

Effective Analysis of the Radio Wave's Diffraction on the Short-Circuited Crack in the Carrying-Out Screen

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Abstract

The reliable and high-speed computing algorithms providing the analysis of diffraction of radio waves on the carrying-out screen with not uniformity are necessary for development and optimization of antennas with flat mirrors and diffraction antennas.

The mathematical model for the analysis of diffraction of radio waves on the short-circuited rectangular crack in the carrying-out screen is offered in this work.

Distinctive feature of this model consists in using of approximation of the front of the disseminated radio wave by the local and flat fragments of the limited extent allowing to consider its any form.

It is reached due to application of the strict solution of a problem of diffraction of a local and flat wave on the screen with a crack.

At the decision representations of a field over a crack Fourier's in integral, fields in a crack – the sum of the waveguide fashions are used.

The system of the functional equations is received by method of partial areas.

The functional equations are reduced to system of the linear algebraic equations of rather complex amplitudes of slot-hole fashions by means of redistribution of modal functions of a field of partial area on basis of adjacent area.

Realization of the main computing procedures is described. On the basis of the analysis of private results reliability of the developed model is shown. The offered algorithm can be used for the analysis and synthesis of antennas with the flat radiating systems punched distributive.

Keywords: the antenna, an electromagnetic wave, a crack in the screen, diffraction, a local and flat wave.

INTRODUCTION

Introduce the problem

The problem of radio wave diffraction on local heterogeneity is one of the key tasks arising in the analysis and synthesis of the microwave ovens functional devices and the distributive radiating systems of certain classes of antennas [1-4].

Thus, the problem of diffraction of a radio wave on a crack in the screen is of special interest and demands the effective decision.

We will emphasize that efficiency of the decision is understood as universality in relation to a type of influence and computing profitability of the algorithm which is at the same time allowing to consider all range of the resonant phenomena in a crack.

In many practical applications the source of a field is placed in close proximity to a crack. The difficult form of the amplitude-phase front of the wave interacting with a crack is a consequence of it. It is obvious that approximation of the field exciting a crack a flat wave in this case is incorrect as reliability of the decision requires the accounting of structure of an exciting field.

The basis of traditional computing algorithms for the analysis of wave diffraction on a crack is made by representation of a field of diffraction of flat waves by continuous spatial Fourier range [8].

At the description of an initial field the wave bunch also applies Fourier's decomposition on flat unlimited waves in space.

As a result of the analysis of diffraction's area the concentrated source is combined with a repeated solution of the flat wave's dispersion's problem of on a crack in the screen [9].

Such way of formalization of a task is most effective at a compact spatial range of primary wave bunch. In antenna, appendices this condition isn't always satisfied by the microwave oven.

Representation of the front of primary wave set of local and

flat fragments of limited extent is the cornerstone of an alternative way [10].

Then primary field can be replaced with the final sum of artificial and local flat waves (with the flat fronts which are artificially limited on extent in space). Such approach for calculation of characteristics of dispersion is based on the principle of superposition, however, demands existence of the solution of a problem of diffraction of a local and flat wave on heterogeneity.

Article purpose – the mathematical description of a problem of wave diffraction with the difficult amplitude-phase front on a rectangular crack in the carrying-out screen and to prove its reliability.

$Q(x)$.

