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Control of Chaotic Jerk Systems using Nonlinear Feedback Control

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Abstract
Chaotic systems that involve a third order ordinary differential equation in a single variable with simple polynomial nonlinearities with one or two parameters is called chaotic jerk systems. In this paper, nonlinear feedback control method which is based on Lyapunov stability theory is proposed to stabilize the chaotic jerk system. By means of applying nonlinear feedback control in Lyapunov concept to stabilize the Sprott chaotic jerk system. Since the Lyapunov exponents are not required for the calculations, by using MATLAB, numerical simulations are also given to validate the proposed control approach.

Index Terms—Chaos; Jerk; nonlinear control; Lyapunov; Stability.

I. INTRODUCTION

Chaotic dynamical systems are described by nonlinear differential equations and it can be strongly sensitive to initial conditions. Third order differential equation in one scalar variable sometimes called jerk dynamics ([1]).

Chaos theory has been applied in wide areas such as mathematics ([2]), ecology, chemistry ([3], [4]), physical science ([5]), population dynamics and robotics, etc.

In recent years, various control techniques have been deployed ([6], [7]) to carry control the chaotic systems such as PC method, OGY method, sample-data feedback method, sliding mode control method control method ([8]-[9]), backstepping control method ([10]-[14]), active nonlinear control method ([15]-[18]), delayed feedback control method, etc.

Recently, nonlinear feedback control techniques have been taken much attention in controlling the chaotic systems. In this method, the controller tracks the unstable periodic orbit to stable periodic orbit. Comparing to other control techniques, nonlinear control design procedure have simpler implementation, fast speed, more accuracy and reduced control energy.

This paper is organized as follows. In section 2, the result of nonlinear feedback control system is derived. In section 3, the chaotic jerk system ([19]) and its application are clearly defined. In section 4, the control of nonlinear feedback control for chaotic jerk system is derived, the nonlinear feedback control is derived using Lyapunov stability theory. The proposed nonlinear feedback control is very simple and effective to implement in application sides. Conclusions are contained in final section.

II. PROBLEM STATEMENT AND METHODOLOGY

Suppose the chaotic jerk system is system described by the dynamics

$$\dddot{x} = J(x, \dot{x}, \ddot{x}) \qquad (1)$$

where $x \in R^n$ is the state of the system and J is a jerk function, Consider the system with the controller u described by the dynamics

$$\dddot{x} = J(x, \dot{x}, \ddot{x}) + u \qquad (2)$$

The problem is to find a nonlinear feedback controller u so as to stabilize the chaotic jerk dynamics (2) for all initial conditions $x(0) \in R^n$ i.e. $\lim_{t\to\infty}\|x(t)\| = 0$ for all initial conditions $x(0) \in R^n$.

Lyapunov function methodology for establishing control of the system (2)

Suppose candidate Lyapunov function is defined as

$$V(x) = x^T P x \qquad (2)$$

where P is a $n \times n$ positive definite matrix. It is assumed that the parameters of the systems are known and that the states of system (2) are measurable.

If a controller u can be found such that

$$\dot{V}(x) = -x^T Q x \qquad (3)$$

where Q is a positive definite matrix.

Which is a negative definite function.

Hence, by Lyapunov stability theory (Hahn, 1967,[20]) the dynamics (2) is globally exponentially stable.

Hence the condition (3) will be satisfied for all initial conditions $x(0) \in R^n$ then the states of the system (2) is globally exponentially stabilized.

III. SYSTEM DESCRIPTION

Sprott introduce the simple function of chaotic jerk system. The equation chosen has the form (Sprott, J. C., 2006, [19])

$$\dddot{y} + a\ddot{y} + b\dot{y} - |x| - 1 = 0$$

The state space representation of the system is described by

$$\dot{y}_1 = y_2$$

$$\dot{y}_2 = y_3$$

$$\dot{y}_3 = |y_1| - 1 - ay_3 - by_2$$

where a and b are positive constants.

When $a = 0.6$ and $b = 1$, the jerk system excites chaotic natures.
The Jerk chaotic system shown in Figure 1, Figure 2, Figure 3 and Figure 4.

