

Structural Analysis Using Shear Deformation Theories Having Nonpolynomial Nature: A Review

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Abstract

A large number of articles are available on behaviour of composite structures, smart structures, and structures of functionally graded materials. Analytical as well as FEM solutions can be observed in existing literatures. The past works are comprised of various shear deformation theories. However, the subject of usage of nonpolynomial shear deformation theories is an emerging area of research and has relatively less number of published literatures. Significant works which are relevant to this area are cited in this paper. Advantages of using nonpolynomial shear deformation theories are discussed. The excellent precision as well as efficacy of generalized formulation involving nonpolynomial shear deformation theories brands this study relevant. Critical observations are made from the review of open literature.

Keywords: shear deformation theory, laminated plate, piezoelectric laminate, functionally graded material.

INTRODUCTION

Analysis of structures is emerging as a significant aspect in the field of design. Many structural engineering applications and improvements of composite structures are becoming an important topic research of in recent days [1-5]. Also, better understanding of structural behaviour is required for structural engineering application. This motivates researchers to investigate bending, buckling, vibration, etc. of laminated composite structures considering various aspects. Many studies have been conducted in the past related to the analysis of structural responses. Analytical as well as FEM solutions can be seen in existing literature. The past works comprised various shear deformation theories. In the present article, a general introduction of various shear deformation theories is discussed first. Motivation of present work and objective are also discussed in this section 1. A detailed literature survey

related to works using nonpolynomial shear deformation theories (NPSDTs), which are relevant to present field of research, are presented in section 2. Critical observations are made from the review of existing literatures. The objective of this paper is to review the significant works using nonpolynomial shear deformation theories. The current paper emphasizes to review the recent works done in this area, especially in the last decade. The main focus is on works using NPSDTs. However, relevant works in polynomial shear deformation theories are included as well in this introduction section for the readers who are learning these FEM concepts. Also this will help to understand how the shear deformation studies are evolving and how the new theories differ from the earlier ones. Investigators working in this area can select the best and appropriate idea for their complete use and researchers can realize about the gaps in this area from this review article.

A few significant works which used FEM are incorporated in this paragraph. These important works using FEM provides helps for the readers who are learning these FEM concepts and implementing formulations using any shear deformation theory. Zienkiewicz [6] studied structural behaviour using FEM. He discussed in detail about von Karman nonlinearity and the geometric stiffness matrix associated with the membrane forces. Reddy [7] has described in detail the laminated composite plates. Analytical and finite element derivations are discussed by Reddy [7] in detail. Solutions for bending, buckling, and vibration are also presented. He presented a good description of the mechanics and associated finite element models of laminated composite structures. Agarwal et al. [8] and Jones [9] presented in detail the fundamental and advance topics related to composite structures. Bhavikatti [10] has discussed the finite element concept and applications to simple structures in detail. Also, application of the isoparametric concept to complex problems is discussed. Finite element formulations are made clear by

solving simple problems by hand calculation. Reddy [11] presented in detail the introductory concepts, noticeably studied the mathematical foundations of FEM, and delivered a general methodology of engineering application areas. Many works are available with descriptions on the computational aspects of FEM. Cook et al. [12], Kwon and Beng [13], Chandrupatla and Belegundu [14], Ferreira [15], Rugarli [16], and Kattan [17] discussed in detail about the finite element coding with numerous examples. Literature with FEM has employed various shear deformation theories for finding solutions as described in the paragraphs below.

Any physical system for solving is firstly converted into a mathematical model using certain technique. Then solution techniques are used to solve the mathematical model. The precision of any numerical model depends mainly upon above conversion and solution techniques. For obtaining the exact responses three-dimensional (3D) elasticity methods can be employed. However, 3D solutions can be used only for specific boundary and geometry conditions. It is better to use two-dimensional (2D) models in the investigation of composite structures due to the above reason. Owing to the large ratio of elastic modulus to shear modulus in a fibre reinforced laminate, effects of shear deformation are noteworthy in composite structures and therefore it plays an important role in modeling composite structures. Various plate theories are existing which combine the effects of shear deformation in typical ways. A brief classification of plate theories is shown in Figure 1. Here the classification is based on the choice of variable description. The laminate theories classified as equivalent single layer theories (EQSL) include Classical Laminated Plate Theory (CLPT), First order Shear Deformation Theory (FSDT) and higher order Polynomial and Nonpolynomial Shear Deformation Theories (PSDT and NPSDT), etc.

