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Vortex Shedding From a Circular Cylinder According to the Mesh-Free Vortex Method

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Abstract- Mesh-free vortex method is used to simulate vortex-shedding (von Karman street) how past a circular cylinder for high Reynolds number (Re=140000). In this method, the primary variable is vorticity of the flow which is discretized into a finite number of vortex particles with the specific strengths. These vortices after generation on the cylinder surface, followed in two separate fractional time step in a Lagrangian reference, convection and diffusion.in order to satisfying the no-slip boundary condition number of vortices are generated on the surface. Because of Lagrangian nature of the scheme there is very good agreement between the numerical scheme and physics of the flow. The time development of the periodic vortex shedding, are illustrated by vorticity field picture. The simulation results are compared with experimental data and satisfactory agreement is found.

Keywords: Vortex shedding, turbulence flow, one cylinder, mesh free, direct solution ,vortex method, random walk

Introduction

One of the fundamental research topics of bluff-body aerodynamics is the flow around the circular cylinder. The flow around a circular cylinder has received considerable attention in the last few decades (for example the reviews by Zdravkovich [12],Williamson,[11]). It has been widely investigated due to the applications in engineering such as airplane, buildings, power transmission lines, bridge, offshore structures, ship and the pipes in a heat exchanger. In many of these engineering applications, vortex shedding (von Karrman street) may be responsible for major problems.

The studies of Von Karman Street or vortex shedding behind circular cylinder which exposed to uniform flow have been the focus of several investigations. A large number of experimental studies have been reported on vortex shedding flows (for example, Bearman [1], Strykowsky and Sreenivasan [8]) and many researchers investigated about vortex shedding by numerical scheme. But simulating high Reynolds flow is difficult for who use mesh base numerical scheme .therefore for simulation at high Reynolds we need mesh free methods. Random Vortex methods are successful and attractive mesh-free methods for the numerical simulation of viscose flow at high Reynolds number (Puckett, E. G. [6], Leonard, [5]).

The Numerical Method

Vorticity equation can be derived from the Navier-Stokes equations (1) .To do that, we first define the vorticity $\vec{\omega}$ to be the curl of the flow velocity (2),

$$\frac{\partial \vec{V}}{\partial t} + \vec{V}.grad\,\vec{V} = -\frac{1}{\rho}gradP + \nu\nabla^2\vec{V} \tag{1}$$

$$\vec{\omega} \equiv \nabla \times \vec{u} \tag{2}$$

By applying Curl Operator on Navier Stocks and merging it with continuity equation we can gain vorticity transfer equation as follows:

$$\frac{\partial \vec{\omega}}{\partial t} + \left(\vec{V}.\vec{\nabla}\right)\vec{\omega} = \nu\Delta\vec{\omega}$$

$$\text{Or,}\quad \frac{\partial \vec{\omega}}{\partial t} + \left(\vec{V}.\vec{\nabla}\right)\vec{\omega} = \frac{1}{\mathrm{Re}}\Delta\vec{\omega} \tag{3}$$

We denote $\vec{V}$ to be the velocity vector, *Re* the Reynolds number and $\Delta \equiv \nabla^2$ the Laplacian. According to the “viscose splitting scheme” which introduced by Chorin [4] vorticity transfer equation can be split into two parts: 1- inviscid part (4), 2- viscose part (5)

$$if\quad \nu = 0 \Rightarrow \frac{\partial \vec{\omega}}{\partial t} + \vec{V}.grad\,\vec{\omega} = 0 \rightarrow \begin{cases} \dfrac{d\vec{r}_p}{dt} = V(\vec{r}_p) \\ \dfrac{D\vec{\omega}_p}{Dt} = 0 \end{cases} \tag{4}$$

$$\frac{\partial \omega}{\partial t} = \nu\nabla^2\omega \rightarrow \begin{cases} \dfrac{d\vec{r}_p}{dt} = 0 \\ \dfrac{d\vec{\omega}_p}{dt} = \nu\Delta\vec{\omega}(r_p) \end{cases} \tag{5}$$