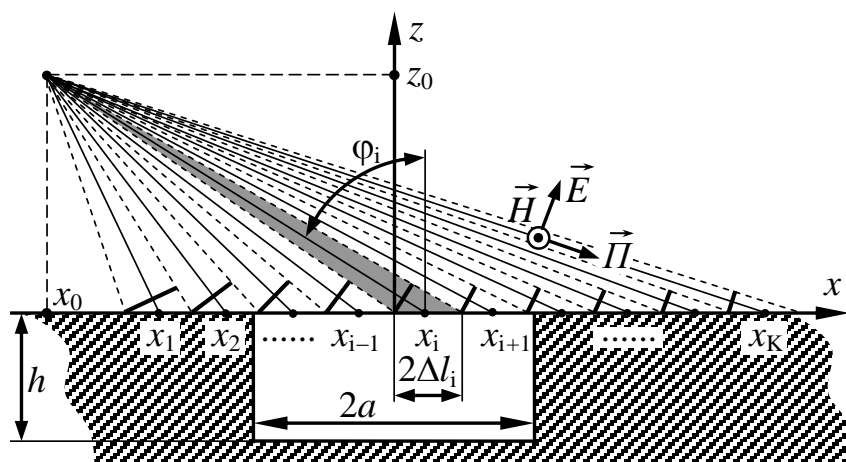


Figure 1

The axis piece Ox symmetric concerning the center of a crack and possessing length $2\Delta = x_K - x_1$ (further – a flare strip), we will segment points with coordinates x_i , where $i = \overline{1, K}$, K - number of points of sampling of the front of a wave. In the vicinity of each point x_i we will consider the front of primary radio wave local and flat. Then the field within a strip of a flare can be described by the final sum of radio waves of linear H-polarization with artificially limited on extent and the flat fronts which aren't crossed in space:

$$H_y^{(n)}(x) = \sum_{i=1}^K Q(x_i) \exp[j\beta_i(x-x_i)] [1(x-x_{i-1}) - 1(x-x_{i+1})], \quad (1)$$

β_i – distribution constant on an axis Ox i- local and flat wave with an amplitude $Q(x_i)$, other than zero on an interval $[x_{i-1}, x_{i+1}]$:

METHODS

Mathematical formalization of a task

The two-dimensional problem of diffraction of a radio wave on a crack in the screen is considered. The crack of a rectangular profile, regular in the direction of a vertical axis, $2a$ and depth of his executed by width in the screen (fig. 1), unlimited in the xOy plane and possessing infinite conductivity. The radio wave possesses the amplitude-phase distribution, known and any in a form, in a crack opens described by function

$$\beta_i = k \sin \varphi_i = \frac{k(x_i - x_0)}{\sqrt{(x_i - x_0)^2 + z_0^2}}; \quad x_{i-1} = x_i - \Delta_i, \quad x_{i+1} = x_i + \Delta_i;$$

$2\Delta_i$ – length of the i -fragment of wave's strip;

$k = 2\pi/\lambda$, λ – constant of distribution and wavelength in free space; $1(x) = \{1, x \geq 0; 0, x < 0\}$

φ_i – the direction of arrival of i -waves from a point (x_0, z_0) to point $(x_i, 0)$.

For achievement of the purpose it is necessary to solve a problem of diffraction of a local and flat wave of linear H-polarization on a crack in the screen (fig. 2).

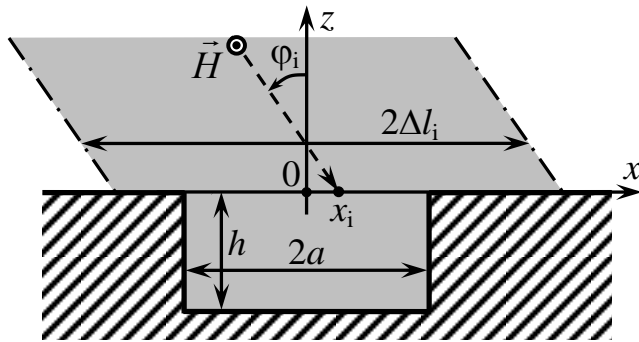


Figure 2

The electromagnetic field of such wave is described excellent from zero H- value in the direction of its uniformity which agrees (1):

$$H_y^{(0)}(x,z) = Q(x_i) [1(x-x_i(z)) - 1(x-x_i(z))] \exp[j\beta_1(x-x_i)] \exp(-j\gamma_1 z), \quad (2)$$

$x_{[.]}(z)$ – abscissae of the left and right current borders of surfaces: $x_{[.]}(z) = x_i + z \operatorname{tg} \varphi_i \mp \Delta L_i$, $2\Delta L_i = x_j - x_l$;

x_i – abscissa of the center of the strip lit by a wave;

$\gamma_i = k \cos \varphi_i$ – cross constant of distribution.

