Informatica 33, pp. 213-220.

[13] Govindarajan Prashanthi., "Signal Sparsification with Discrete Rajan Transform (DRT) Principles, Properties and Applications", M. Sc, Staffordshire University, UK.