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Lehmann-Type Laplace Distribution-Type II Software Reliability Growth Model

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Abstract
In this paper, Lehmann-Type Laplace Type II reliability growth model is proposed for early detection of software failure based on time between failure observations. Cumulative time between failures of the software data is assumed to follow Lehmann-Type Laplace distribution-Type II (LLD-II). The parameters are estimated using Profile Likelihood Method. In terms of AIC and BIC, this distribution is found to be a better fit for the software failure data than the other distributions which are commonly used in reliability analysis. A LLD-II control mechanism is used to detect the failure points of a software data.

Keywords Statistical Process Control (SPC), Non-Homogeneous Poisson Process (NHPP), Goel-Okumoto distribution, Weibull distribution, Exponential Geometric distribution (EG), Pareto type-III, Lehmann-Type Laplace distribution Type I (LLD-I), Lehmann-Type Laplace distribution Type II (LLD-II), Profile Likelihood, Akaike Information Criterion(AIC), Bayesian Information Criterion (BIC).

1.Introduction

Assessing software is important to evaluate and predict the reliability and performance of software system. Software Reliability [4] is mainly focused on identifying the failures in software and helps to build a reliable model by which the identified failures can be removed. To perform Software Reliability testing, Software Reliability Growth Models (SRGM) are used.

The Non-Homogeneous Poisson Process (NHPP) based SRGM are proved to be successful in practical software engineering. The main issue in NHPP model is to determine the mean value function $m(x)$ which is the expectation of the number of failures experienced upto a certain point. This number of failures is assumed to follow Poisson distribution. The failure intensity function $\lambda(x)$ is proportional to the residual fault content.

Software Reliability process can be monitored by using Statistical Process Control (SPC) [5]. In testing software reliability, control charts can be used as an efficient and appropriate SPC tool. Mean value control chart is used here which takes failure number along X-axis, successive differences of $m(x)$ along Y-axis and three parallel lines to X-axis for Lower Control Limit(LCL), Upper Control Limit(UCL) and Control Limit(CL). A point below LCL indicates an alarming signal. A point above UCL indicates better quality. If the points are falling within the control limits, it indicates the software process is in stable condition.

In recent years, several authors framed SRGM based on NHPP models. Out of them the commonly used are Goel-Okumoto [6], Weibull, Exponential Geometric, Pareto type III [9], Lehmann Type Laplace Distribution-Type I (LLD-I) [3] models.

In this paper, it is proved that Lehmann-Type Laplace distribution Type II (LLD-II) [7] has a better fit when compared to other distributions for a Software failure data [10][7] using Akaike Information Criterion (AIC) [2] and Bayesian Information Criterion (BIC) [8]. A control mechanism is developed using LLD-II for the software failure data.

The paper is organized as follows. Section 2 describes the models viz., Goel-Okumoto, Weibull, Exponential Geometric, Pareto type III, LLD- I and LLD-II along with their parameter estimation. In section 3, two set of software failure data are considered to show that LLD-II is a better fit than other distributions. In section 4, LLD-II NHPP model is given and LLD-II control mechanism is used to find the control limits and failure detection points of the software. Conclusion is given in Section 5.

2. Software Reliability Growth Models

2.1 Goel-Okumoto or Exponential Growth Model

The Goel-Okumoto model [6] is a simple NHPP model.
The probability density function of this model is

$$f(x) = \lambda e^{-\lambda x} \quad (2.1.1)$$

where $x \geq 0, \lambda \geq 0$.

Its corresponding cumulative distribution function is

$$F(x) = 1 - e^{-\lambda x} \quad (2.1.2)$$

Parameter estimation

Method of Maximum Likelihood is used to estimate the parameter λ.

The likelihood function is given by

$$l = \prod_{i=1}^{n} \lambda e^{-\lambda x_i} \quad (2.1.3)$$

The log-likelihood function is

$$\log l = n \log \lambda - \sum_{i=1}^{n} \lambda x_i \quad (2.1.4)$$

Partially differentiating (2.1.4) with respect to λ, and equating it to zero, it is found that

$$\lambda = \frac{n}{\sum_{i=1}^{n} x_i} \qquad (2.1.5)$$

2.2 Weibull Growth Model

Weibull distribution is considered to be more flexible and able to deal with different types of aging phenomenon in reliability.
The probability density function of Weibull distribution with two parameters is

$$f(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1} e^{-(x/\eta)^{\beta}} \qquad (2.2.1)$$

with $x \in (0,\infty), \beta > 0$ is the shape parameter and $\eta > 0$ is the scale parameter.
The cumulative distribution function is

$$F(x) = (1 - e^{-(x/\eta)^{\beta}}) \qquad (2.2.2)$$

2.2.1 Parameter Estimation

The likelihood function of the Weibull distribution is

$$l = \prod_{i=1}^{n} \frac{\beta}{\eta}\left(\frac{x_i}{\eta}\right)^{\beta-1} e^{-\left(x_i/\eta\right)^{\beta}} \qquad (2.2.3)$$

