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Improving the Performance of Current Controller in a Grid Connected Renewable PV System using Fractional Order Controller

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Abstract

This paper focuses on the implementation of fractional order control techniques in real world control system for improving the performance of current controller used in grid connected PV system. Two types of controller are usually used in the grid connected system they are Linear and non-linear controller. PI-controller being a linear controller is basically used in the grid connected PV system. Therefore Fractional order controller with Linear PI controller is used in this paper to enhance the performance of current controller. MATLAB Simulink software is used to implement and test the controller. Fractional order operator function and codes are used to optimise and tuning of the controller.

Keywords Fractional order Operator; PI-controller; Fractional Order PI; Bounded system

INTRODUCTION

Today's in digital era, Fractional order controller has many applications such as in Digital Signal Processing, Control engineering, Instrumental applications and in Biological applications too. Fractional order calculus are the advanced application version of Integer and derivative function which allows the user to describe the real world system more accurately. A varying solar radiation can be treated as a Fractional Order system which clearly shows the application of controller to a variety of engineering applications.

Fractional Order system is more flexible because most of the real world applications possess some degree of fractionality in its characteristics and operation [1]. However in most of the cases fractionality is not enough to affect the behaviour of the system. Therefore fractional order controller based on Integration & derivative is applied to the real world system for changing its characteristics. There are many definitions for fractional order controller however Grunwald-Letnikov, Cauchy Integral Formula, Caputo and Riemann-Liouville are more commonly used definitions. Some special functions used in the fractional order controller are represented below.

A. Gamma Function

One of the most important function of fractional order controller is the Gamma Function. The function is represented as

$$\Gamma(n)=\int_0^\infty t^{n-1}e^{-t}dt \tag{1}$$

This is the general function for fractional order controller [4,5].

B. Mittag-Leffler Function

MittagLeffler function [6] plays an important role in the exponential function in integer order calculus. Mathematically it can be represented as

$$E_{\alpha,\beta}=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+\beta)} \tag{2}$$

Where R(α)>0 and R(β)>0

Some of the important Fractional order Mittag-Leffler function are as follows

$$E_{1,1}=e^z \tag{3}$$

$$E_{0,1}=\frac{1}{1-z} \tag{4}$$

$$E_{2,1}=\cosh\left(\sqrt{z}\right) \tag{5}$$

$$E_{0,5,1}=e^{z^2}crfc(-z) \tag{6}$$

All the Mittag-Leffler function are bounded by [1, 1].

C. Grunwald-Letnikov Fractional Order Integral & Derivative Controller

Real world system consisting of higher order derivatives can be written as

$$f^{'}(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \tag{7}$$

Or in more compact manner

Grunwald-Letnikov Fractional Order Integral and Derivative for real world system can be represented as

$${}_{a}D_{t}^{\alpha} f(t) = \lim_{h \to 0} h^{-\alpha} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^{j} \binom{\alpha}{j} f(t - jh) \tag{8}$$

Where α,j represents

$$\binom{\alpha}{j} = \frac{\Gamma(\alpha+1)}{\Gamma(j+1)\Gamma(\alpha - j + 1)} \tag{9}$$

An alternative way of representing the Grunwald-Letnikov Fractional Order Integral and Derivative [7] is as follows

$${}_{a}D_{t}^{\alpha} f(t) = \sum_{k=0}^{n} \frac{f^{(k)(0^{+})t^{k-\alpha}}}{\Gamma(n+1-\alpha)} + \frac{1}{\Gamma(n+1-\alpha)} \int_{0}^{t} \frac{f^{(n+1)}(\tau)}{(t-\tau)^{\alpha-n}} d\tau \tag{10}$$

for n+1≥α≥n

D. Reimann-Liouville Fractional Order Integral and Derivative

Reimann-Liouville Fractional Order Integral and Derivative [7,8] for real world system can be represented as

$${}_{a}I_{t}^{\alpha} f(t) = {}_{a}D_{t}^{-\alpha} f(t) = \frac{1}{\Gamma(-\alpha)} \int_{0}^{t} \frac{f(\tau)}{(t-\tau)^{\alpha+1}} d\tau \tag{11}$$

Derivative order function for Reimann-Liouville Fractional Order system can be represented as

$${}_{a}D_{t}^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^{n}}{dt^{n}} \int_{\alpha}^{t} \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \tag{12}$$

Fractional order equation for Reimann-Liouville Fractional Order system can be written as

$${}_{a}D_{t}^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d}{dt} \int_{\alpha}^{t} \frac{f(\tau)}{(t-\tau)^{\alpha}} d\tau \tag{13}$$

E. Caputo Fractional Order Derivatives

Caputo Fractional order derivatives can be modelled by modifying the Reimann-Liouville Fractional Order Integral and Derivative [10] for real world system. For a real world system it can be written as

$${}_{a}D_{t}^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{0}^{t} \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \tag{14}$$