The field disseminated by a crack in space over a crack is expedient to present in the form integrated Fourier decomposition on classical flat waves with unlimited fronts [11]:

$$H_y^{(A)}(x,z) = \int_{-\infty}^{+\infty} A(\beta) \exp[j(\beta x + \gamma(\beta)z)] d\beta, \quad (3)$$

β , $\gamma(\beta) = \sqrt{k^2 - \beta^2}$ – continuous distribution of a partially component of a spatial range of dispersion;

$A(\beta)$ – complex spectral density of intensity of a field.

The field in a crack – the short-circuited flat wave guide – makes sense to approximate discrete set of the waveguide fashions [11]:

$$H_y^{(k)}(x,z) = \sum_{m=0}^{+\infty} D_{mi} \cos[\eta_m(z+h)] f_m(x), \quad (4)$$

D_{mi} – complex amplitude of m-mode from a cross constant of distribution $\eta_m = \sqrt{k^2 - [m\pi/(2a)]^2}$;

$f_m(x)$ – the modal function providing performance of boundary conditions on crack walls:

$$f_m(x) = \begin{cases} \cos\left[\frac{m\pi}{2a}(x+a)\right], & |x| \leq a, \\ 0, & |x| > a. \end{cases} \quad (5)$$

Components of electric field over a crack and in a crack are defined according to Maxwell's equations.

As a result of the sewing's together procedure tangential a field component at $z=0$ it is simple to receive system of the functional equations:

$$\int_{-\infty}^{+\infty} \left[\{A(\beta) - \delta(\beta - \beta_i)\} Q(x_i) \exp(-j\beta x_i) \times \right. \\ \left. \times [1(x-x_i) - 1(x-x_j)] \right] \gamma(\beta) \exp(j\beta x) d\beta = \\ = j \sum_{m=0}^{+\infty} D_{mi} \eta_m \sin(\eta_m h) f_m(x), \quad -\infty < x < \infty, \quad (6)$$

$$\int_{-\infty}^{+\infty} \left[\{A(\beta) + \delta(\beta - \beta_i)\} Q(x_i) \exp(-j\beta x_i) \times \right. \\ \left. \times [1(x-x_i) - 1(x-x_j)] \right] \exp(j\beta x) d\beta = \\ = \sum_{m=0}^{+\infty} D_{mi} \cos(\eta_m h) f_m(x), \quad |x| \leq a, \quad (7)$$

$\delta(\beta - \beta_i)$ – delta function for the account as a part of the spectral density of intensity of a field over a crack of components of a field of primary wave.

We will increase both members of equation (6) on $\exp(-j\beta'x)$, also we will integrate on a variable x в бесконечных пределах [12]. As a result we will receive expression for density $A(\beta)$, which taking into account replacement β' to β will have an appearance:

$$A(\beta) = \frac{1}{2\pi \gamma(\beta)} \left\{ Q(x_i) \gamma_i K_i(\beta) + j \sum_{m=0}^{+\infty} D_{mi} \eta_m \sin(\eta_m h) I_m(\beta) \right\}, \quad (8)$$

$$\text{Где } I_m(\beta) = \int_{-\infty}^{+\infty} f_m(x) e^{-j\beta x} dx = a e^{-j\beta a} \left[\operatorname{sinc}\left(\beta a + \frac{m\pi}{2}\right) + (-1)^m \operatorname{sinc}\left(\beta a - \frac{m\pi}{2}\right) \right]; \quad (9)$$

$$K_i(\beta) = \exp(-j\beta_i x_i) \int_{x_i}^{x_j} e^{-jx(\beta - \beta_i)} dx = 2\Delta L_i e^{-j\beta_i x_i} \operatorname{sinc}[\Delta L_i(\beta - \beta_i)]; \quad (10)$$

$\operatorname{sinc}(u) = \sin(u)/u$.