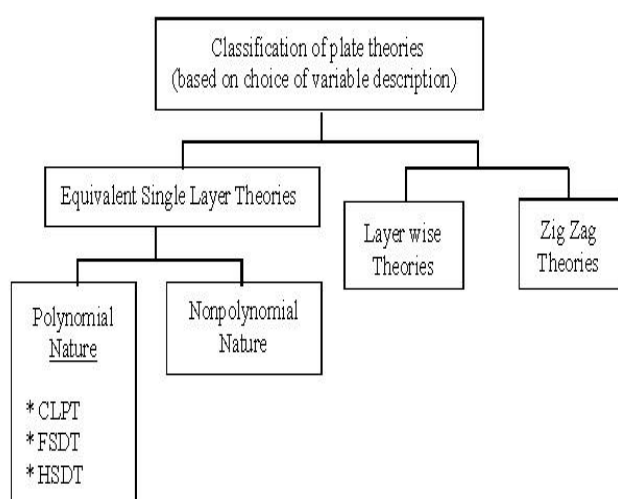


Figure 1: Classification of plate theories based on choice of variable description (presenting subdivisions of EQSL theories)

Researchers have established many theories based on equivalent single layer theory (EQSL). Number of field variables of EQSL theories depends only on displacement field and are independent of number of layers. EQSL theories include CLPT, FSDT, higher order PSDT and NPSDT, etc. In the early days of plate theories, Love [18] proposed the concept now known as Love first approximation theory. This theory assumed that the normal to the reference plane before deformation remain normal after deformation without any variation in length. CLPT is the Love first approximation theory applied on to a multilayered composite plate structure. Ashton and Whitney [19] used CLPT to analyze composite plates. It should be noted that shear deformation was not taken into account as the transverse shear effects are considered negligible here. Reissner [20, 21] and Mindlin [22] studied on first order shear deformation theories where the effects of shear deformation are included. They assumed the first order displacement function. A shear correction factor is introduced in this theory to satisfy transverse shear stress conditions on the top and bottom surface of plate. Several researchers used Taylor's series coefficient to characterize the effect of shear deformation which are generally discussed to be PSDTs. Ambartsumian [23] proposed a higher order shear deformation theory involving transverse shear stress function. Whitney and Pagano [24] used FSDT to predict free vibration and bending response of anisotropic plates. Levinson [25] and Murthy [26] presented studies with higher order polynomial shear deformation theories. Their theories contained parabolic distribution of transverse shear stresses across the plate thickness. Reddy [27] presented a simple higher order theory for laminated plates. Conditions of traction free boundary were used by him. He reduced the number of unknowns from 9 to 5 by imposing conditions of transverse shear stresses on top and bottom. He used higher order shear deformation theory with same number of unknowns as in FSDT without using shear correction factor. Later, different higher order shear deformation theories having nonlinear shear deformation and satisfying zero transverse shear stress conditions on the plate surfaces were developed by various researchers.

Shear deformation theories having nonpolynomial nature

A large number of articles are available on the behaviour of structures. However, the subject of NPSDTs in numerical formulation has relatively less number of published literatures. NPSDTs are those in which function of thickness coordinate is used to denote the shear deformation. As observed from recent literatures, the overall percentage error is relatively less with new NPSDTs (shear deformation theory based upon secant function (SFSDT), an inverse-trigonometric shear deformation theory (ITSDT), and an inverse hyperbolic shear deformation theory (IHSDT)) compared to other existing shear deformation theories. So, special attention must be given for the investigation of the

structures using NPSDTs. Important works which are relevant to present study are cited in this section. Certain restrictions were applied for narrowing the present review article and are summarized below. The work reviewed here includes only those papers dealing with NPSDTs in laminated composites, piezolaminated structures, and FGMs. Hence, in an effort to assess the available literature shortly and critically concerning the current research problem, the entire review process is separated into several key modules as follows:

1. Laminated composite structures.
2. Smart composite laminates.
3. Structures of FGM.

Each category is reviewed in detail in the subsequent subsections.