One of the greatest advantages of Caputo Fractional order derivatives is that initial condition for integer order differential equation holds good for Caputo Fractional order derivatives. Both Caputo Fractional order derivatives and Reimann-Liouville Fractional Order Integral and Derivative becomes equal for homogeneous condition of real world system. Relationship between Caputo Fractional order derivatives and Reimann-Liouville Fractional Order Integral and Derivative under homogeneous condition becomes

$${}_{a}^{RL}D_{t}^{\alpha} f(t) = {}_{a}^{C}D_{t}^{\alpha} f(t) + \sum_{k=0}^{n-1} \frac{(t-a)^{(k-\alpha)}}{\Gamma(k+1-\alpha)} f^{(k)} d\tau \tag{15}$$

Where ^{RL}D and ^{C}D represents the derivative function for Reimann-Liouville Fractional Order and Caputo Fractional order system respectively.

PROPERTIES OF FRACTIONAL ORDER CONTROLLER

A. Property -1

From the geometrical Interpretation point of view for a fractional order control function (g,f), g always scales f based on the following equation

$$g_{t}(\tau) = \frac{1}{\Gamma(\alpha+1)} \left\{ t^{\alpha} - (t-\tau)^{\alpha} \right\} \tag{16}$$

B. Property -2

Based on short memory principle of fractional order controller for t > a+L,

$${}_{a}D_{t}^{\alpha} f(t) \approx {}_{t-L}D_{t}^{\alpha} f(t) \tag{17}$$

Where L represents the length of memory or in general Fractional order controller works on the recent past values such as on function f(t). Error associated with Fractional order controller based on short memory principle can be written as

$${}_{a}D_{t}^{\alpha} f(t) - {}_{t-L}D_{t}^{\alpha} f(t) \le \frac{M L^{-\alpha}}{\Gamma(1-\alpha)} \tag{18}$$

Here M represents the Upper boundary of the function.

C. Property -3

For an analytical function of f(t), ${}_{a}D_{t}^{\alpha} f(t)$ is also analytical for all t and α.

D. *Property -4*

Fractional order derivatives are the common form of integer order derivatives. Therefore for a fractional order integer and derivatives the output result is also same as integer and derivative operator.

E. *Property-5*

Fractional order operators are linear operators for all constant a and b.

$$ {}_0D_t^{\alpha}(af(t)+bg(t)) = {}_{a0}D_t^{\alpha} f(t) + {}_{b0}D_t^{\alpha} g(t) \quad (19)$$

F. *Property-6*

With reference to Caputo definition of system, both fractional order and integer order derivatives are interchangeable i.e..

$$ {}_a^cD_t^{\alpha}({}_a^cD_t^{m} f(t)) = {}_a^CD_t^{m}({}_a^cD_t^{\alpha}) = {}_a^cD_t^{\alpha+m} f(t) \quad (20)$$

Similarly for both mixed and integer order derivative

$$ {}_aD_t^{\alpha}({}_aI_t^{t} f(t)) = \begin{cases} {}_aD_t^{\alpha-\beta} f(t) \\ {}_aI_t^{\beta-\alpha} f(t) \end{cases}, \alpha>\beta \quad (21)$$

for condition-1 and β>α for condition-2

G. *Property-7*

For all continuous function of f(t) and g(t) in the boundary [a,t], Leibniz’s rule becomes

$$ {}_0D_t^{\alpha}(f(t)+g(t)) = \sum_{k=0}^{\infty} \binom{r}{k} f^{(k)}(t)\, {}_aD_t^{\alpha-k} g(t) \quad (22)$$

H. *Property-8*

Caputo derivative for a constant term is always Zero, however Riemann-Liouville derivative of a constant results into a nonzero term.

$$ {}_aD_t^{\alpha} K = \frac{K(t-a)^{\alpha}}{\Gamma(1-\alpha)} \quad (23)$$

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FRACTIONAL ORDER DYNAMIC SYSTEM

Al the real world phenomena possess some degree of fractionality in their behavior some of these fractionality are dominant and some are negligible in their behavior and recognition. Hence Integer Order differential equation is the closet approximated value for fractional order dynamics. The general form of fractional order differential equation can be written as

$$ {}_{an}D_t^{\alpha n} y(t) + {}_{an-1}D_t^{\alpha n-1} y(t) + \dots\dots + {}_{a0}D_t^{\alpha 0} y(t) = {}_{bm}D_t^{\beta m} g(t) + {}_{bm-1}D_t^{\beta m-1} g(t) + \dots\dots + {}_{b0}D_t^{\beta 0} g(t) \quad (24)$$

Where a_k and b_k are the constants and y(t) , g(t) represents the input and output of the system. Applying Laplace transformation to the above differential equation results into a transfer function as follows

$$ G(s) = \frac{{}_{bm}D_t^{\beta m} g(t) + {}_{bm-1}D_t^{\beta m-1} g(t) + \dots\dots + {}_{b0}D_t^{\beta 0} g(t)}{{}_{an}D_t^{\alpha n} y(t) + {}_{an-1}D_t^{\alpha n-1} y(t) + \dots\dots + {}_{a0}D_t^{\alpha 0} y(t)} \quad (25)$$

