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Some Graph Operations Of Even Vertex Odd Mean Labeling Graphs

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Abstract

A graph with p vertices and q edges is said to have an even vertex odd mean labeling if there exists an injective function f:V(G)→{0, 2, 4, ... 2q-2,2q} such that the induced map f*: E(G)→ {1, 3, 5, ... 2q-1} defined by f*(uv)= $\frac{f(u)+f(v)}{2}$ is a bijection. A graph that admits an even vertex odd mean labeling is called an even vertex odd mean graph. In this paper we pay our attention to prove some graph operations of even vertex odd mean labeling graphs

Keywords: Even vertex odd mean labeling, even vertex odd mean graph

AMS subject classification (2010): 05C78

INTRODUCTION

Throughout this paper we restrict our attention to finite, simple and undirected graphs.The set of vertices and the set of edges of a graph G will be denoted by V(G) and E(G) respectively and let $p = |V(G)|$, $q = |E(G)|$.For general graph theorectic notations we follow F.Harary[7].A graph labeling is a mapping that carries a set of elements(usually vertices and /or edges) into a set of numbers .Many kinds of labeling have been studied an excellent survey of graph labeling can be found in[2].Most of the graph labeling techniques found their origin with graceful labeling which was introduced by Rosa.A(1967) . Let G(V,E) be a graph with p vertices and q edges.

The concept of mean labeling was introduced and studied by Somasundaram and Ponraj [11]. Further some more results on mean graphs are discussed in [4,6]. A graph G is said to be a mean graph if there exists an injective function f: V(G) → {0, 1, 2, …q} such that the induced map f*: E(G)→{1, 2, …q} defined by f* (uv)= $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a bijection.

Manickam and Marudai [10] have introduced the concept of odd mean labeling of a graph. A graph G is said to be odd mean if there exists an injective map f:V(G)→{0, 1, …2q–1} defined by f*(uv) = $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a bijection. The concept of even mean labeling was introduced and studied by Gayathri and Gopi [3]. A graph G is said to be even mean if there exists an injective function

f: V(G)→{0,1,…2q} such that the induced map f*:E(G)→{2,4,…2q} defined by f*(uv) = $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a bijection.

A graph G is said to have an even vertex odd mean labeling if there exists an injective function f:V(G)→{0, 2, …2q–2, 2q} such that the induced map f*:E(G)→{1, 3, … 2q–1} defined by f*(uv) = $\frac{f(u)+f(v)}{2}$ is a bijection. A graph that admits an even vertex odd mean labeling is called even vertex odd mean graph [1, 16].

In this paper, we proved that the even vertex odd meanness of some graph operations of even vertex odd mean labeling graphs

Definition :1.1[8]

The shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' and G''. Join each vertex u' in G' to the neighbours of the corresponding vertex v' in G''.

Definition :1.2[15]

Consider a cycle C_n and let $e_k = v_k\, v_{k+1}$ be an edge in it with $e_{k-1} = v_{k-1}\, v_k$ and $e_{k+1} = v_{k+1}\, v_{k+2}$ be its incident edges and $e_k^{'} = v_k^{'}\ v_{k+1}^{'}$ be a new edge. The duplication of an edge e_k by an edge $e_k^{'}$ produces a new graph G in such a way that $N(v_k)\cap N(v_k^{'})=\{v_{k-1}\}$ and $N(v_{k+1})\cap N(v_{k+1}^{'})=\{v_{k+2}\}$ which is called edge duplication of C_n and denoted by $ED(C_n)$ where $N(v_k)$ denotes the set of vertices adjacent to v_k

Definition :1.3[14]

For a bipartite graph with G with partite sets v_1 and v_2. Let G' be the copy of G and $v_1^{'}$ and $v_2^{'}$ be the copies of v_1 and v_2. The mirror graph M(G) of g is obtained from G and G' by joining v_2 to its corresponding vertex in $v_2^{'}$ by an edge.