The inviscid (Euler) equation is solved using the point vortex method. In point vortex method we can divide the fluid containing vorticity into distinct fluid particles of constant vorticity; then the motion of these fluid particles determines the evolution of the vorticity and the diffusion equation is solved considering the effect of fluid viscosity by using random walks. Vortices will move according to the Euler's equations as follows [3]:

$$x_i^{n+1} = x_i^n + \Delta t u_i^n \tag{6}$$

$$y_i^{n+1} = y_i^n + \Delta t \upsilon_i^n \tag{7}$$

$$u_i^n = -\frac{1}{2\pi}\sum_{r_{ij}>\sigma}\Gamma_j \frac{y_i^n - y_j^n}{r_{ij}^2} - \frac{1}{2\pi}\sum_{r_{ij}\le\sigma}\Gamma_j \frac{y_i^n - y_j^n}{\sigma r_{ij}} \tag{8}$$

$$\upsilon_i^n = +\frac{1}{2\pi}\sum_{r_{ij}>\sigma}\Gamma_j \frac{x_i^n - x_j^n}{r_{ij}^2} + \frac{1}{2\pi}\sum_{r_{ij}\le\sigma}\Gamma_j \frac{x_i^n - x_j^n}{\sigma r_{ij}} \tag{9}$$

$$r_{ij} = \left[\left(x_i^n - x_j^n\right)^2 + \left(y_i^n - y_j^n\right)^2\right]^{1/2} \qquad t = n\Delta t$$

Where $\left(x_i^n, y_i^n\right)$ is the location of the i^{th} vortex element at time t, Δt is time step, $\sigma = h/\pi$ is the cutoff , and Γ is vortex strengths. Vortices move to new locations using the equations above at each time step then add the diffusion process (for inducing the viscosity effect) to the vortices movement. Equation (5) is the heat equation. Its solution can be obtained by green function. The Green's function for the heat equation in two dimensions is given by,

$$G\left(\vec{r}, t\right) = \frac{1}{4\pi \nu t}\exp\left[-\frac{r^2}{4\nu t}\right] \tag{10}$$

Therefore the solution to equation (5) is given as,

$$\vec{\omega}\left(\vec{r}, t\right) = \frac{1}{4\pi \nu t}\int_{R_2} \exp\left[-\frac{\left(r - r'\right)^2}{4\nu t}\right]\vec{\omega}\left(\vec{r'}\right) dx' dy' \tag{11}$$

Several methods have been presented to solve this equation. In this work we used the random walk method to solve the diffusion part of vorticity equation. Random vortex method solve diffusion by letting the individual vortex blobs undergo independent random walks with the displacement obtained from a Gaussian distribution having zero mean and variance $2\nu\Delta t$. Thus the discrete approximations to vorticity equation are

$$x_i^{n+1} = x_i^n + \Delta t u_i^n + \eta_i^1 \tag{12}$$

$$y_i^{n+1} = y_i^n + \Delta t \upsilon_i^n + \eta_i^2 \tag{13}$$

Where η_i^1 , η_i^1 are random variable with Gaussian distribution. When we impose a circular cylinder into the flow, we need to satisfy the no- penetration and no-slip boundary conditions:

$$\vec{V}\left(\vec{r}, t\right).\hat{e}_n = \vec{V}_B.\hat{e}_n \quad \text{Normal condition: (no-penetration)} \tag{14}$$

$$\vec{V}\left(\vec{r}, t\right).\hat{e}_t = \vec{V}_B.\hat{e}_t \quad \text{Tangential condition: (no-slip)} \tag{15}$$

Where $\hat{e}_n$ is the unit vector normal to the solid body and $\hat{e}_t$ is unit vector tangent to the solid body. No- penetration boundary condition is satisfied by using source and sink on the surface of body .No-slip boundary condition is satisfied by creation of vortex on the boundary every time.