In general equation (25) can be written as

$$ G(s) = \frac{\sum_{k-0}^{m} b_k s^{\alpha k}}{\sum_{k-0}^{m} a_k s^{\alpha k}} \quad (26)$$

For pseudo rational function equation (26) can be written as

$$ G(\lambda) = \frac{\sum_{k-0}^{m} b_k \lambda^{k}}{\sum_{k-0}^{m} a_k \lambda^{k}} \quad (27)$$

Above transfer function can be written as

$$ G(s) = K_0 \sum_{k=1}^{n} A_k t^{\alpha} E_{\alpha,\alpha}(-p_k t^{\alpha}) \quad (28)$$

A. *Fractional Order PI-Controller*

In the real world system First order system with time delay is wisely used to model the physical system. A generalized model with Fractional order pole plus time delay can be represented as

$$ P(s) = \frac{K}{Ts^{\alpha}+1} e^{-Ls}, \alpha \in (0,1] \quad (29)$$

Where K, L and T represents the constant for fractional order system. Transfer function for Fractional order open PI and closed bounded [PI] controller can be written as

$$ C_2(s) = K_P\left(1+\frac{K_i}{S^{\lambda}}\right) \quad (30)$$

$$ C_2(s) = K_P\left(1+\frac{K_i}{S^{1}}\right)^{\lambda} \quad (31)$$

Here it can be assumed that gain cross over frequency and phase cross over frequency are the two constraints for designing and tuning the controller parameters. These constraints are as follows

- Constraint for Phase Margin

$$ Arg\left[G(j\omega_C)\right] = Arg\left[C(j\omega_C)P(j\omega_C)\right] = \angle C(j\omega_C) + \angle P(j\omega_C) = -\pi + \phi_m \quad (32)$$

Where $G(j\omega_C)$ is the open loop transfer function and $C(j\omega_C)$ is the controller transfer function.

- Similarly constraint for gain cross over frequency

$$\left|G(j\omega_C)\right| = \left|C(j\omega_C)P(j\omega_C)\right|_{dB} = \left|C(j\omega_C)\right|_{dB}\left|P(j\omega_C)\right|_{dB} = 0 \tag{33}$$

- Constraints for robustness of the controller

$$\left.\frac{d\left(Arg\left[G(j\omega)\right]\right)}{d\omega}\right|_{\omega=\omega_C} = 0 \tag{34}$$

B. *Fractional Order PI Controller, Unbounded System*

Transfer function for open loop system based on Fractional Order PI Controller can be written as

$$G_2(s) = C_2(s)P(s) = K_P\left(1+\frac{K_i}{s^\lambda}\right)\left(\frac{K}{Ts^\alpha+1}e^{-Ls}\right) \tag{35}$$

Or

$$C_2(s) = K_P\left(1+\frac{K_i}{s^\lambda}\right) = K_P\left(1+\frac{K_i}{(j\omega)^\lambda}\right) = K_P\left(1+\frac{K_i(\omega)^{-\lambda}}{(j)^\lambda}\right) \tag{36}$$

Where C_2 represents the Fractional order controller. Fractional order controller can be written as

$$C_2(s) = K_P\left(1+\frac{K_i\omega^{-\lambda}}{\cos(\frac{\lambda\pi}{2})+j\sin(\frac{\lambda\pi}{2})}\right) \tag{37}$$

Now open loop phase at gain cross over frequency can be written as

$$Arg\left[G_2(j\omega_C)\right] = -\tan^{-1}\left[\frac{K_i\omega_c^{-\lambda}\sin\left(\frac{\lambda\pi}{2}\right)}{1+K_i\omega_c^{-\lambda}\cos\left(\frac{\lambda\pi}{2}\right)}\right] - \tan^{-1}\left[\frac{B}{A}\right] - L\omega_C \tag{38}$$

Where $A = 1+T\omega_c^\alpha\cos\left(\frac{\alpha\pi}{2}\right)$ and $B = 1+T\omega_c^\alpha\sin\left(\frac{\alpha\pi}{2}\right)$.

Phase margin constraint for the above transfer function can be written as

$$\tan^{-1}\left[\frac{K_i\omega_c^{-\lambda}\sin\left(\frac{\lambda\pi}{2}\right)}{1+K_i\omega_c^{-\lambda}\cos\left(\frac{\lambda\pi}{2}\right)}\right] - \tan^{-1}\left[\frac{B}{A}\right] - L\omega_C = -\pi+\phi_m \tag{39}$$

Or

$$\frac{K_i\omega_c^{-\lambda}\sin\left(\frac{\lambda\pi}{2}\right)}{1+K_i\omega_c^{-\lambda}\cos\left(\frac{\lambda\pi}{2}\right)} = \tan\left(\tan^{-1}\left[\frac{B}{A}\right]+L\omega_C+\phi_m\right) \tag{40}$$