The log-likelihood function is

$$\log l = \sum_{i=1}^{n} \log\left[\frac{\beta}{\eta}\left(\frac{x_i}{\eta}\right)^{\beta-1} e^{-\left(x_i/\eta\right)^{\beta}}\right] \qquad (2.2.4)$$

Equating to zero after partially differentiating (2.2.4) with respect to β,

$$\frac{\partial \log l}{\partial \beta} = \frac{n}{\beta} + \sum_{i=1}^{n} \log x_i - \frac{1}{\eta}\sum_{i=1}^{n} x_i^{\beta} \log x_i = 0 \qquad (2.2.5)$$

Similarly partially differentiating (2.2.4) with respect to η and equating to zero,

$$\frac{\partial \log l}{\partial \eta} = \frac{-n}{\eta} + \frac{1}{\eta^2}\sum_{i=1}^{n} x_i^{\beta} = 0 \qquad (2.2.6)$$

Eliminating η between equations (2.2.5) and (2.2.6), the following equation is obtained.

$$\frac{\sum_{i=1}^{n} x_i^{\beta} \log x_i}{\sum_{i=1}^{n} x_i^{\beta}} - \frac{1}{\beta} - \frac{1}{n}\sum_{i=1}^{n} \log x_i = 0 \qquad (2.2.7)$$

By solving (2.2.7) using Newton-Raphson method, β is obtained. The value of β is substituted in (2.2.6) to get the value of η as follows.

$$\eta = \frac{\sum_{i=1}^{n} x_i^{\beta}}{n} \qquad (2.2.8)$$

2.3 Exponential Geometric Growth Model

Exponential Geometric distribution [1] is a decreasing failure rate distribution used for lifetime behavior.

The probability density function is given by

$$f(x) = \beta(1-p)e^{-\beta x}(1 - pe^{-\beta x})^{-2} \qquad (2.3.1)$$

where $\beta > 0$ and $p \in (0,1)$

Its cumulative distribution function is given by

$$F(x) = (1 - e^{-\beta x})(1 - pe^{-\beta x})^{-1} \qquad (2.3.2)$$

Parameter estimation

Expectation Maximization (EM) algorithm is used to find the maximum likelihood estimates.
After using E-step and M-step of the EM algorithm, the parameters β and p are estimated as,

$$\beta^{(t+1)} = n\left\{\sum_{i=1}^{n} \frac{x_i(1 + p^{(t)}e^{-\beta^{(t)}x_i})}{(1 - p^{(t)}e^{-\beta^{(t)}x_i})}\right\}^{-1} \qquad (2.3.3)$$

and $$p^{(t+1)} = 1 - n\left\{\sum_{i=1}^{n} \frac{(1 + p^{(t)}e^{-\beta^{(t)}x_i})}{(1 - p^{(t)}e^{-\beta^{(t)}x_i})}\right\}^{-1} \qquad (2.3.4)$$

2.4 Pareto type III Growth Model

Pareto type III distribution [9] is more advantageous in order to estimate the failure rate exactly.
The probability density function of this distribution is

$$f(x) = \frac{\alpha s^{\alpha}}{x^{\alpha+1}} \qquad (2.4.1)$$

where $x \geq s, s > 0$ is the scale parameter and $\alpha > 0$ is the shape parameter.
Its cumulative distribution function is

$$F(x) = 1 - \left(\frac{s}{x}\right)^{\alpha} \qquad (2.4.2)$$

Parameter estimation

The likelihood function of Pareto type III distribution is

$$l = \prod_{i=1}^{n} \frac{\alpha s^{\alpha}}{x_i^{\alpha+1}} \qquad (2.4.3)$$

The log-likelihood function is

$$\log l = n\log\alpha + n\alpha\log s - (\alpha+1)\sum_{i=1}^{n}\log x_i \qquad (2.4.4)$$

It can be seen that $\log l$ is monotonically increasing with s since $x \geq s$. Thus

$$s = \min_{i} x_i \qquad (2.4.5)$$

Partially differentiating (2.4.4) with respect to α and equating to zero, α is obtained as

$$\alpha = \frac{n}{\sum_{i=1}^{n} (\log x_i - \log s)} \qquad (2.4.6)$$

2.5 Lehmann-Type Laplace Type I Growth Model

Lehmann-Type Laplace Type I distribution [7] can be used in lifetime data analysis.