Using function decomposition $\exp(-j\beta x)$ on orthogonal on an interval $|x| \leq a$ to system of functions $f_m(x)$ [12], it is simple to get rid of the current coordinate x in the functional equation (7):

$$\int_{-\infty}^{+\infty} A(\beta) I_s^*(\beta) d\beta + (-D_{si}) \cos(\eta_s h) (1 + \Delta_s^0) a = -Q(x_i) J_{si}, \quad s = \overline{0, \infty}, \quad (11)$$

Δ_s^t – Kronecker's symbol;

$I_s^*(\beta)$ – complex interface to $I_m(\beta)$ taking into account replacement m to s ;

J_{si} – coefficients of the excitement caused by direct impact

of a local and flat wave on a crack:

$$J_{si} = \int_{-\infty}^{\infty} [1(x-x_{li})-1(x-x_{li})] \exp[j\beta_1(x-x_i)] [1(x+a)-1(x-a)] \cos\left[\frac{s\pi}{2a}(x+a)\right] dx.$$

If $x_{li} \leq -a$ or $x_{li} \geq +a$ (the crack isn't lit"), coefficients are equal to zero, for the other x_{li}, x_{ji} can be fined by the analysis:

$$J_{si} = \int_{\rho_{li}}^{\rho_{ji}} \cos\left[\frac{s\pi(x+a)}{a}\right] \exp[j\beta_1(x-x_i)] dx, \quad (12)$$

$$\rho_{li} = \begin{cases} -a, & x_{li} \leq -a, \\ x_{li}, & -a < x_{li} < a; \end{cases} \quad \rho_{ji} = \begin{cases} a, & x_{ji} \geq a, \\ x_{ji}, & -a < x_{ji} < a. \end{cases}$$

After an exception of the equation (11) spectral density $A(\beta)$, described by expression (8), we receive the system of the linear algebraic equations (SLAE) of rather complex amplitudes of fashions of a crack:

$$\sum_{m=0}^{\infty} D_m [\eta_m \sin(\eta_m h) \sigma_{ms} + j\Delta_m^s (1 + \Delta_m^0) a \cos(\eta_m h)] = jQ(x_i)(J_{si} + \gamma_i P_{si}), \quad (13)$$

σ_{ms} – coefficients of communication of fashions across the field over a crack:

$$\sigma_{ms} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{1}{\gamma(\beta)} I_m(\beta) I_s^*(\beta) d\beta = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{k^2 - \beta^2}} \int_0^{2a} \cos\left(\frac{m\pi}{2a}x\right) \exp(-j\beta x) dx \int_0^{2a} \cos\left(\frac{s\pi}{2a}x\right) \exp(j\beta x) dx d\beta; \quad (14)$$

P_{si} – coefficients of the excitement caused by electrodynamic communication of fashions of a crack with the excited disclosure site:

$$P_{si} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{1}{\gamma(\beta)} K_i(\beta) I_s^*(\beta) d\beta = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{\exp[-j(\beta a + \beta_i x_i)]}{\sqrt{k^2 - \beta^2}} \int_{x_i}^{x_j} \exp[-jx(\beta - \beta_i)] dx \int_0^{2a} \cos\left(\frac{s\pi}{2a}x\right) \exp(j\beta x) dx d\beta. \quad (15)$$

The problem of diffraction of set of local and flat waves (1) on a crack in the screen is a linear task for which the principle of imposing is fair.

The system of the linear equations (13) defining result of diffraction of a single local and flat wave can be used for calculation of a response of a crack for the sum of local and flat waves. For this purpose, is enough to modify the right part of SLAE determined by excitement conditions, way of introduction of the additional composed:

$$\sum_{m=0}^{\infty} D_m [\eta_m \sin(\eta_m h) \sigma_{ms} + j\Delta_m^s (1 + \Delta_m^0) a \cos(\eta_m h)] = j \sum_{i=1}^K Q(x_i)(J_{si} + \gamma_i P_{si}), \quad (16)$$

D_m – complex amplitude of m-mode, determined at

excitement of a crack by set of local and flat waves.

Complex amplitudes of the waveguide fashions D_m are used for calculation of spectral density of intensity of a field over a crack:

$$A(\beta) = \frac{1}{2\pi\sqrt{k^2 - \beta^2}} \left\{ \sum_{i=1}^K Q(x_i) \gamma_i K_i(\beta) + j \sum_{m=0}^{+\infty} D_m \eta_m \sin(\eta_m h) I_m(\beta) \right\}. \quad (17)$$

At research of a direct contribution of a crack to formation of a field of diffraction first composed in (17) it is necessary to lower.