Hence the relationship between K_i and λ can be written as

$$K_i = \frac{-D_2}{\omega_c^{-\lambda}\sin\left(\frac{\lambda\pi}{2}\right)+\omega_c^{-\lambda}\cos\left(\frac{\lambda\pi}{2}\right)} \tag{41}$$

Where D_2 becomes $D_2 = \tan\left[\tan^{-1}(B/A)+\phi_M+L\right]$

Gain of fractional order controller at open loop frequency becomes

$$\left|G_2(j\omega_C)\right| = \frac{K.K_P\sqrt{\left(1+K_i\omega_c^{-\lambda}\cos\left(\frac{\lambda\pi}{2}\right)\right)^2+\left(1+K_i\omega_c^{-\lambda}\sin\left(\frac{\lambda\pi}{2}\right)\right)^2}}{\sqrt{A^2+B^2}} \tag{42}$$

Now with reference to above equation and constraint for gain cross over frequency, the Proportional gain can be written as

$$K_P = \sqrt{\frac{A^2+B^2}{K^2\left(1+K_i\omega_c^{-\lambda}\cos\left(\frac{\lambda\pi}{2}\right)\right)^2+\left(K_i\omega_c^{-\lambda}\sin\left(\frac{\lambda\pi}{2}\right)\right)^2}} \tag{43}$$

Now with reference to the third constraint for checking the robustness of the Fractional Order Controller

$$\left.\frac{K_i\omega_c^{\lambda-1}\sin\left(\frac{\lambda\pi}{2}\right)}{\omega^{2\lambda}+2K_i\omega^\lambda\cos\left(\frac{\lambda\pi}{2}\right)+K_i^2}\right|_{\omega=\omega_C} - E_2 = 0 \tag{44}$$

Where $E_2 = \frac{\alpha T\omega_c^{\alpha-1}}{A^2+B^2}\left(A\sin\left(\frac{\alpha\pi}{2}\right)-B\cos\left(\frac{\alpha\pi}{2}\right)\right)+L$

(45)

Now relation between K_i and λ can be written as

$$K_i = \frac{-F_2\pm\sqrt{F_2^2-4E_2^2\omega_c^{-2\lambda}}}{2E_2\omega_c^{-2\lambda}} \tag{46}$$

From the above equation K_i, λ and K_P can be found out either by using graphical method or by minimum search algorithm.

C. Fractional Order PI Controller, Bounded System

Transfer function for bounded Fractional order PI-controller can be written as

$$G_3(j\omega)=\left(K_P+\frac{K_i}{s}\right)^{\lambda}\left(\frac{K}{Ts^{\alpha}+1}e^{-Ls}\right) \tag{47}$$

Open loop phase gain at cross over frequency can be written as

$$Arg\left[G_3(j\omega_C)\right]=-\lambda\tan^{-1}\left(\frac{K_i}{K_P\omega_C}\right)-\tan^{-1}\left(\frac{B}{A}\right)-L\omega_C \tag{48}$$

Now with reference to 1st constraint of stability

$$-\lambda\tan^{-1}\left(\frac{K_i}{K_P\omega_C}\right)-\tan^{-1}\left(\frac{B}{A}\right)-L\omega_C=-\pi+\phi_M \tag{49}$$

Or

$$\frac{K_i}{K_P\omega_C}=D_3 \tag{50}$$

Or

$$D_3=\tan\left[\left(\pi-\phi_m-\tan^{-1}(B(\omega_C)/A(\omega_C))-L\omega_C\right)/\lambda\right] \tag{51}$$

Again with reference to 2nd constraint of Stability the gain cross over frequency must satisfy the stability criteria and under such condition

$$\frac{K\left[K_P^2+\left(\frac{K_i}{\omega_C}\right)^2\right]}{\sqrt{A^2+B^2}}=1 \tag{52}$$

Similarly with respect to the 3rd constraint of stability gain variation can be written as

$$\frac{\lambda K_iK_P}{\left(K_P\omega_C\right)^2+K_i^2}=E_3 \tag{53}$$

Or

$$E_3=\frac{\alpha T\omega_C^{\alpha-1}}{A^2+B^2}\left(A\sin\left(\frac{\alpha\pi}{2}\right)-B\cos\left(\frac{\alpha\pi}{2}\right)\right)+L \tag{54}$$

From the above three constraints of equations K_i and K_P can be written as

$$K_i=\sqrt{\frac{E_3}{\lambda}\omega^3D_3\left(A^2\omega_C\right)+B^2\omega_C^{\frac{1}{\lambda}}} \tag{55}$$

$$K_P=\sqrt{\frac{E_3\omega_C\left(A^2\omega_C\right)+B^2\omega_C^2}{\lambda D_3}} \tag{56}$$

DESIGNING AND TUNING OF CURRENT CONTROLLER

Let us consider a plant system with transfer function

$$G(s)=\frac{a_1S^{\alpha 1n}+a_0}{b_2S^{\alpha 2n-2}+b_1S^{\alpha 2n-1}+b_0} \tag{57}$$