The probability density function of LLD-I is

$$f(x) = \begin{cases} \dfrac{\alpha}{2^{\alpha}\phi} e^{\alpha\left(\frac{x-\theta}{\phi}\right)} & x \le \theta \\ \dfrac{\alpha}{2\phi} e^{-\left(\frac{x-\theta}{\phi}\right)} \left(1 - \dfrac{1}{2} e^{-\left(\frac{x-\theta}{\phi}\right)}\right)^{\alpha-1} & x \ge \theta \end{cases} \qquad (2.5.1)$$

where $\theta \in (-\infty, \infty)$ is a location parameter, $\alpha > 0$ is the shape parameter and $\phi > 0$ is a scale parameter.

The cumulative distribution function is

$$F(x) = \begin{cases} \dfrac{1}{2^{\alpha}} e^{\alpha\left(\frac{x-\theta}{\phi}\right)} & x \le \theta \\ \left(1 - \dfrac{1}{2} e^{-\left(\frac{x-\theta}{\phi}\right)}\right)^{\alpha} & x \ge \theta \end{cases} \qquad (2.5.2)$$

2.5.1 Parameter Estimation

Profile Likelihood method is used to estimate the parameters of LLD-I. The log-likelihood function for the observed data can be written as

$$\log l = n \log \alpha - n_1 \log 2\phi - n_2 \log \phi$$

$$+ \sum_{i \in I_2} \log\left(1 - \frac{1}{2} e^{\frac{\theta - x_i}{\phi}}\right) + \frac{\alpha+1}{\phi} \sum_{i \in I_1} (x_i - \theta) + \frac{\alpha}{\phi} \sum_{i \in I_2} (\theta - x_i) \qquad (2.5.3)$$

where $I_1 = \{i \mid x_i \le \theta\}$ and $I_2 = \{i \mid x_i > \theta\}, |I_1| = n_1, |I_2| = n_2$ and $n = n_1 + n_2$.

Keeping θ fixed, the equations $\frac{\partial \log l}{\partial \alpha} = 0$ and $\frac{\partial \log l}{\partial \phi} = 0$ give

$$\alpha = \frac{n}{n_1 \log 2 - \dfrac{n_1 \overline{X_1}}{\phi} + \dfrac{n_1 \theta}{\phi} - \sum_{i=1}^{n_2} \log\left(1 - \dfrac{1}{2} e^{-\left(\frac{x_i - \theta}{\phi}\right)}\right)}$$

(2.5.4)

$$n\phi + \alpha(n_1 \overline{X}_1 - n_1 \theta) - n_2 \overline{X}_2 + n_2 \theta + \frac{(\alpha-1)}{2} \sum_{i=1}^{n_2} \frac{(x_i - \theta) e^{-\left(\frac{x_i - \theta}{\phi}\right)}}{1 - \frac{1}{2} e^{-\left(\frac{x_i - \theta}{\phi}\right)}} = 0 \qquad (2.5.5)$$

where $\overline{X_1} = \dfrac{\sum_{i=1}^{n_1} x_i}{n_1}$ and $\overline{X_2} = \dfrac{\sum_{i=1}^{n_2} x_i}{n_2}$.

By evaluating,

$$\max_{\alpha,\theta,\phi} \log\ l(\alpha, \theta, \phi \mid x) = \max_{\theta}\left[\max_{\alpha,\phi} \log l(\alpha, \phi \mid x)\right]$$

(2.5.6)

using numerical techniques and MATLAB tools, the parameters α, θ, ϕ are estimated.