For an assessment of the amplitude chart of dispersion of a wave the crack representing dependence of amplitude of a field of diffraction in a distant zone from a supervision corner it is necessary to use a formula [13]:

$$f(\Theta) = 2\pi\sqrt{30k} |A(k \sin \Theta)| \cos \Theta, \quad (18)$$

Θ – the supervision corner counted clockwise from a normal to a point of opening of a crack.

Realization of the main computing procedures.

Truncation of SLAE (16) is made by simple restriction of number of the fashions considered in a crack ($m, s = \overline{0, M}$).

The analysis shows that for providing an error of performance of balance of the power of 0.5% and it is less enough to consider all fashions and two-three fading extending in a crack.

The dimension of SLAE received thus is small: SLAE contains in resonant range ($\lambda \sim 2\Delta = x_K - x_1$) no more than 3-5 equations.

This size, at least, on one - two orders is less than number of the equations in the methods based on sampling of space or time.

The main time expenditure when forming SLAE (16) falls on calculations of coefficients. A formula (14) for the σ_{ms} is suitable for calculation.

At the same time, in [12] it is shown that initial expression (14) for coefficients σ_{ms} it can be reduced to the look which was more adapted for numerical calculations. If to consider (14) as transformation Fourier (range) of work of two functions, that, believing that the range of work of functions is equal to integrated convolution of ranges of these functions, it is simple to receive:

$$\sigma_{ms} = \frac{1}{2} \int_{-2a}^{2a} H_0^{(1)}(k|\xi|) \int_{\mu_1}^{\mu_2} \cos\left[\frac{m\pi(\mu + \xi)}{2a}\right] \cos\left(\frac{s\pi\mu}{2a}\right) d\mu d\xi, \quad (19)$$

$H_0^{(1)}(x)$ – Hankel's function first sort of a zero order;

$$\mu_l = \begin{cases} -\xi, & \xi \leq 0, \\ 0, & \xi > 0; \end{cases} \quad \mu_r = \begin{cases} 2a, & \xi \leq 0, \\ 2a - \xi, & \xi > 0. \end{cases}$$

The internal integral in (19) pays off analytically, external – in number. We will notice that sub-integral function has logarithmic feature at $\xi=0$. For its elimination the interval of integration should be broken so that feature fell on one of boundary points of partial intervals of integration, and for calculation of partial integrals to use a quadrature formula of Gauss. Besides, owing to property of the ratio (14) which is that $\sigma_{ms} = \sigma_{sm}$, calculation of a half of the specified number of coefficients cannot be made.

The right part of SLAE (16) is defined by the size of coefficients J_{si} and P_{si} . In formula (15) calculation P_{si} needs a lot of time.

Uniformity of expressions (14) and (15) gives the chance of modification (15). Really, due to use of properties of transformation of Fourier it is simple to receive expression, more suitable for numerical calculation of coefficients in P_{si} :

$$P_{si} = \frac{1}{2} \int_{-x_i-2a}^{x_i} H_0^{(1)}(k|\xi+a|) \int_{v_l}^{v_r} \cos\left(\frac{s\pi\mu}{2a}\right) \exp(j\beta_i(\mu+\xi)) d\mu d\xi, \quad (20)$$

$$v_l = \begin{cases} x_{ij} - \xi, & \xi \leq x_{ij}, \\ 0, & \xi > x_{ij}; \end{cases} \quad \mu_l = \begin{cases} 2a, & \xi \leq x_{ij} - 2a, \\ x_{ij} - \xi, & \xi > x_{ij} - 2a. \end{cases}$$

If the area of direct interaction of a radio wave with a crack has extent $2\Delta l_i = x_{ji} - x_{li} \gg \lambda$, that for ensuring high precision of stay P_{si} it's strip fragment $2\Delta l_i$ it makes sense to segment $[x_{lij}, x_{rij}]$ with a binding to a crack. So, it is reasonable to allocate an interval $[-2a, 2a]$, falling on area of a direct flare of a crack, and part at the left and to the right of a crack extent no more than $\lambda/2$. If the j-part of a fragment of a strip is considerably removed from a crack, for example $x_{lij} \geq a + (1...2)\lambda$, than j-part of P_{si} calculation it is possible to execute on an approximate formula. The last is received from initial replacement of function of Hankel with its asymptotic approach and the subsequent analytical integration with freezing of argument of a root of a denominator of sub-integral function [15].