Laminated composite structures

It is essential to explore the recent studies conducted for composite structures using shear deformation theories having nonpolynomial nature. A variety of shear deformation theories with nonpolynomial nature and expressed in terms of shear-strain function have been proposed in the past, especially in the last few years. Touratier [28] proposed a trigonometric shear strain function for higher order theory. Some of the nonpolynomial higher order theories were presented by Aydogdu [29], Karama et al. [30], Meiche et al. [31], and Mantari et al. [32, 33]. Demasi [34] revealed that a sequence of trigonometric functions of the thickness coordinate may be also considered in his generalized unified formulation.

Grover et al. [35] proposed the new nonpolynomial shear deformation theories and employed for finding the responses of laminated composite and sandwich plates. They demonstrated that it shows enhanced performance similar to all prevailing higher order shear deformation theories containing a shear strain function. This theory fulfills zero transverse shear stress at the top and the bottom surfaces of the plate and the transverse shear stress distribution is non-linear across the thickness. It is observed from the literature that the overall percentage error is less with new NPSDTs compared to other existing shear deformation theories with relatively less computational effort. Grover et al. [36, 37] explained in detail the inverse hyperbolic shear deformation theory for static and dynamic analysis of laminated composite and sandwich plates using C1 and C0 continuity in numerical formulation. Displacement model given in Eq. (1) was proposed by Grover et al. [36].

$$\begin{aligned}
 u(x, y, z) &= u_0(x, y) - z \frac{\partial w_0}{\partial x} + \left[\sinh^{-1}(rz/h) - z \left(\frac{2r}{h\sqrt{(r^2+4)}} \right) \right] \theta_x(x, y) \\
 v(x, y, z) &= v_0(x, y) - z \frac{\partial w_0}{\partial y} + \left[\sinh^{-1}(rz/h) - z \left(\frac{2r}{h\sqrt{(r^2+4)}} \right) \right] \theta_y(x, y) \\
 w(x, y, z) &= w_0(x, y)
 \end{aligned}
 \tag{1}$$

In the above displacement field, u_0 , v_0 , and w_0 are the midplane displacements while θ_x , θ_y are the shear displacements. The transverse shear stress factor, represented by r , is fixed as 3 per the reverse technique in post processing phase [36]. The displacement model is highlighted above as it was used to calculate different structural behaviors by first author [38-40]. Sreehari and Maiti [38] explained in detail the finite element formulation for handling structural analysis of laminated composite plates subjected to mechanical and hygrothermal loads using the inverse hyperbolic shear deformation theory. Geometric nonlinearity had been included in the von Karman sense. Numerous numerical results were presented as validation and parametric studies. By this investigation, the validation of IHSDT was proved for buckling and post buckling analysis in hygrothermal environment. They also employed IHSDT to find out the response of a damaged composite plate [39] and a smart plate [40]. It was critically noted that results with IHSDT was showing less deviation from exact solutions compared to other higher order theories. General layout of FEM implementation used in Ref. [38-40] in which formulations were based on NPSDTs is presented in Figure 2.

The displacements and strains are continuous through the thickness of laminate in EQSL theories leading to a discontinuous interlaminar stress field at layer interfaces due to different elastic coefficients of the neighboring layers. Layer-wise theories help to overcome these restrictions. A rise in number of works with layerwise theories using nonpolynomial shear deformation theories is observed in the last few years. Suganyadevi and Singh [41] conducted studies on nonpolynomial theories that can satisfy interlaminar continuity. The number of unknowns in layerwise model increases as the number of layers increases. But this makes the computational calculations tedious for these theories in the analysis of cases with higher number of layers.

The unknowns taken at all the interfaces are specified in terms of those at the reference planes by fulfilling the condition of transverse shear stress continuity at the layer interfaces and also follow a zigzag variation of in-plane displacement in zigzag theories. A new inverse hyperbolic zigzag theory for the static analysis of laminated composite and sandwich plates were developed by Sahoo and Singh [42]. Later a new trigonometric zigzag shear deformation theory was developed by Sahoo and Singh [43]. They applied their proposed theory for buckling and free vibration analysis of composite sandwich plate. The number of unknown field variables remains same as that of FSDT and this makes the solution computationally more efficient. They used a C⁰ continuous isoparametric serendipity element to solve the discrete eigenvalue equations. The accuracy of the model is validated through numerical examples on the free vibration and stability analysis of laminated composite and sandwich plates. Suganyadevi and Singh [44] developed improved zigzag theories for the flexural analysis of laminated plates using various nonpolynomial shear deformation theories. They used

hyperbolic, inverse trigonometric and trigonometric shear strain functions in their analysis. Using the principle of virtual work, the governing differential equations and boundary conditions of the structural system were acquired. A generalized Navier closed form solution technique was used for the analysis. They exposed the potency and performance of models. Numerical comparisons were made with the 3D elasticity solution and other numerical methods. They concluded that their models perform very well for the static behavior of laminated plates.