2.6 Lehmann-Type Laplace Type II Growth Model

The probability density function of LLD-II [7] is

$$f(x) = \begin{cases} \dfrac{\alpha}{2\phi} e^{(\alpha+1)\left(\frac{x-\theta}{\phi}\right)} & x \le \theta \\ \dfrac{\alpha}{\phi}\left(1 - \dfrac{1}{2} e^{\left(\frac{\theta - x}{\phi}\right)}\right) e^{\alpha\left(\frac{\theta - x}{\phi}\right)} & x \ge \theta \end{cases}$$

(2.6.1)

where $\alpha > 0$ is the shape parameter, $\phi > 0$ is the scale parameter, $\theta > 0$ is the location parameter.

The cumulative distribution function is

$$F(x) = \begin{cases} \dfrac{\alpha}{2(\alpha+1)} e^{(\alpha+1)\left(\frac{x-\theta}{\phi}\right)} & x \le \theta \\ 1 - e^{\alpha\left(\frac{\theta - x}{\phi}\right)} + \dfrac{\alpha}{2(\alpha+1)} e^{(\alpha+1)\left(\frac{\theta - x}{\phi}\right)} & x \ge \theta \end{cases} \qquad (2.6.2)$$

Parameter estimation

Profile Likelihood method is used to estimate the parameters of LLD-II. The log-likelihood function for the observed data can be written as

$$\log l = n_1 \log \alpha - n_1 \log 2\phi + \frac{\alpha+1}{\phi} \sum_{i=1}^{n_1} (x_i - \theta) + n_2 \log \alpha - n_2 \log \phi + \sum_{i=1}^{n_2} \log\left(1 - \frac{1}{2} e^{\left(\frac{\theta - x_i}{\phi}\right)}\right) + \frac{\alpha}{\phi} \sum_{i=1}^{n_2} (\theta - x_i)$$

(2.6.3)

where $I_1 = \{i \mid x_i \le \theta\}$ and $I_2 = \{i \mid x_i > \theta\}, |I_1| = n_1, |I_2| = n_2$ and $n = n_1 + n_2$.

Keeping θ fixed, the equations $\frac{\partial \log l}{\partial \alpha} = 0$ and $\frac{\partial \log l}{\partial \phi} = 0$ give

$$\alpha = \frac{n\phi}{(n_1 - n_2)\theta + (n_2 \overline{X}_2 - n_1 \overline{X}_1)} \qquad (2.6.4)$$

(2.6.5)

$$-(n_1 + n_2)\phi + \left[(\alpha+1) n_1 \theta - (\alpha+1) n_1 \overline{X}_1 + \alpha n_2 \overline{X}_2 - \alpha n_2 \theta\right] + \frac{1}{2} \sum_{i=1}^{n_2} \frac{(\theta - x_i) e^{\left(\frac{\theta - x_i}{\phi}\right)}}{1 - \frac{1}{2} e^{\left(\frac{\theta - x_i}{\phi}\right)}} = 0$$

where $\overline{X_1} = \frac{\sum_{i=1}^{n_1} x_i}{n_1}$ and $\overline{X_2} = \frac{\sum_{i=1}^{n_2} x_i}{n_2}$.

By evaluating,

$$\max_{\alpha,\theta,\phi} \log\, l(\alpha,\theta,\phi \mid x) = \max_{\theta}\left[\max_{\alpha,\phi} \log l(\alpha,\phi \mid x)\right] \quad (2.6.6)$$

using numerical techniques and MATLAB tools, the parameters α,θ,ϕ are estimated.

3. Estimation and Goodness of fit

3.1 Data Set 1

Let the random variable X be defined as cumulative time between failures. Table 3.1.1, [10] gives the cumulative time between failures of a software product.

Table 3.1.1 Cumulative Time between failures

FAILURE NUMBER	TIME BETWEEN FAILURES (hrs)	TIME BETWEEN FAILURES (hrs) (CUMULATIVE)
1	30.02	30.02
2	1.44	31.46
3	22.47	53.93
4	1.36	55.29
5	3.43	58.72
6	13.2	71.92
7	5.15	77.07
8	3.83	80.9
9	21	101.9
10	12.97	114.87
11	0.47	115.34
12	6.23	121.57
13	3.39	124.97
14	9.11	134.07
15	2.18	136.25
16	15.53	151.78
17	25.72	177.5
18	2.79	180.29
19	1.92	182.21
20	4.13	186.34
21	70.47	256.81
22	17.07	273.88
23	3.99	277.87
24	176.06	453.93
25	81.07	535
26	2.27	537.27
27	15.63	552.9
28	120.78	673.68
29	30.81	704.49
30	34.19	738.68

For the data set 1 in Table 3.1.1, the parameters, log likelihood values, AIC and BIC of corresponding distributions are given in the following Table 3.1.2.