Where a_1,a_0 and b_2,b_1,b_0 are the coefficients. The fractional order transfer function shown in (57) corresponds to time domain representation of PI controller used in current control strategies of a voltage source inverter. The corresponding differential equation to equation (57) can be written as

$$b_1y^{\alpha 2n2}(t)+b_2y^{\alpha 2n}(t)+b_0y(t)=u(t) \tag{58}$$

The validity of equation (57) holds good only when all the initial condition sets to zero i.e.

$$Y(0)=0,\ Y'(0)=0 \text{ and } Y''(0)=0 \tag{59}$$

Unit step response to equation (58) can be found out by transferring the equation (58) as

$$y(t)=\frac{1}{b_2}\sum_{k=0}^{\infty}\frac{(-1)^K}{k}\left(\frac{b_0}{b_1}\right)^{k-2}\left(\frac{b_1}{b_2}\right)^{k-1}\varepsilon_{k-2}+\frac{1}{b_1}\sum_{k=0}^{n-1}\frac{(-1)^K}{k}\left(\frac{b_0}{b_1}\right)^{k}\varepsilon_{k-1} \tag{59}$$

Real time transfer function for PI-controller can be written as (extracted from MATLAB signal Processor)

$$G(s)=\frac{0.001093s-1.264e-06}{S^2+0.001376s+4.945e-07} \tag{60}$$

Controlling parameters for equation (60) are extracted using frequency domain analysis tools. Bode plot for equation (60) is shown in fig-1. Stability criteria parameters extracted from bode plot is shown in table-1

Table 1: Stability Criterion Parameters

Sl. No.	Parameter	Magnitude
01	Proportional Gain (K_P)	-0.10717
02	Integral Gain (K_i)	-0.0019911
03	Rise Time	2.35e03sec
04	Settling Time	4.31e04sec
05	Over Shoot	5.81%
06	Gain Margin	4.47dB@ 0.00597 rad/sec
07	Phase Margin	22^0 @ 0.00417 rad/sec
08	Closed loop Stability	Stable

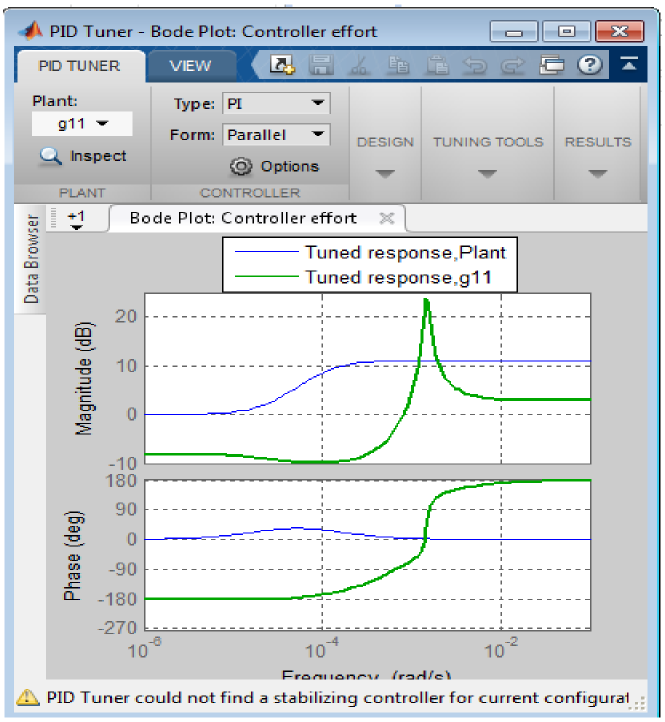

Figure 1: Frequency response- Bode plot extracted from Signal analysis of plant transfer function

In the above transfer function and its corresponding frequency domain bode plot analysis it can be seen that the cross over frequency is maintained at 1000rad/sec for λ=0.637. Applying tuning condition to the controller transfer function under the three different constraint as discussed above normalised open loop transfer function becomes

$$G(s) = \frac{1}{0.01S + 1} \qquad (61)$$

And the corresponding fractional order transfer function after pass-1 with q=0.002 becomes

0.0049956s^{2.63}+3.9948e-05s^{2}+0.00019982s^{1.63}+1.5979e-06s+1.9982e-06s^{0.63}+1.5979e-08

G(s) = --

s^{3.5}+s^{3}+0.0049956s^{2.63}+s^{2.5}+s^{2}+0.00019982s^{1.63}+s^{1.5}+s+1.9982e-06s^{0.63}+1.5979e-08

(62)

Stability Criteria parameters for Fractional order transfer function (62) can be found out from superimpose graphical analysis of (62) as shown in fig-2

Table 2: Stability Criteria parameters for Fraction order transfer function after pass-1

Sl. No.	Parameter	Magnitude
01	Proportional Gain (K_P)	-0.10717
02	Integral Gain (K_i)	-0.0019911
03	Rise Time	2.35e03sec
04	Settling Time	4.31e04sec
05	Over Shoot	5.81%
06	Gain Margin	4.47dB@ 0.00597 rad/sec
07	Phase Margin	22^0 @ 0.00417 rad/sec
08	Closed loop Stability	Stable