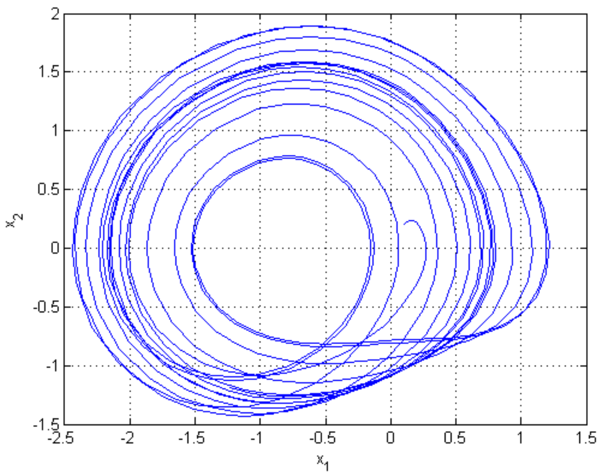

Figure 1 Portrait of Chaotic Jerk System

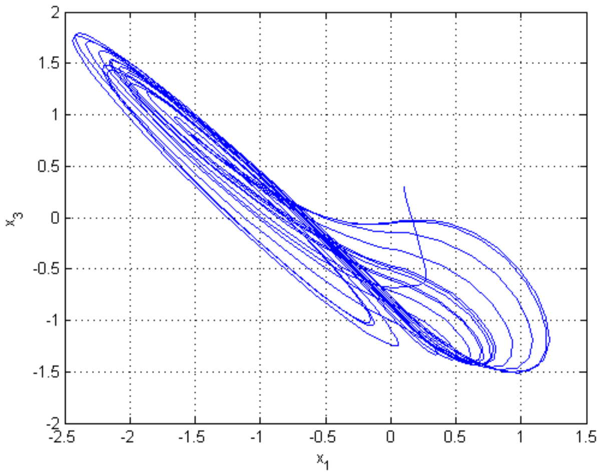

Figure 2 Portrait of Chaotic Jerk System

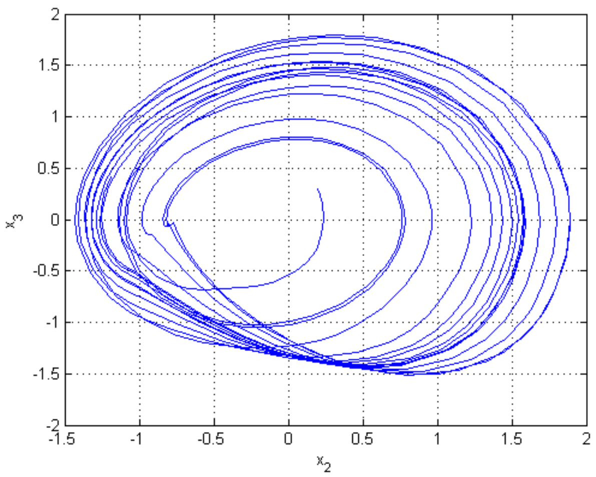

Figure 3 Portrait of Chaotic Jerk System

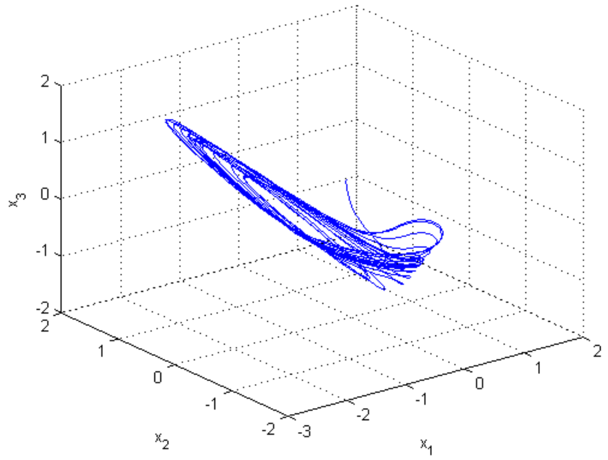

Figure 4 Portrait of Chaotic Jerk System

IV. Control of Chaotic Jerk Systems

In this section, the nonlinear control method is applied for the control of chaotic jerk system (Sprott, J. C., 2006, [19]).
The chaotic jerk system (2006) is described by the equations

$$\begin{aligned} \dot{y}_1 &= y_2 + u_1 \\ \dot{y}_2 &= y_3 + u_2 \\ \dot{y}_3 &= |y_1| - 1 - a y_3 - b y_2 + u_3 \end{aligned} \tag{4}$$

where and a, b are positive real constants.

Case :1 when $y_1 < 0$

To determine the controller, we first define

$$\begin{aligned} u_1 &= u_{1a} + u_{1b}, \text{where } u_{1b} = -k_1 y_1 \\ u_2 &= u_{2a} + u_{2b}, \text{where } u_{2b} = -k_2 y_2 \\ u_3 &= u_{3a} + u_{3b}, \text{where } u_{3b} = -k_3 y_3 \end{aligned} \tag{5}$$

Substitution of (5) into (4) yields

$$\begin{aligned} \dot{y}_1 &= y_2 - k_1 y_1 + u_{1a} \\ \dot{y}_2 &= y_3 - k_2 y_2 + u_{2a} \\ \dot{y}_3 &= y_1 - 1 - a y_3 - b y_2 - k_3 y_3 + u_{3a} \end{aligned} \tag{6}$$

The candidate Lyapunov function is taken as

$$V(y) = \frac{1}{2} y^T y = \frac{1}{2}\; y_1^2 + y_2^2 + y_3^2 \tag{7}$$

A simple calculation gives

$$\begin{aligned} \dot{V} = {} & y_1(y_2 - k_1 y_1 + u_{1a}) + y_2(y_3 - k_2 y_2 + u_{2a}) \\ & + y_1(y_1 - 1 - a y_3 - b y_2 - k_3 y_3 + u_{3a}) \end{aligned} \tag{8}$$

We choose

$$u_{1a} = -y_2$$
$$u_{2a} = -y_3 \qquad (9)$$
$$u_{3a} = -y_1 + 1 + by_2$$

Substitution of (9) into (8) yields

$$\dot{V}(y) = -k_1 y_1^2 - k_2 y_2^2 - (k_3 + a) y_3^2 \qquad (10)$$

which is a negative definite function on R^3 since $k_1, k_2, k_3 \geq 0$.