Table 3.1.2. Goodness of fit using AIC and BIC

Distributions	Number of Parameters	Parameter values	Log likelihood value	AIC	BIC
Goel-Okumoto	1	$\lambda = 0.0042$	-198.3813	390.9054	395.5649
Weibull	2	$\beta = 255.5402$ $\eta = 1.1883$	-1605.9	3216.3	3218.6
EG	2	$\beta = 0.0042$ $p = 0.0024$	-194.3835	393.2114	395.5694
Pareto type III	2	$\alpha = 0.5913$ $s = 30.02$	-198.549	401.5424	403.9003
LLD-I	3	$\theta = 31.45$ $\alpha = 3.6781$ $\phi = 148.953$	-195.1961	397.3153	397.1946
LLD-II	3	$\theta = 30$ $\alpha = 1.3922 \times 10^{-5}$ $\phi = 0.0029$	-88.9842	**184.8915**	**184.7708**

3.2 Data Set 2

Table 3.2.1, [7] gives the cumulative time between failures of a software product.

Table 3.2.1 Time between failures and their cumulative

FAILURE NUMBER	TIME BETWEEN FAILURES(hrs)	TIME BETWEEN FAILURES (hrs) (CUMULATIVE)
1	81	81
2	48	129
3	9	138
4	4.5	142.5
5	4.5	147
6	60	207
7	24	231
8	21	252
9	21	273
10	12.6	285.6
11	12	297.6
12	3	300.6
13	90	390.6
14	6	396.6
15	24	420.6
16	6	426.6
17	150.6	577.2
18	1.2	578.4
19	3.6	582
20	12	594
21	3	597

22	12	609
23	12	621
24	33	654
25	198	852
26	30	882
27	6	888
28	96	984
29	84	1068
30	81	1149
31	156	1305
32	18	1323
33	54	1377
34	39	1416
35	24	1440
36	12	1452
37	795	2247
38	90	2337

For the data set 2 in Table 3.2.1, the parameters, log likelihood values, AIC and BIC of corresponding distributions are obtained and given in the following Table 3.2.2.

Table 3.2.2. Goodness of fit using AIC and BIC

Distributions	**Number of Parameters**	**Parameter values**	**Log likelihood value**	**AIC**	**BIC**
Goel-Okumoto	1	$\lambda = 0.0014$	-288.414	578.9391	584.1032
Weibull	2	$\beta = 798.063$ $\eta = 1.3604$	-11123	22249	22252
EG	2	$\beta = 0.0014$ $p = 0.0023$	-288.421	581.1852	584.1175
Pareto type III	2	$\alpha = 0.5318$ $s = 81$	-300.437	605.2175	608.1498
LLD-I	3	$\theta = 129$ $\alpha = 3.7944$ $\phi = 432.258$	-287.124	580.9551	581.5244
LLD-II	3	$\theta = 80$ $\alpha = 1.7721 \times 10^{-4}$ $\phi = 0.1148$	-151.565	**309.8361**	**310.4054**

Tables 3.1.2 and 3.2.2 show that LLD-II is a better fit for both the set of software failure data when compared to other distributions.

4. Software failure data analysis

4.1 NHPP Model

The mean value function $m(x)$ and intensity function $\lambda(x)$ for finite value NHPP models are given as follows

$$m(x) = aF(x)$$
$$\lambda(x) = af(x) \qquad (4.1.1)$$

where 'a' is the number of faults in the software.

For LLD-II, the mean value function and intensity function using (2.6.1) and (2.6.2) are

$$m(x) = \begin{cases} \dfrac{a\alpha}{2(\alpha+1)} e^{(\alpha+1)\left(\frac{x-\theta}{\phi}\right)} & x \le \theta \\ a\left(1 - e^{\alpha\left(\frac{\theta-x}{\phi}\right)} + \dfrac{\alpha}{2(\alpha+1)} e^{(\alpha+1)\left(\frac{\theta-x}{\phi}\right)}\right) & x \ge \theta \end{cases} \qquad (4.1.2)$$

$$\lambda(x) = \begin{cases} \dfrac{a\alpha}{2\phi} e^{(\alpha+1)\left(\frac{x-\theta}{\phi}\right)} & x \le \theta \\ \dfrac{a\alpha}{\phi} e^{\alpha\left(\frac{\theta-x}{\phi}\right)}\left(1 - \dfrac{1}{2} e^{\left(\frac{\theta-x}{\phi}\right)}\right) & x \ge \theta \end{cases} \qquad (4.1.3)$$