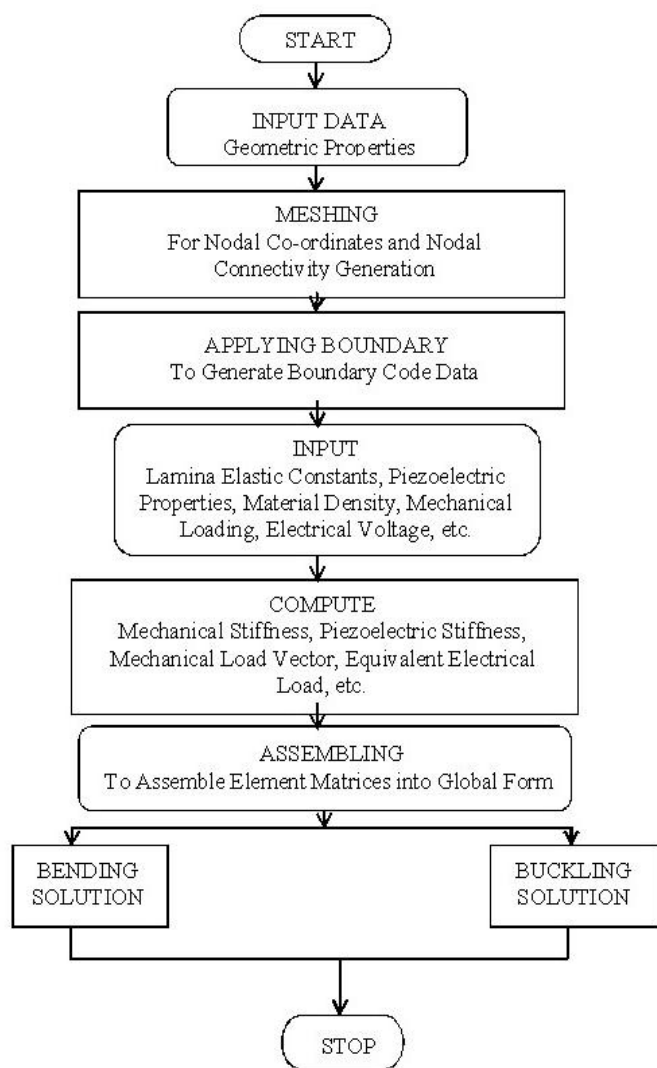


Figure 2: General layout of FEM implementation used in Ref. [38-40] in which formulations were based on NPSDTs

Smart composite laminates

Smart materials combine the information of dissimilar streams of science and engineering. Their application assists mankind to govern the environment better and increases the effectiveness of appliances. They are capable for rectifying themselves from the exterior forces acting on them. Smart

structure comprises constituents executing various functions like sensing, actuation, and control. Based on their response to stimuli and the speed of response, the ‘smartness’ of a structure is explained. These smart materials are anticipated to imitate naturally existing systems. Roles of smart material are self-correcting features, improvement in efficiency and sustainability, high reliability, damage detection capability, health and integrity monitoring, etc. Their applications include reduction of vibration and noise, aerospace applications, medical applications, etc. The main reasons behind it are limitations in weight, space, and positioning in many applications. Many types of smart materials exist now. Also, investigations are going on for various types of smart materials. Materials are made smart by incorporating any of the materials like piezoelectric materials, electrostrictive materials, magnetostrictive materials, rheological materials, shape memory alloys, thermoresponsive materials, electrochromic materials, fullerenes, biomimetic materials, smart gels, etc. Materials that generate an electric potential when subjected to mechanical deformation are called piezoelectric materials. Designing smart composite structures includes bonding piezoelectric layers to the bottom and top of a substrate material which acts as distributed actuators and sensors to control and monitor the structural responses.