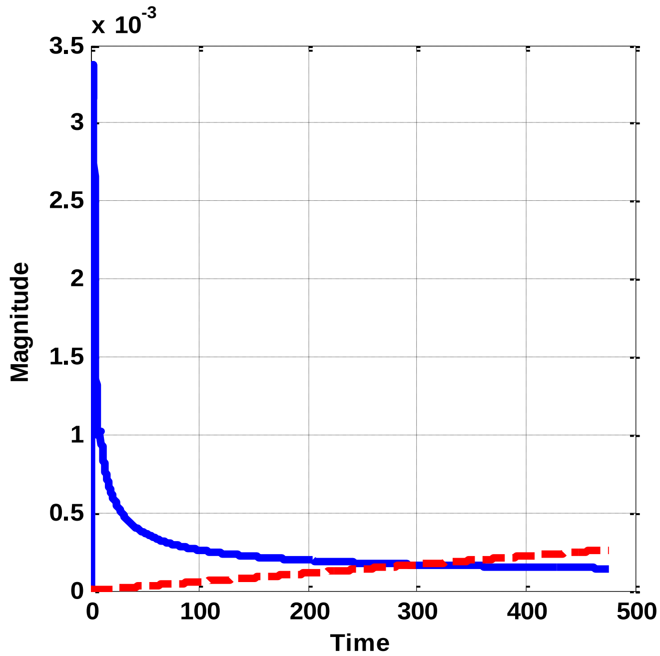

Figure 2: Graphical method for evaluation of FO-PI controller parameters after pass-1 with q=0.001

After applying the auto tuner for Pass-2 with q=0.0047

0.004856s^{2.63}+3.4983e-05s^{2.501}+0.00025794s^{1.63}+1.7824e-06s+2.7419e-06s^{0.839}+1.78209e-11

G(s) = --

s^{2.395}+s^{3.1104}+0.0049978s^{1.92}+s^{2.7}+0.00139578s^{2.49}+s^{1.28}+s+1.0183e-03s^{0.47}+1.569e-11

(63)

0.018635s^{2.613}+2.458e-07s^{2.37}+0.00187307s^{1.528}+1.8745e-08s+1.45993e-07s^{0.59}+1.8295e-06

G(s) = ---

s^{3.538}+s^{3.17}+0.0039471s^{2.413}+s^{2.47}+s^{2.01}+0.00015671s^{1.57}+s^{1.62}+s+1.3487e-06s^{0.58}+1.4581e-08

(64)

0.018635s^{2.613}+2.458e-07s^{2.37}+0.00187307s^{1.528}+1.8745e-08s+1.45993e-07s^{0.59}+1.8295e-06

G(s) = ---

s^{3.538}+s^{3.17}+0.0039471s^{2.413}+s^{2.47}+s^{2.01}+0.00015671s^{1.57}+s^{1.62}+s+1.3487e-06s^{0.58}+1.4581e-08

(65)

0.0051913s^{1.33}+4.1513e-05s^{0.7}+1.557e-06s^{0.63}+1.245e-08

G(s) = --

s^{3.2}+s^{2.7}+s^{2.2}+s^{1.7}+0.0051913s^{1.33}+s^{1.2}+s^{0.7}+1.557e- 06s^{0.63}+1.245e-08

(66)

0.0049956s^{2.53}+3.9948e-05s^{1.9}+0.00022897s^{1.33}+1.831e-06s^{0.7}+2.0707e-06s^{0.63}+1.6558e-08

G(s)= --

s^{3.2}+s^{2.7}+0.0049956s^{2.53}+s^{2.2}+3.9948e-05s^{1.9}+s^{1.7}+0.00022897s^{1.33}+s^{1.2}+s^{0.7}+2.0707e-06s^{0.63}+1.6558e-08

(67)

0.0052454s^{2.58}+4.1945e-05s^{1.95}+0.00022897s^{1.35}+1.831e-06s^{0.72}+2.0707e-06s^{0.63}+1.6558e-08

G(s)= --

s^{3.22}+s^{2.72}+0.0052454s^{2.58}+s^{2.22}+4.1945e-05s^{1.95}+s^{1.72}+0.00022897s^{1.35}+s^{1.22}+s^{0.72}+2.0707e-06s^{0.63}+1.6558e-08

(68)

0.0052704s^{2.645}+4.2145e-05s^{2.015}+0.00031223s^{1.412}+2.4967e-06s^{0.782}+2.0707e-06s^{0.63}+1.6558e-08

G(s)= ---

s^{3.282}+s^{2.782}+0.0052704s^{2.645}+s^{2.282}+4.2145e-05s^{2.015}+s^{1.782}+0.00031223s^{1.412}+s^{1.282}+s^{0.782}+2.0707e-06s^{0.63}+1.6558e-08

(69)