The joint density function or likelihood function for NHPP models can be written as

$$L = e^{-m(x_n)} \sum_{i=1}^{n} \lambda(x_i) \qquad (4.1.4)$$

The log-likelihood function for NHPP model of LLD-Type I using (4.1.2) and (4.1.3), is

$$\log L = \begin{cases} \sum_{i=1}^{n} \log \dfrac{a\alpha}{2\phi} e^{(\alpha+1)\left(\frac{x_i-\theta}{\phi}\right)} - \dfrac{a\alpha}{2(\alpha+1)} e^{(\alpha+1)\left(\frac{x_n-\theta}{\phi}\right)} & x \le \theta \\ \sum_{i=1}^{n} \log \dfrac{a\alpha}{\phi} e^{\alpha\left(\frac{\theta-x_i}{\phi}\right)}\left(1 - \dfrac{1}{2} e^{\left(\frac{\theta-x_i}{\phi}\right)}\right) - a\left(1 - e^{\alpha\left(\frac{\theta-x_n}{\phi}\right)} + \dfrac{\alpha}{2(\alpha+1)} e^{(\alpha+1)\left(\frac{\theta-x_n}{\phi}\right)}\right) & x \ge \theta \end{cases} \qquad (4.1.5)$$

Partially differentiating (4.1.5) with respect to 'a' and equating to zero, it is found that,

$$a = \frac{n}{\dfrac{\alpha e^{(\alpha+1)\left(\frac{x_{n_1}-\theta}{\phi}\right)}}{2(\alpha+1)} + \left(1 - e^{\alpha\left(\frac{\theta-x_{n_2}}{\phi}\right)} + \dfrac{\alpha}{2(\alpha+1)} e^{(\alpha+1)\left(\frac{\theta-x_{n_2}}{\phi}\right)}\right)} \qquad (4.1.6)$$

From (4.1.6), the value of 'a' for the data set 1 is 28.8803 and data set 2 is 36.4897.

4.2 LLD-II Control Mechanism

A control chart for data sets 1 and 2 would be based on 0.9973 probability limits of the cumulative time between failures. These limits and central line are respectively the solutions of the following equations taking equitailed probabilities

$$F(x) = 0.99865$$
$$F(x) = 0.5 \qquad (4.2.1)$$
$$F(x) = 0.00135$$

Let X_u, X_c, X_l be respectively the solutions of these equations in standard form. Then

$$X_u = F^{-1}(0.99865)$$
$$X_c = F^{-1}(0.5) \qquad (4.2.2)$$
$$X_l = F^{-1}(0.00135)$$

The graph between the serial number of the failures and corresponding successive differences of $m(x)$ together with the 3 curves for X_u, X_c, X_l gives the control chart.

For the data set 1, by equating the mean value function (4.1.2) to equitailed probabilities 0.99865, 0.5, 0.00135 control limits are given as

$$m(X_u)/UCL = 28.8413$$
$$m(X_c)/CL = 14.4401 \qquad (4.2.3)$$
$$m(X_l)/LCL = 0.0390$$

Table 4.2.1 Mean Value function and its Successive Difference for 30 failures

FAILURE NUMBER	MEAN VALUE $m(x)$	SUCCESSIVE DIFFERENCES OF $m(x)$
1	0.0028	0.1989
2	0.2017	2.9326
3	3.1343	0.1675
4	3.3018	0.4177
5	3.7196	1.5449
6	5.2645	0.5767
7	5.8412	0.4197
8	6.2610	2.1692
9	8.4301	1.2345
10	9.6646	0.0433
11	9.7079	0.5649
12	10.2729	0.3013
13	10.5741	0.7825
14	11.3566	0.1824
15	11.5391	1.2458
16	12.7849	1.8696
17	14.6545	0.1893
18	14.8437	0.1288
19	14.9725	0.2730
20	15.2456	3.9135
21	19.1590	0.7649
22	19.9239	0.1699
23	20.0938	5.0130
24	25.1068	1.2166
25	26.3233	**0.0277**
26	26.3510	0.1828
27	26.5339	1.0324
28	27.5663	0.1807
29	27.7470	0.1716
30	27.9185	-

Figure 4.2.2 gives the failure control chart for LLD-II by taking failure number along X-axis, successive differences of $m(x)$ along Y-axis and 3 lines parallel to X-axis for the 3 control limits.