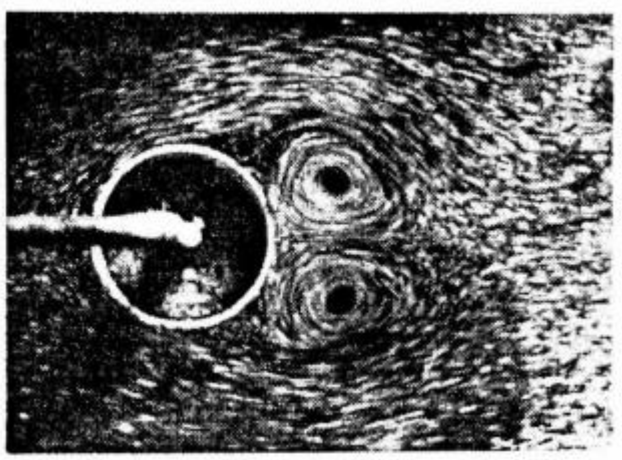
Result and Discussion

In regards to the case of the flow about a bluff symmetrical body, it is considered that the flow starts from rest and it is assumed that the initial motion is irrotational. With the diffusion of vorticity from the body surface there is a thickening of the boundary layer for a while but later the flow gets separated. For circular cylinder, the separation will begin at the rear stagnation point of cylinder before moving toward downstream until a stationary position is reached. During this process, two thin vortex layers form symmetrically and curl up with each other due to diffusion out from the body surface. With the shedding of more vorticity from the surface, there is growth in size and strength of this vortex pair and it extends itself further downstream as represented in Fig .1.

This arrangement is stable for Reynolds numbers, up to about *Re=50,* the structure of flow is stable and the vortex pair grows until reaching steady point at that time there is a balance between the vorticity which diffusing out from the vortex pair toward downstream and the vorticity being added to the pair of vortex from the shear layer.

This stability cannot be continued if the Reynolds number is increased beyond *Re=50.* This is due to the development of an asymmetry where one vortex rolls up over the other and is shed into the wake as a starting vortex. The alternate shedding of vortices of opposite sign into the wake results the Karman vortex street pattern.

(a)



(b)

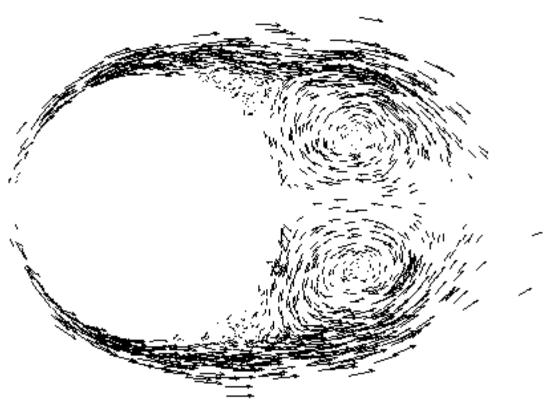


Fig.1. Growth of vortex pair after impulsive start of flow, a) experimental result of Goldstein [5], b) Ours numerical results.