0.0054452s^{2.725}+4.3543e-05s^{2.095}+0.00031514s^{1.392}+2.5201e-06s^{0.762}+2.9033e-06s^{0.63}+2.3216e-08

G(s)= --

s^{3.262}+s^{2.762}+0.0054452s^{2.725}+s^{2.262}+4.3543e-05s^{2.095}+s^{1.762}+0.00031514s^{1.392}+s^{1.262}+s^{0.762}+2.9033e-06s^{0.63}+2.3216e-08

(70)

0.0056117s^{2.681}+4.4875e-05s^{2.051}+0.00032347s^{1.292}+2.5866e-06s^{0.662}+2.9713e-06s^{0.63}+2.376e-08

G(s)= ---

s^{3.162}+0.0056117s^{2.681}+s^{2.662}+s^{2.162}+4.4875e-05s^{2.051}+s^{1.662}+0.00032347s^{1.292}+s^{1.162}+s^{0.662}+2.9713e-06s^{0.63}+2.376e-08

(71)

0.0057782s^{2.621}+4.6207e-05s^{1.991}+0.00032455s^{1.312}+2.5953e-06s^{0.682}+2.9705e-06s^{0.63}+2.3754e-08

G(s)= ---

s^{3.182}+s^{2.682}+0.0057782s^{2.621}+s^{2.182}+4.6207e-05s^{1.991}+s^{1.682}+0.00032455s^{1.312}+s^{1.182}+s^{0.682}+2.9705e-06s^{0.63}+2.3754e-08

(72)

0.0066108s^{2.638}+5.2865e-05s^{2.008}+0.00033271s^{1.319}+2.6605e-06s^{0.689}+3.2202e-06s^{0.63}+2.5751e-08

G(s)= ---

s^{3.189}+s^{2.689}+0.0066108s^{2.638}+s^{2.189}+5.2865e-05s^{2.008}+s^{1.689}+0.00033271s^{1.319}+s^{1.189}+s^{0.689}+3.2202e-06s^{0.63}+2.5751e-08

(73)

The solver have reached a local minimum, but cannot be certain because the first-order optimality measure is not less than the TolFun tolerance (1e-4*TolFun for the Levenberg-Marquardt algorithm).

Table 3: Successful Iteration Function Parameters after Pass-1

Iteration No.	Function Count	F(x)	Norm of Step	1st Order Optimality
00	4	4.821e11	0	9.64e09
01	8	3.05e11	10	7.56e09
02	12	1.744e10	20	3.72e09
03	16	0.0066017	20.82	74.6
04	20	0.006183	3.91	4.33
05	24	0.005965	7.89	34.4
06	28	00005919	1.31	1.16
07	32	0.005911	1.07	0.626
08	36	0.005910	0.9807	0.622387
09	40	0.005908	0.9784	0.6214
10	44	0.005908	0.9781	0.6214

For tuning the Current controller with fractional order controller Result obtained from equation (73) is ploted with integer order controller as obtained in chapter-3 of PI-controller in a frequency domain plot shown in fig:-3,4 & 5. Intersection of the two plot clearly determines the order and gain of the controller. Putting this value in equation 58 will determine the proportional gain of the controller.