Definition :1.4[13]

Le $P_m + \overline{K_n}$ be the graph with the vertex set V (G) $=\{u_i,v_j : 1 \le i \le m, 1 \le j \le n\}$ and the edge set E(G) $= \{u_iu_{i+1}, u_1v_j, u_mv_j : 1 \le i \le m-1, 1 \le j \le n\}$.

Definition :1.5[17]

Let G = (V,E) be a simple graph and $G^0 = (V^0,E^0)$ be another copy of graph G. Join each vertex v of G to the corresponding vertex v^0 of G^0 by an edge.The new graph thus obtained we call 2- tuple graph of G. We denote 2-tuple graph of G by the notation $T^2(G)$. Further we note that if G = (p,q) then $|V(T^2(G))| = 2p$ and $|(T^2(G))| = 2q + p$

Definition :1.6[5]

$T_{t,n,m}$ is a graph obtained by joining the centers of $K_{1,n}$ and $K_{1,m}$ by a path P_t. It consists of t+n +m vertices and t+n +m -1 edges.

MAIN RESULTS

Theorem:2.1

The graph $D_2(k_1, n)$, n≥ 2 is an even vertex odd mean graph.

Proof :

Let {v,v_i, 1≤i≤n, u,u_i,1≤ i ≤n} be the vertices and {a_i, 1≤ i ≤ 4n} be the edges which are denoted as in figure 1.1

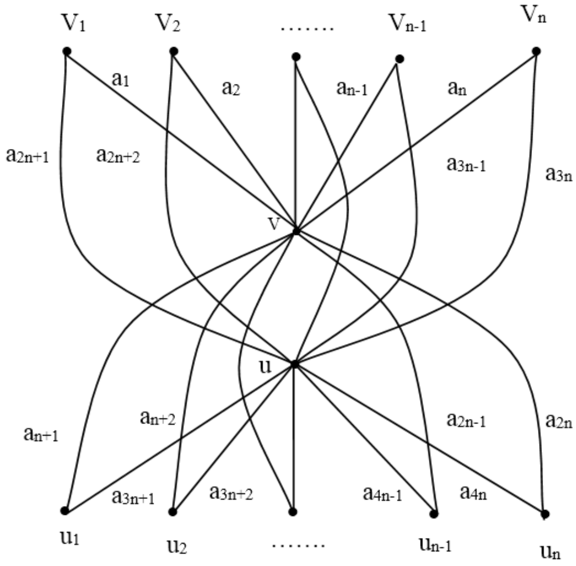

Figure 1.1 : Ordinary labeling of $D_2(k_1, n)$

First we label the vertices as follows :

Define f : v→{0, 2, … 2q} by

Let f(v) = 0 and f(u) = 8n

For 1 ≤ i ≤ n

$f(v_i) = 4i-2$

$f(u_i) = 4n+4i-2$

Then the induced edge labels are :

For 1 ≤ i ≤ 4n $f^*(a_i) = 2i–1$

Therefore f*(E) = {1, 3, 5, ...2q–1}. So, f is an even vertex odd mean labeling and hence, the graph $D_2(k_1, n)$, n≥ 2 is an even vertex odd mean graph.Even vertex odd mean labeling of $D_2(k_1, 4)$ is shown in Figure 1.2 :

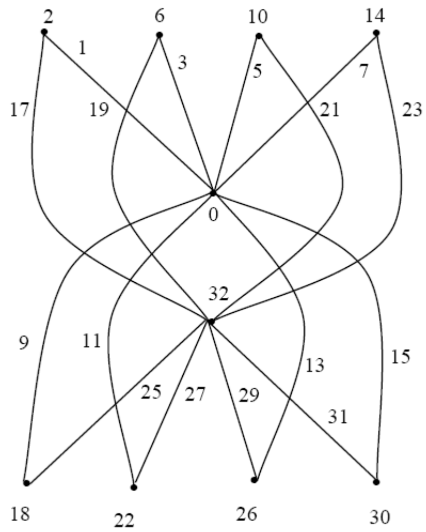

Figure 1.2 : Even vertex odd mean labeling of $D_2(k_1, 4)$

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Theorem :2.2

The mirror graph M(P_n) ,n≥ 3 is an even vertex odd mean graph.