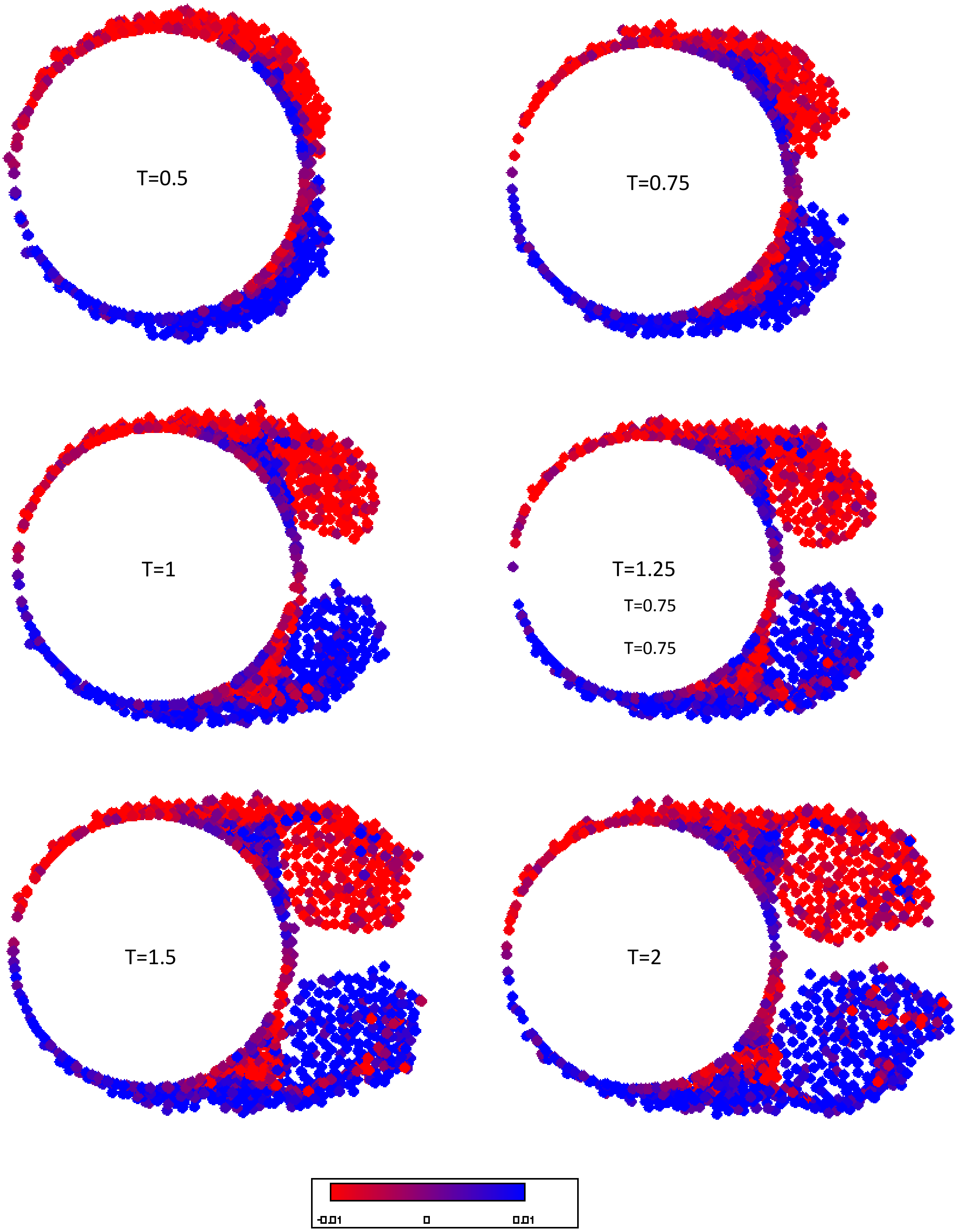


Fig.2. Vorticity fields at different time, Re=140000

The first one to analyses the stability of vortex streets after interpreting the vortex street as an intrinsic property of the wake structure was Von Karman and he did this as early as 1912. The reason behind why the obstacle wake may stretch for a considerable distance downstream is that the growth rates of these instabilities is very small. Likewise, long after an obstacle has passed through that location, there may be a persistence of the high levels of vorticity associated with the wake region. Because of this phenomenon, particularly in the case of an aircraft, assurance of suitable time periods elapse between aircraft landings is considered important.

Vorticity Field

When the circular cylinder is impulsively imposed into motion, the initial flow around it is potential (irrotational). All circulation occurs in a vortex sheet on the surface of the cylinder. This vortex sheet diffuses into the flow for later times.

The evolution of the vorticity field is shown in fig.2 for Re= 140000. At time t>0 the flow at the rear of the cylinder is in the upstream direction. This upstream flow (or flow reversal) have shown by a vorticity sublayer of opposite sign (red and blue color in the figures) to that of the main boundary layer above it. At later times, the boundary layer vorticity rolls up into two discrete eddies at the rear of the cylinder. This is clear at fig.2.

Such separation of vorticity layers from solid surface play important roles in flow control; for example, for control the dynamic stall of aircraft wings. This was investigated by Wang [10] that studied flow control techniques by using vortex method.