Carrera and Brischetto [45] concludes that use of refined theories is mandatory in case of smart structures to obtain correct value of certain variables like in-plane stresses, when the plate is multilayered and moderately thick with significant anisotropy. Carrera et al. [46] conducted a detailed study of smart structures employing basic principles and numerous advanced models. Several plate/shell models were made and examined. Both analytical and finite element solutions were provided by them. The precision of the models were improved through the use of an efficient framework known as Carrera Unified Formulation. The Carrera Unified Formulation was extended to comprise nonpolynomial shear strain shape function in their numerical formulation and made a new general formulation. Work indicates the requirement of appropriate nonpolynomial shear strain shape function for possible better performance. Carrera et al. [46] also conveys through their vibration analysis of piezoelectric shells that refined models are necessary for higher frequency values.

Structures of FGM

Modeling of functionally graded substrate structures instead of laminated structures is an emerging field and several researchers are studying these currently. Functionally graded structures can withstand higher elevated hygrothermal environment because of its continuous variation of material properties across the thickness. So the analysis of aerospace structures using functionally graded materials is important for some specific applications.

Carrera et al. [47] investigated the effects of thickness stretching in functionally graded plates (FGP) and shells and presented the significance of the transverse normal strain effects in the estimation of stresses for functionally graded structures. Neves et al. [48, 49] and Ferreira et al. [50] brought notable progress in this subject. They developed a quasi-3D hybrid (trigonometric as well as polynomial) type shear deformation theory. They used different nonpolynomial displacement fields for in-plane displacements and polynomial displacement field for the out-of-plane displacement. Thus they obtained the structural responses of functionally graded plates (FGPs) by using meshless numerical method. The Carrera Unified Formulation was extended to include nonpolynomial shear strain shape function in their formulation. This is an innovative general formulation. Work shows the need of new suitable nonpolynomial shear strain shape function for possible superior performance. The structural analysis of FGPs with new hyperbolic (nonpolynomial) shear strain shape function was performed by Mechab et al. [51]. Thai and Kim [52] used a trigonometric plate theory considering stretching effect and 5 unknown variables which shown good accuracy. Analysis of functionally graded structures by using new nonpolynomial HSDTs was performed by Mantari and Soares [53-55]. The number of unknowns was varied to 4 and 5 respectively in addition to the incorporation of stretching effects [55, 56]. They observed better results of displacement and in-plane normal stresses for functionally graded structures. Kulkarni et al. [57] has demonstrated the accuracy and efficiency of inverse trigonometric shear deformation theory in modeling and studying of FGPs. They presented an analytical solution for a simply supported FGP. They calculated displacements, stresses, and the critical buckling load for various conditions using the proposed methodology.

As seen in the preceding section, several studies are being conducted in plate theories for the structural analysis. Recent works for structural analysis using NPSDTs are thus reviewed.

CONCLUSION

Finally, the review of the state-of-the art on the present topic is concluded here with some more emphasize on opinion from the authors and the assessments on this work. Structural analysis with various shear deformation theories is a broad research topic. It is understood from literature review carried out in introduction section that shear deformation theories chosen will have large effect on modeling of structures. As seen in the observations made in introduction section, certain gaps exist for the structural analysis of laminated composite plates. Only very few works using NPSDTs exist in the past. The limited studies on the behaviour of structures using NPSDTs provide a scope for such investigation considering more realistic shear deformation. In the course of present review work, many interesting problems have been found

related to this area. There are several problems remains to be solved. The scope of the present article is limited to the review of recent works on structural analysis (of laminated composite structures, smart composite laminates, and structures of FGM) using NPSDTs. In particular, the use of non-polynomial functions to describe the through-thickness deformation field is discussed. Critical assessment of the literature and good insight into the field are provided in present article. A lot of works and applications are concisely discussed. The critical observations from literature study conclude that some areas of future research that need special attention are structural analysis with NPSDTs. It has been observed that higher efficiencies are obtained at similar computational cost of other higher order theories. Modeling and computational abilities improve when NPSDTs are used. But the lack of experimental supports providing for these NPSDTs is a disadvantage in research in this direction. Structure can be made in experiments, but its complex to design structure experimentally according to various displacement fields. But the important point is that employment of NPSDTs can simplify the displacement terms instead of complex expansions of Taylors series. From the whole work, authors' viewpoint is that this can be appropriately used in complex analytical studies that address the modelling of composite beam and plate bending for obtaining efficient formulation and precise results.

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