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Detection of the Fluctuating Pulse with Unknown Time of Arrival and Intensity

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Abstract

We introduce the algorithms for the detection of the rapidly fluctuating radio pulse with the rectangular modulating function, if the pulse time of arrival and intensity are unknown. We establish that the optimal detection algorithm synthesized by means of the maximum likelihood method leads to multichannel detector structure. Therefore, it is sufficiently complex for practical implementation. For the case of a weak signal, we propose the simple and asymptotically optimal detection algorithm implemented through the single-channel detector. We also show that the characteristics of the proposed detector can be improved by the detector input filter bandwidth widening when a large signal is received.

Keywords: Signal detection, Fluctuating pulse, Maximum likelihood method, Quasi-optimal reception, Local Markov approximation, False alarm and missing probabilities.

INTRODUCTION

Detection of the pulse signals against random interferences is often required in various applications of radio physics and radio engineering [1-3]. Such problem is the common one in the fields of radio- and hydrolocation, radio control, radio communication and navigation, telemetry, technical diagnostics, etc. The task of the pulse signals detection also arises in radio astronomy, hydroacoustics, seismology and geology, spectroscopy and other areas of physics and engineering.

In practice, the received signals may have random amplitude and phase modulation, i.e. they represent the realizations of the random processes. Such signal modulation is often caused by the random fading in the propagation medium, by the influence of multiplicative (modulating) distortions, or it arises due to the stochastic nature of the radio source. The fluctuating signals are often superpositions of a great number of independent elementary signals and, therefore, they can be considered as the Gaussian random processes.

One of the types of fluctuating pulse signals is the Gaussian random pulse [4-7]

$$s(t) = a(t) I[(t-\lambda_0)/\tau] \cos[\nu t - \varphi(t)] = \xi(t) I[(t-\lambda_0)/\tau]. \quad (1)$$

Here $\xi(t) = a(t)\cos[\nu t - \varphi(t)]$ is the high-frequency stationary centered Gaussian random process (stochastic carrier); $a(t)$ and $\varphi(t)$ are the random processes describing the amplitude and phase fluctuations and obeying Rayleigh and uniform distribution laws [3], correspondingly; $I(x)$ is the modulating function specifying the squared shape of the pulse envelope in the absence of the fluctuations: $I(x) = 1$, if $|x| \le 1/2$, and $I(x) = 0$, if $|x| > 1/2$; λ_0 is the time of arrival; τ is the duration; ν is the carrier frequency of the pulse.

In general terms, the spectral density $G(\omega)$ of the process $\xi(t)$ can be presented as [4]

$$G(\omega) = \frac{\gamma_0}{2}\left[g\left(\frac{\nu - \omega}{\Omega}\right) + g\left(\frac{\nu + \omega}{\Omega}\right)\right], \quad (2)$$

where $\Omega = \int_0^\infty G(\omega)\, d\omega / \max G(\omega)$ is the equivalent bandwidth, γ_0 is the spectral power (intensity), and $g(x)$ is the function describing the form of the spectral density, and it is normalized so that $\max\ g(x) = 1$, $\int_{-\infty}^{\infty} g^2(x)\, dx = 1$.

We presuppose that the signal (1) fluctuations are "fast" [4], i.e. the pulse (1) duration τ exceeds essentially the correlation time of the process $\xi(t)$. Then [4-7]

$$\mu = \tau\Omega/2\pi >> 1. \quad (3)$$

In fulfilling the condition (3), the γ_0 and Ω parameters characterize the intensity and the bandwidth of the fluctuating pulse (1).

The stochastic model (1), for example, enables us to describe the signals in radio- and hydroacoustic channels with random fades [8, 9] and the pulses distorting by the modulated interference [10], or reflected from the compound physical object with a lot of bright spots [1, 11]. The pulse signals (1) occur in astronomy, geophysics, spectrography and other areas of physics. The random pulse (1) sequence can be used as the noise carrier in the hidden data communication systems [12].

Usually, the signal detection is implemented in the presence of random noises (interferences). Thus, the mix of signal and noise is available for processing only. The elementary but sufficiently common noise model is the additive Gaussian white noise [1-3].

Let the fluctuating pulse $s(t)$ (1) is received against Gaussian white noise $n(t)$ with the one-sided spectral density N_0 within the time interval $[T_1, T_2]$, while the signal and the noise are statistically independent. If the signal (1) is present at the detector input (H_1 hypothesis), then the additive mix of signal and noise

$$x(t) = s(t) + n(t), \quad t \in [T_1, T_2] \quad (4)$$

is to be processed, while in the absence of the signal (1) (H_0 hypothesis) there is only noise present in the observed realization:

$$x(t) = n(t), \quad t \in [T_1, T_2]. \quad (5)$$

As a rule, the pulse detection is carried out when the signal time of arrival is unknown. This can be a result of the signal emission period uncertainty and (or) the random signal delay during its propagation from the radiating object to the receiver. In certain communication systems, the pulse time of arrival is modulated by the transmission and is a priori unknown also. Only the interval of the possible values of the time of arrival is known then, and its length may be great. Besides, the signal intensity (average power) is often unknown too in this case. This is due to the random signal fading while the signal distribution in the physical environments, or through the communication channels.