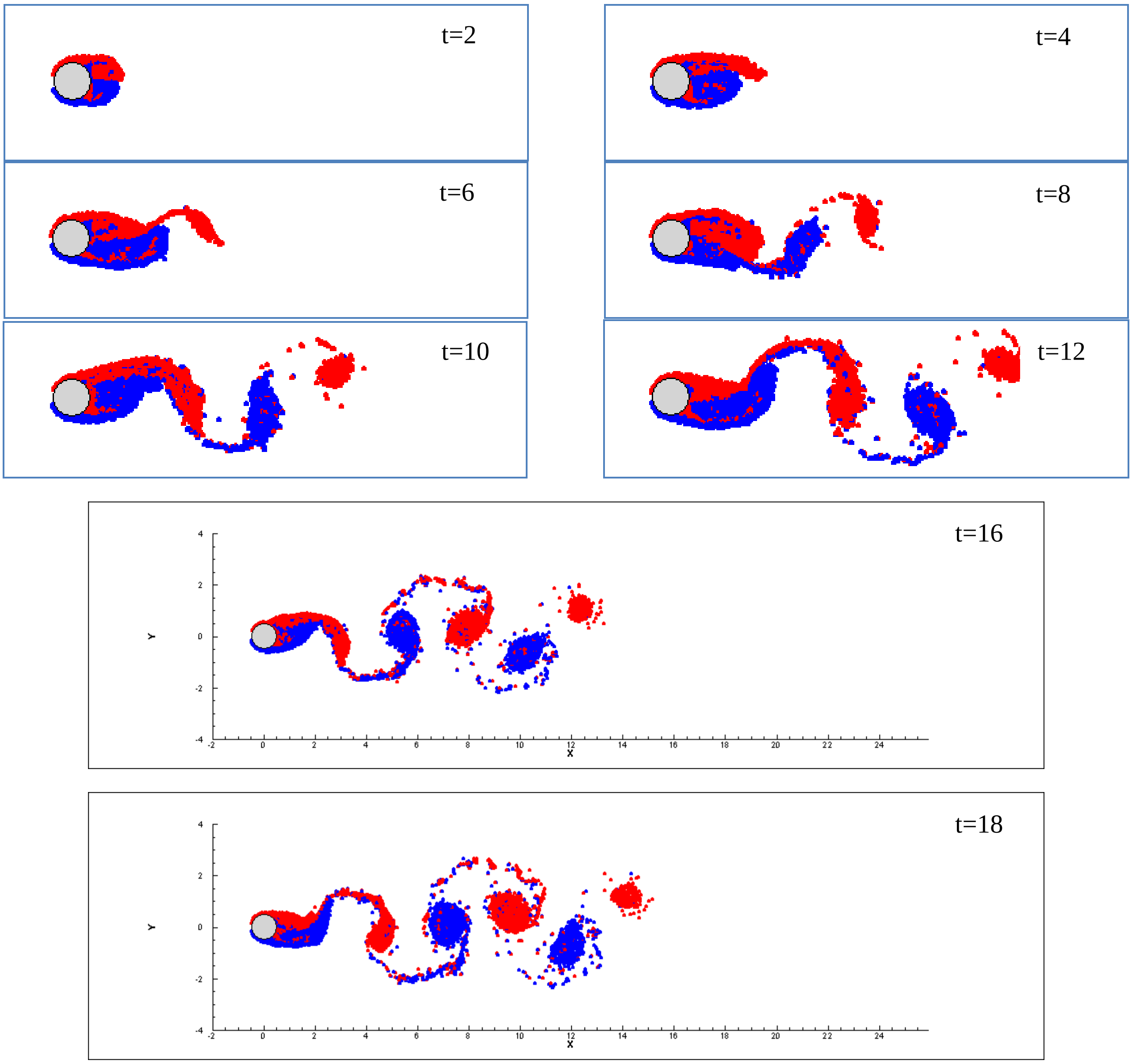


Fig.3. Vorticity field around impulsively started cylinder at a) t=2 to t=18, Re=140000