Hence, by Lyapunov stability theory [18], then the dynamics (4) is globally exponentially stable.

Combining (5) and (9), the nonlinear controller u is obtained as

$$u_1 = -y_2 - k_1 y_1$$
$$u_2 = -y_3 - k_2 y_2 \qquad (11)$$
$$u_3 = 1 - y_1 + by_2 - k_3 y_3$$

Thus, we have proved the following result.

Theorem 1

The chaotic jerk systems (4) is exponentially and globally for any initial conditions with the nonlinear controller u defined by (11).

Case :2 when $y_1 > 0$

To determine the controller, we first define

$$u_1 = u_{1a} + u_{1b}, \text{ where } u_{1b} = -k_1 y_1$$
$$u_2 = u_{2a} + u_{2b}, \text{ where } u_{2b} = -k_2 y_2 \qquad (12)$$
$$u_3 = u_{3a} + u_{3b}, \text{ where } u_{3b} = -k_3 y_3$$

Substitution of (12) into (4) yields

$$\dot{y}_1 = y_2 - k_1 y_1 + u_{1a}$$
$$\dot{y}_2 = y_3 - k_2 y_2 + u_{2a} \qquad (13)$$
$$\dot{y}_3 = -y_1 - 1 - ay_3 - by_2 - k_3 y_3 + u_{3a}$$

The candidate Lyapunov function is taken as

$$V(y) = \frac{1}{2} y^T y = \frac{1}{2} \; y_1^2 + y_2^2 + y_3^2 \qquad (14)$$

A simple calculation gives

$$\dot{V} = y_1(y_2 - k_1 y_1 + u_{1a}) + y_2(y_3 - k_2 y_2 + u_{2a}) + y_1(-y_1 - 1 - ay_3 - by_2 - k_3 y_3 + u_{3a}) \qquad (15)$$

We choose

$$u_{1a} = -y_2$$
$$u_{2a} = -y_3 \qquad (16)$$
$$u_{3a} = 1 + y_1 + by_2$$

Substitution of (16) into (15) yields

$$\dot{V}(y) = -k_1 y_1^2 - k_2 y_2^2 - (k_3 + a) y_3^2 \qquad (17)$$

which is a negative definite function on R^3 since $k_1, k_2, k_3 \geq 0$.

Hence, by Lyapunov stability theory [18], then the dynamics (4) is globally exponentially stable.

Combining (16) and (12), the nonlinear controller u is obtained as

$$u_1 = -y_2 - k_1 y_1$$
$$u_2 = -y_3 - k_2 y_2 \qquad (18)$$
$$u_3 = 1 + y_1 + by_2 - k_3 y_3$$

Thus, we have proved the following result.

Theorem 2

The chaotic jerk systems (4) is exponentially and globally for any initial conditions with the nonlinear controller u defined by (11).

Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method is used to solve the system using MATLAB.

For the chaotic jerk system (4), the parameter values are taken as those which result in chaotic behaviour of the system. When $a = 0.6,\; b = 1$

The initial values of the system (4) are taken as $x_1(0) = 1$, $x_2(0) = 9$

Figure 5 shows that stabilization of the gyros system (4).

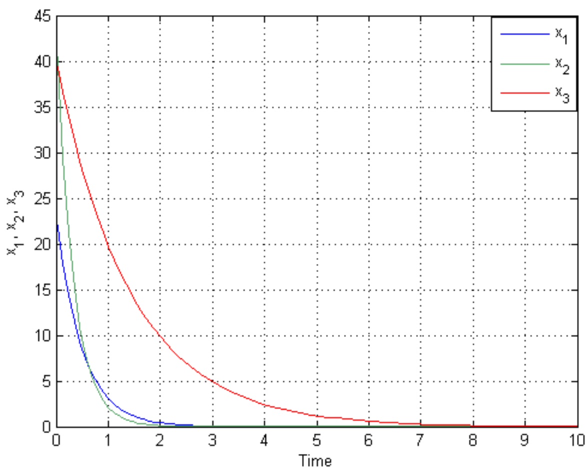

Figure 5. Control of chaotic jerk system

V. CONCLUSIONS

In this paper, nonlinear feedback control method based on Lyapunov stability theory is proposed to stabilize the jerk system. Numerical simulations are also given to validate the proposed control approach. Since the Lyapunov exponents are not required for the calculation, the nonlinear feedback control method is effective and convenient stabilize the jerk systems.

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