Proof:

Let { v_i,v_i', 1≤ i ≤n} be the vertices and {e_i,e_i' 1≤ i ≤ n-1, a_i ,1≤ $i \leq n$} be the edges which are denoted as in figure 1.3

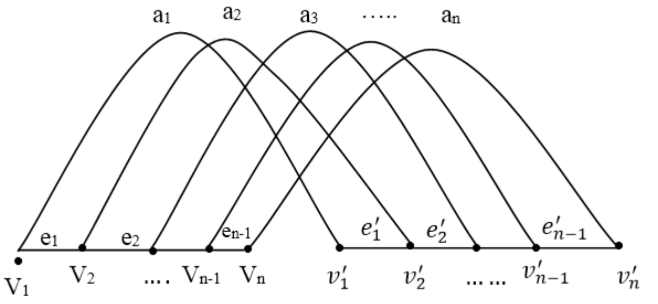

Figure 1.3: Ordinary labeling of M(P_n)

First we label the vertices as follows :

Define f : v→{0, 2, … 2q} by

For 1 ≤ i ≤ n f(v_i) = 6(i-1)

For 1 ≤ i ≤ n f(v_i') = 6i-4

Then the induced edge labels are :

For 1 ≤ i ≤ n f*(a_i) = 6i–5

For 1 ≤ i ≤ n -1 f*(e_i) = 6i–3

For 1 ≤ i ≤ n-1 f*(e_i') = 6i–1

Therefore f*(E) = {1, 3, 5, ...2q–1}. So, f is an even vertex odd mean labeling and hence the graph M(P_n), n≥ 3 is an even vertex odd mean graph.The below Figure 1.4 provides the better idea about the above defined mean labeling

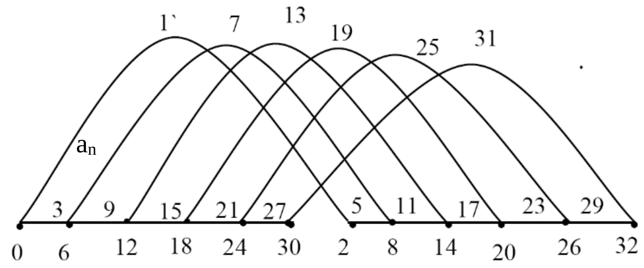

Figure 1.4: Even vertex odd mean labeling of M(P_6)

Theorem 2.3

The graph $D_2(B_{n,n})$, n≥ 3 is an even vertex odd mean graph

Proof :

Let {v,v_i 1≤ i ≤ n,u,u_i, 1≤ i ≤n ,x,x_i, 1≤ i ≤n ,w,w_i, 1≤ i ≤n } be the vertices and {a_i, 1≤ i ≤ 2n, a_i' ,1≤ $i \leq n, a', a'', a, b_i$ 1≤ i ≤2n, $b_{i,}', 1 \leq i \leq n$ } be the edges which are denoted as in figure 1.5