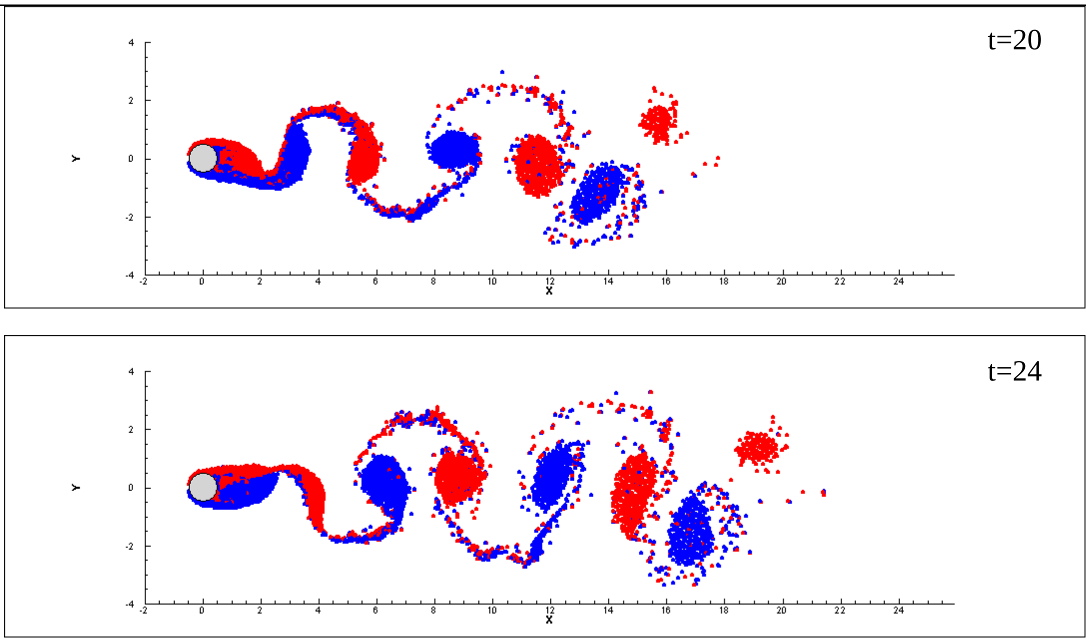


Fig.4. Vorticity field around impulsively started cylinder at a) t=20 and t=24, Re=140000

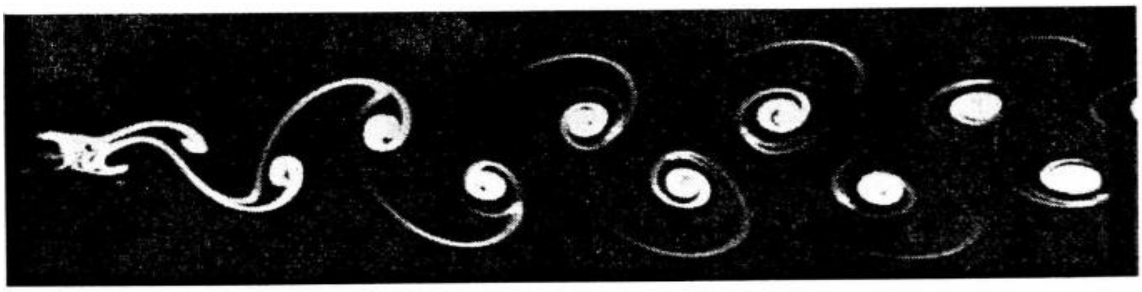


Fig.5. Von Karman street behind a circular cylinder at Re=10^5, Van Dyke [9].

Vortex Shedding

In this section the time histories of vorticity field at Reynolds 140000 is plotted. Keeping track of the vortex elements in a Lagrangian reference, the formation, growth, convection and diffusion of the elements can very well be Shown .Vortex shedding and Von Karman street can be seen clearly .The flow is potential at t=0. Immediately after impulsive start, vortex elements are created on the boundary of the cylinder in order to satisfy the tangential boundary conditions.

The sign of the vorticity generated is positive (blue color) in the lower half of the cylinder and negative (red color) in the upper half. Ultimately there is more vorticity of each sign at the rear of the cylinder than is needed to satisfy the no-slip condition there and a backflow is induced near the surface. The backflow counters the forward-moving fluid and deflects it away from the rear of the cylinder. The shedding process of the Karman vortex is well illustrated in fig.3 and fig.4. Basically, from one of the wake shear layers a vortex grows until it is large enough to reach across the wake behind the cylinder and entrain the shear layer from the opposite side. Then the vortex formation stops and vortex detaches from the shear layer and forms von Karman vortex shedding downstream. As it can be seen in fig.5, there is similarity between our simulation and experimental result.

U-Velocity for One Cylinder

U-velocity (Component of the velocity vector along x direction) around one cylinder are calculated for several section which proportional to the flow direction .the results obtained for one cylinder are compared with experimental data of Cantwell and Coles [2]. It can be seen very good agreement between calculated result and experimental result. Re=140000, $\Delta t = 0.05$. And result obtained for 300 iteration. (See Fig.6).