It is inferred that LLD-II detect the failure of the software at the failure number 25.

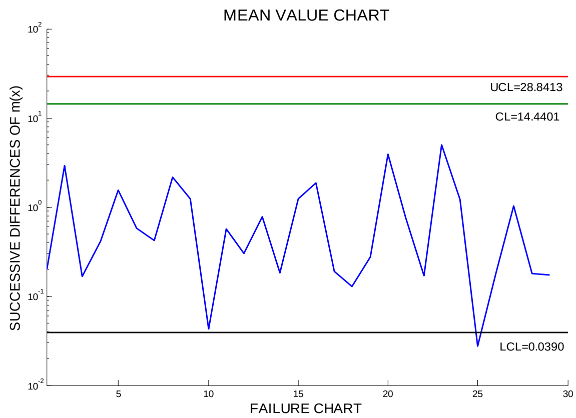

Figure 4.2.2 Mean Value chart for LLD-II

Similarly for the data set 2, the control limits are

$$m(X_u)/UCL = 36.4405$$
$$m(X_c)/CL = 18.2449 \qquad (4.2.4)$$
$$m(X_l)/LCL = 0.0493$$

Table 4.2.3 gives the values of $m(x)$ at 38 cumulative failure times and their successive differences for LLD-II for the Software failure data set 2.

Table 4.2.3 Mean Value function and its Successive Difference for 38 failures

FAILURE NUMBER	MEAN VALUE $m(x)$	SUCCESSIVE DIFFERENCES OF $m(x)$
1	0.0563	2.6019
2	2.6582	0.4668
3	3.1250	0.2310
4	3.3560	0.2294
5	3.5853	2.9107
6	6.4960	1.0909
7	7.5869	0.9219
8	8.5088	0.8925
9	9.4013	0.5218
10	9.9230	0.4876
11	10.4106	0.1205
12	10.5311	3.3671
13	13.8982	0.2083
14	14.1065	0.8141
15	14.9205	0.1988
16	15.1194	4.4328
17	19.5522	**0.0313**
18	19.5835	0.0937
19	19.6772	0.3086
20	19.9858	2.6019
21	20.0626	0.3015

22	20.3635	0.2960
23	20.6595	0.7862
24	21.4457	3.9618
25	25.4075	0.5015
26	25.9091	0.0975
27	26.0066	1.4439
28	27.4505	1.0993
29	28.5497	0.9332
30	29.4830	1.4995
31	30.9825	0.1509
32	31.1334	0.4284
33	31.5618	0.2879
34	31.8497	0.1688
35	32.0184	0.0821
36	32.1005	3.1027
37	35.2032	0.1669
38	35.3701	-

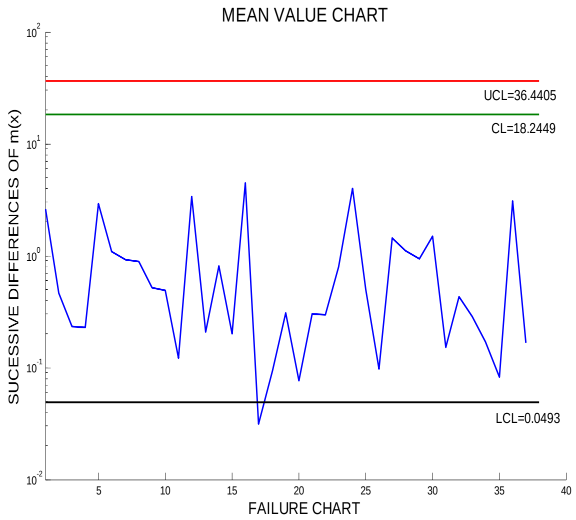

Figure 4.2.4 Mean Value Chart for LLD-II

Figure 4.2.4 gives the failure control chart for LLD-II. It is inferred that LLD-II detects the failure of the software at the failure number 17.

5.Conclusion

This paper assumes that a set of two software failure data follows Goel-Okumoto, Weibull, Exponential Geometric, Pareto type III, LLD-I and LLD-II. The parameters are estimated for both the sets of data. It is proved that LLD-II for the two sets of data is a better fit when compared with the other distributions. Then a control mechanism for LLD-II is framed and the failure points are detected for both the software failure data.

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