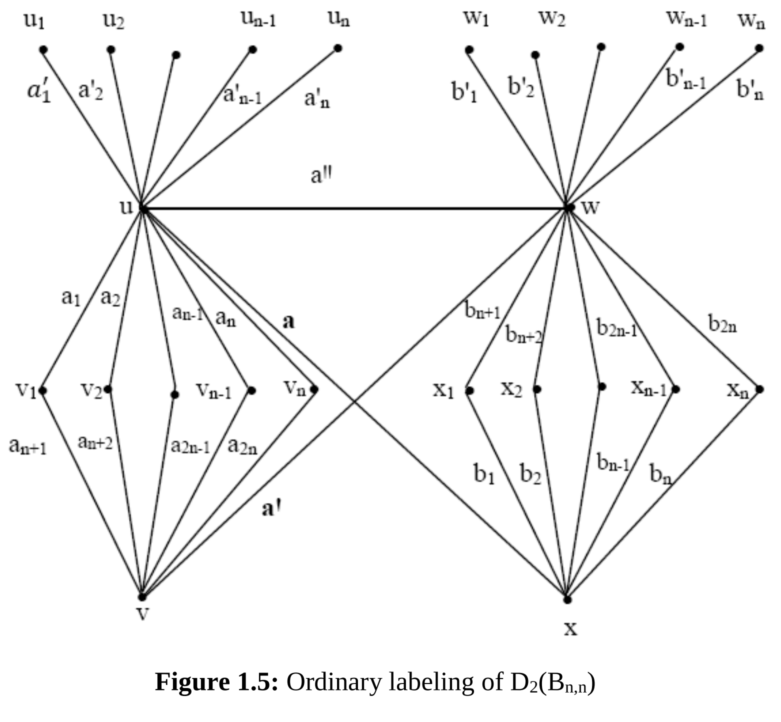

Figure 1.5: Ordinary labeling of $D_2(B_{n,n})$

First we label the vertices as follows :

Define f : v→{0, 2, … 2q} by

f(u)= 8n+6

For 1 ≤ i ≤ n-1 $f(u_i)$ = 4n+4i , $f(u_n)$ = 8n+4

For 1 ≤ i ≤ n $f(v_i)$ = 8n+4(i +1) , f(v) = 12n+6

f(w) = 4n , $f(w_1)$ = 4n+2

For 2 ≤ i ≤ n $f(w_i)$ = 4n+4i+2

f(x) =0

For 1 ≤ i ≤ n $f(x_i)$ = 4i-2

Then the induced edge labels are :

For1 ≤ i ≤ n-1 $f^*(a_i')$ = 6n+2i+3 $f^*(a_n')$ = 8n+5

For 1 ≤ i ≤ 2n $f^*(a_i)$ = 8n+2i+5

f*(a) = 4n+3 , f*(a′) = 8n+3 , f*(a″) = 6n+3,$f^*(b_1')$) = 4n+1

For 2 ≤ i ≤ n $f^*(b_i')$ = 4n+2i+1

Fo1 ≤ i ≤2 n $f^*(b_i)$ = 2i–1

Therefore f*(E) = {1, 3, 5, ...2q–1}. So, f is an even vertex odd mean labeling and hence the graph $D_2(B_{n,n})$,n≥ 3is an even vertex odd mean graph. Even vertex odd mean labeling of $D_2(B_{5,5})$ is shown in figure 1.6

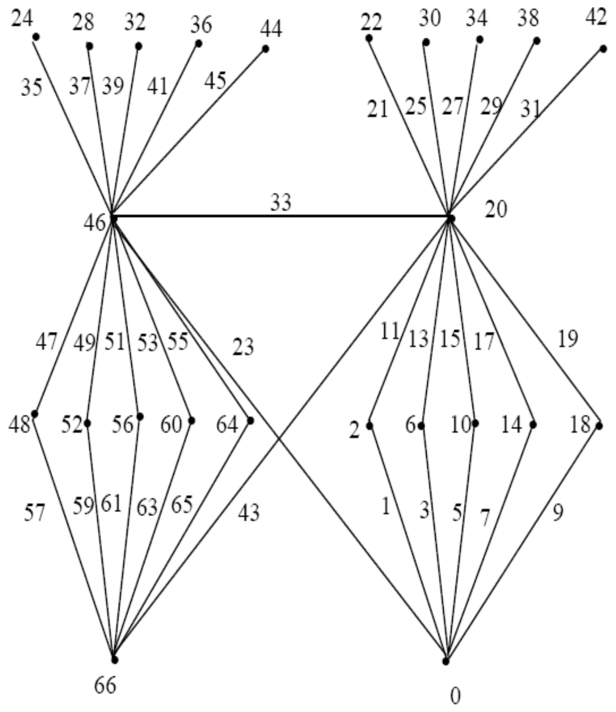

Figure 1.6: Even vertex odd mean labeling of $D_2(B_{5,5})$

Theorem:2.4:

The graph $T^2(P_n \times P_2)$, n≥ 3 is an even vertex odd mean graph

Proof:

Let { v_i,u_i, u_i', v_i',1≤i≤n} be the vertices {a_i, a_i', bi , b_i', 1≤i≤n − 1, c_i, c_i', 1≤i≤n} d_i 1≤i≤2n } be the edges which are denoted as in figure 1.7

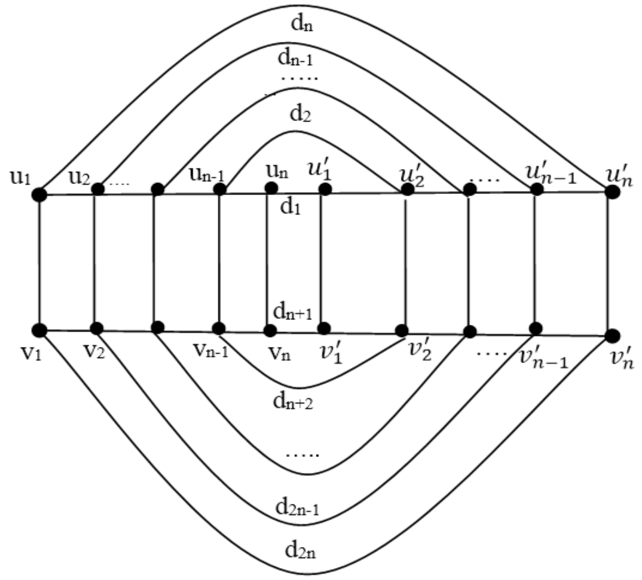

Figure 1.7: Ordinary labeling of $T^2(P_n \times P_2)$

First we label the vertices as follows :

Define f : v→{0, 2, … 2q} by

For 1 ≤ i ≤ n $f(u_i)$ = 2(i-1) , $f(u_i')$) = 10(n-1)+6i

For 1 ≤ i ≤ n $f(v_i)$ = 4n+2i-4 , $f(v_i')$ = 10n+6i-8

Then the induced edge labels are :

For 1 ≤ i ≤ n-1 $f^*(a_i)$ = 2i-1

For 1 ≤ i ≤ n $f^*(b_i)$ = 2n+2i-3

For 1 ≤ i ≤ n-1 $f^*(c_i)$ = 4n+2i-3

For 1 ≤ i ≤ 2n $f^*(d_i)$ = 6n+2i-5

For 1 ≤ i ≤ n-1 $f(a_i')$) = 10n+6i-7

For 1 ≤ i ≤ n $f(b_i')$) = 10n+6i-9

For 1 ≤ i ≤ n-1 $f(c_i')$) = 10n+6i-5

Therefore f*(E) = {1, 3, 5, ...2q–1}. So, f is a even vertex odd mean labeling and hence the graph $T^2(P_n \times P_2)$, n≥ 3 is an even vertex odd mean graph. The below illustration provides the better undestanding of defined labeling pattern in the above theorem

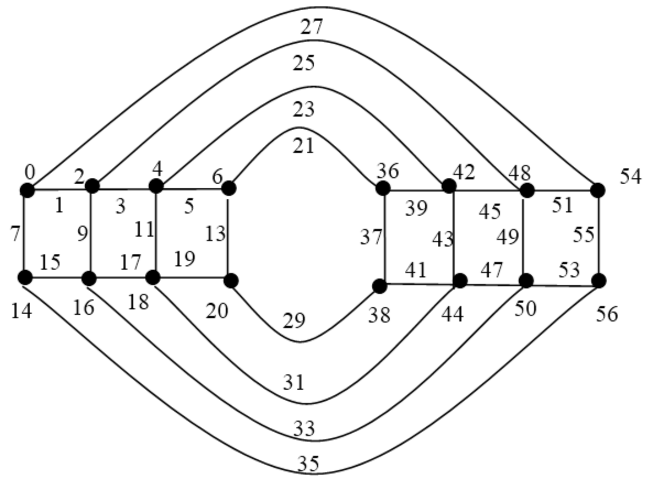

Figure 1.8: Even vertex odd mean labeling of $T^2(P_4 \times P_2)$

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