Thus, the received model of diffraction of a radio wave with the uncommon amplitude-phase front on a crack in the screen favourably differs from traditional model.

At a wave bunch's traditional approach Fourier decomposition on flat waves that assumes the repeated solution of a problem of diffraction of the flat wave coming under different corners on the screen with a crack [9] is used.

The last is followed by considerable computing expenses.

In this model the number of splittings the radio wave's front

into local and flat fragments influences only number of SLAE (16) composed the right part and, as a result, – is only insignificant on a counting duration within the solution of a task.

At the same time, coefficients of the main matrix of SLAE don't depend on quality of the field exciting approximation a crack and pay off once.

SLAE is also solved once that as a result guarantees reduction of computing expenses in comparison with traditional approach.

The prize considerably increases in the presence in the screen of tens-hundreds cracks that is characteristic for the distributive radiating systems of edge type of certain antennas classes [5-7].

Assessment of model's reliability.

In fig. 3 settlement rated amplitude charts of dispersion by a crack are provided in the screen of a two-dimensional Gaussian bunch of waves with an effective size 1.5λ , coming at an angle $\varphi=0^\circ$ and lighting a strip extent $2\Delta=10\lambda$. At calculation first composed in (17), and also a factor $\cos\Theta$ in (18) are lowered. The number of points of splitting the front of a wave bunch is taken at calculations equal 10^3 . Crack depth $h = \lambda/4$, width $2a - 1.5$ (line 1), 2 (2) and 5λ (3). The corresponding charts of dispersion calculated on the basis of traditional approach can be found in [14]. Imposing of settlement charts with charts from [14] testifies to their identity within graphic accuracy. Dashed lines in fig. 3 showed charts of dispersion by a crack of a local and flat wave with a length of a strip of a flare equal 10λ . As the strip lit by a local and flat wave considerably exceeds crack width, such mode is almost equivalent to a case of excitement of a crack a flat wave with the unlimited front in space. Comparison with charts, similar on sense, in [14] also testifies to their identity.

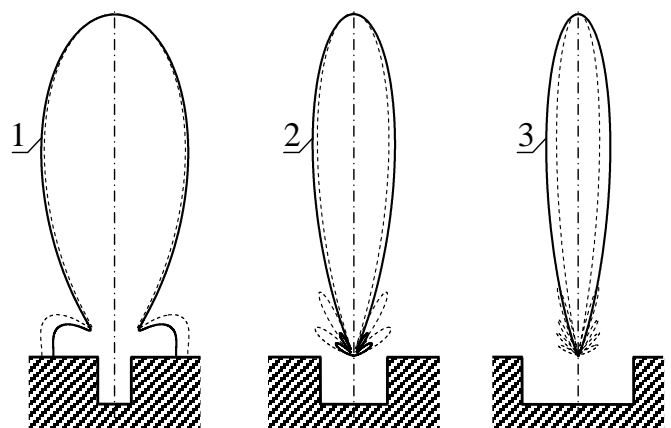


Figure 3

CONCLUSION

The mathematical model of diffraction of a wave of linear H-polarization with the difficult amplitude-phase front on a rectangular crack in the carrying-out unlimited screen is developed. Approximation of the front of primary wave is used by set of local and flat fragments. The task is reduced to final SLAE which main matrix is identical to SLAE matrix in a problem of diffraction on a crack in the screen of a local and flat wave with the limited front. The presented decision can quite be considered strict, as well as the decision [14] as it uses approximations of computing character, and as any strict decision, can be applied in a wide interval of a variation of parameters of primary field and a crack in the screen. It is shown that the developed model demands smaller computing and time expenditure in comparison with the models constructed on the basis of traditional approach. Verification of computing algorithm which showed its adequacy and reliability is executed. The developed algorithm can be used for the analysis of properties of final edge lattices at their excitement by the concentrated source of radio waves and synthesis on this basis of antennas with the flat radiating systems punched distributive.

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