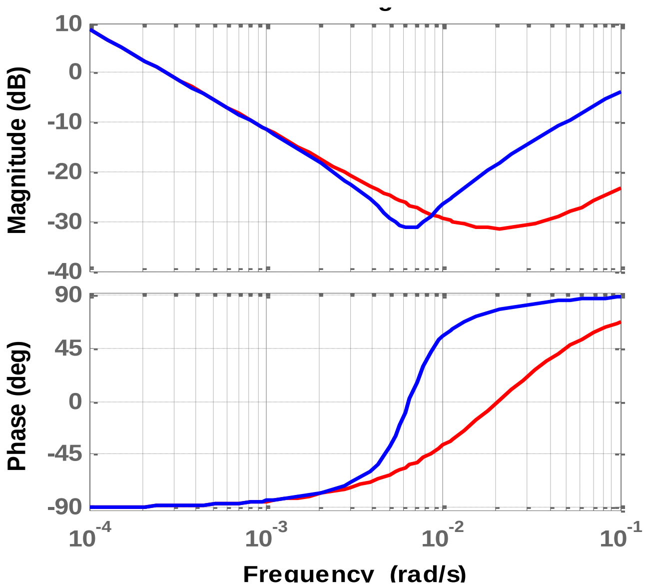

Figure 3: Comparative analysis of FO-PI with PI Controller

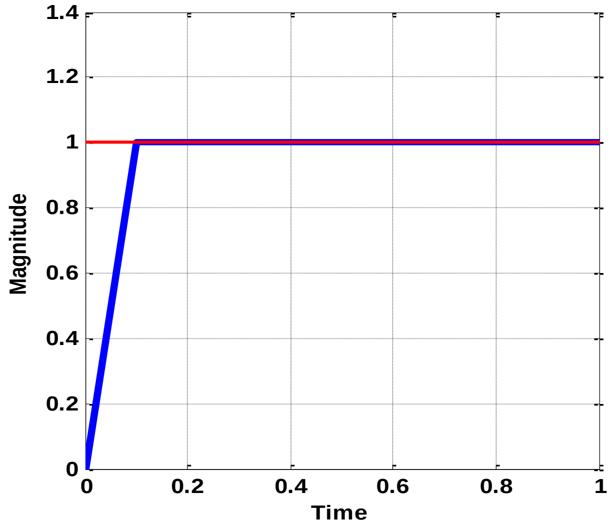

Figure 4: Time domain analysis of FO-PI controller

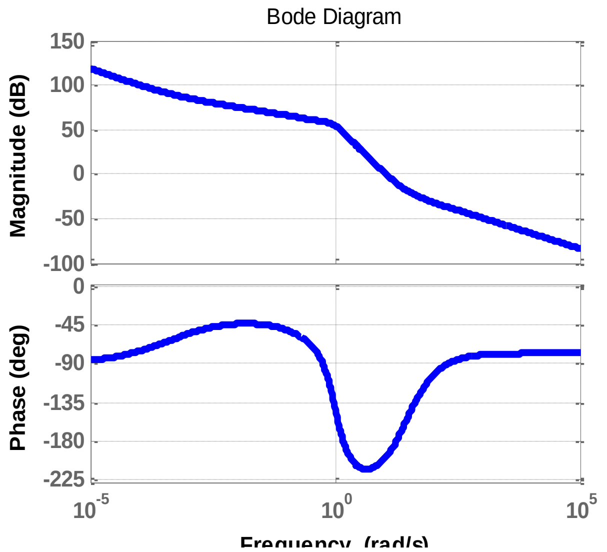

Figure 5: Bode Plot response for FO-PI Identified model

Parameters extracted from fig-4 & 5 using stability tuner is summarized in table-4.

Table 4: Extracted Parameters from Stability Tuner

Sl. No.	Parameter	Magnitude
01	Proportional Gain (K_P)	-1.386
02	Integral Gain (K_i)	-3.84e-05
03	Rise Time	1.19e03
04	Settling Time	1.29e-05
05	Over Shoot	9.99%
06	Gain Margin	1.17dB@ 0.0756 rad/sec
07	Phase Margin	51.3^0 @ 0.0692 rad/sec
08	Closed loop Stability	Marginally Stable

CONCLUSION

Fractional order controller shows better performance as compared to linear PI-controller. This can be easily understood from the Bode plot characteristics. Fractional order controller limits the maximum signal there by increasing the efficiency by reducing the error. It signifies that higher order controller can be easily realised with the help of lower order controllers.

REFERENCES

[1]. J. Mendiola-Fuentes and E. Campos-Cantón, "Algorithm for designing fractional order PD^{μ} controller via integer order controller," *2014 IEEE Central America and Panama Convention (CONCAPAN XXXIV)*, Panama City, 2014, pp. 1-5.

[2]. Ben Hmed, M. Amairi and M. Aoun, "Fractional order controller design using time-domain specifications," *2014 15th International Conference on Sciences and Techniques of Automatic Control and Computer Engineering (STA)*, Hammamet, 2014, pp. 462-467.

[3]. R. Hilfer, P. Butzer, U. Westphal, J. Douglas, W. Schneider, G. Zaslavsky, T. Nonnemacher, A. Blumen, B. West, Applications of fractional calculus in physics, World Scientific, vol. 5, 2000

[4]. M. Amairi, M. Aoun, S. Najar, M. N. Abdelkrim, "Guaranteed frequency-domain identification of fractional order systems: application to a real system", *International Journal of Modelling Identification and Control*, vol. 17, no. 1, pp. 32-42, 2012.

[5]. Saidi, M. Amairi, S. Najar, M. Aoun, "Bode shaping-based design methods of a fractional order pid controller for uncertain systems", *Nonlinear Dynamics*, pp. 1-22, 2014.

[6]. J. Sabatier, M. Moze, C. Farges, "Lmi stability conditions for fractional order systems", *Computers & Mathematics with Applications*, vol. 59, no. 5, pp. 1594-1609, 2010.

[7]. Matignon, "Stability properties for generalized fractional differential systems", *ESAIM: proceedings*, vol. 5, pp. 145-158, 1998.

[8]. M. Moze, J. Sabatier, A. Oustaloup, "Lmi tools for stability analysis of fractional systems", *International Design Engineering Technical Conferences and Computers and Information in Engineering Conference. American Society of Mechanical Engineers*, pp. 1611-1619, 2005.

[9]. J. Sabatier, O. Cois, A. Oustaloup, "Modal placement control method for fractional systems: application to a testing bench", *International Design Engineering Technical Conferences and Computers and Information in Engineering Conference (ASME)2003. American Society of Mechanical Engineers*, pp. 633-639, 2003.

[10]. R. Malti, M. Xavier, F. Khemane, A. Oustaloup, "Stability and resonance conditions of elementary fractional transfer functions", *Automatica*, vol. 47, no. 6, pp. 2462-2467, 2011.