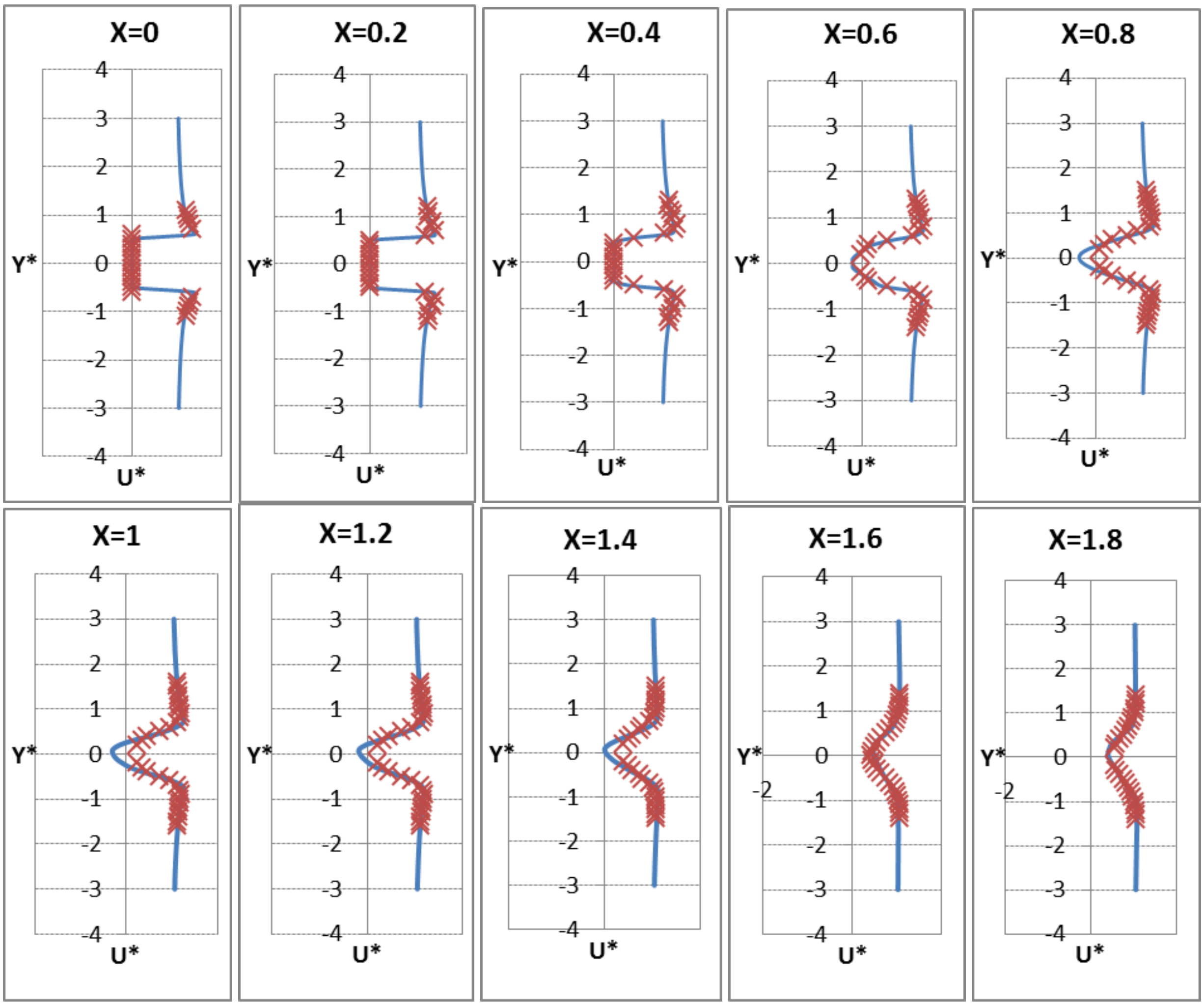


Fig.6. Obtained results (solid line) for distribution of u-velocity around one cylinder and compared with experimental results (symbol), $itr = 300$, Re=140000, X=0 to X=1.8

Conclusion

Vortex shedding and flow pattern around a circular cylinder are modeled numerically by using random vortex method. The diameter of the cylinder was fixed at 1 and the Reynolds number based on the circular cylinder diameter is fixed at 140000.The formation of Von Karman Street can be seen obviously. We dare to say there is a few methods like RVM which able to precisely simulate fluid vorticity distribution because the procedure of solution has a strong physical appeal. It can be seen that vortices are created symmetrically around cylinders and remain symmetric until t=2. The simulation results are compared with experimental result of Cantwell and Coles [2] at Re=14000, and very good agreement is found.

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