Thus, we take the time of arrival λ_0 of the signal (1) as a priori unknown, but with the interval of its possible values $[\Lambda_1, \Lambda_2]$ specified. And the observation (processing) interval $[T_1, T_2]$ is chosen in accordance with the conditions $T_1 < \Lambda_1 - \tau/2$, $T_2 > \Lambda_2 + \tau/2$. It means that the signal (1) is completely situated within the observation interval $[T_1, T_2]$ for any time of arrival $\lambda_0 \in [\Lambda_1, \Lambda_2]$. The signal (1) intensity γ_0 is also assumed as the unknown value satisfying the condition $\gamma_0 > 0$.

While detecting the fluctuating signal (1) with unknown time of arrival and intensity, we find it necessary to make the decision in favor of one of the hypotheses (H_1 or H_0) concerning presence or absence of the signal $s(t)$ (1) in the observed realization $x(t)$. In [1, 11, 13], the algorithms based on the maximum likelihood (ML) method are studied for the detection of the fluctuating signal (1) with a priori known parameters, as well as the asymptotic expressions for the detection characteristics. In [4-7], the synthesis and the analysis are carried out of the ML algorithm for the detection of the fluctuating pulse (1) with unknown time of arrival. Below, we present the case when both time of arrival and intensity of the fluctuating signal (1) are unknown.

THE OPTIMAL DETECTION ALGORITHM

The General Case

To synthesize the detection algorithm we apply the ML method and the adaptive approach [4]. According to the ML method [1-3, 13], we must use the observed data $x(t)$ to devise the logarithm of the functional of the likelihood ratio (FLR) $L(\lambda, \gamma)$ as the function of all the possible values λ and γ of the time of arrival λ_0 and the intensity γ_0 of the signal that are a priori unknown. Further, according to the adaptive approach [4], we should determine the magnitude L_M of the absolute (greatest) maximum of the logarithm of FLR $L(\lambda, \gamma)$ by the variables λ and γ within the prior intervals $\lambda \in [\Lambda_1, \Lambda_2]$ and $\gamma > 0$. Then the detection of the signal (1) is made through the comparison of the magnitude L_M with the specified detection threshold *h*. If the threshold *h* is exceeded, then the decision is made in favor of H_1 hypothesis declaring the signal presence in the observed data. If it is not, the decision is made in favor of H_0 hypothesis stating the signal absence. Then, the generalized ML signal (1) detection algorithm is written down in the form of [4]

$$L_M = \sup_{\lambda \in [\Lambda_1, \Lambda_2], \gamma > 0} L(\lambda, \gamma) \underset{H_0}{\overset{H_1}{\gtrless}} h. \quad (6)$$

The value of the *h* threshold is determined on the basis of the accepted detection optimality criterion [1-4]. For example, Neumann-Pirson criterion can be used where *h* threshold is chosen by the permissible false alarm probability [1-4].

If the condition (3) is satisfied, then, by applying the results from [4-6], we obtain

$$L(\lambda,\gamma)=M(\lambda,q)/N_0-\Gamma(q), \tag{7}$$

$$M(\lambda,q)=\int_{\lambda-\tau_0/2}^{\lambda+\tau_0/2} y^2(t,q)\,dt\,,\ \ \Gamma(q)=\mu_0\int_{-\infty}^{\infty}\ln[1+qg(x)]\,dx\,, \tag{8}$$

where $q=\gamma/N_0$, and $y(t,q)=\int_{-\infty}^{\infty}x(t')h(t-t',q)\,dt'$ is the response of the filter with the transfer function $H(\omega,q)$ satisfying the condition

$$|H(\omega,q)|^2=\frac{qg[(\nu-\omega)/\Omega]}{1+qg[(\nu-\omega)/\Omega]}+\frac{qg[(\nu+\omega)/\Omega]}{1+qg[(\nu+\omega)/\Omega]} \tag{9}$$

concerning the realization of the observed data $x(t)$. Whereupon, the value $q_0=\gamma_0/N_0$ which is equal to the relation between the signal and noise spectral densities can be considered as the input signal-to-noise ratio.

We now see that, for the implementation of the detection algorithm (6), there should be formed the logarithm of FLR $L(\lambda,\gamma)$ (7) as the function of the two variables λ and γ, and that leads to the multichannel structure of the ML detector.

In Figs. 1, 2 the block diagram is shown of the multichannel ML detector of the fluctuating pulse (1) with time of arrival and intensity unknown. The received signal $x(t)$ is passed to the inputs of N detector channels (Fig. 1). Each detector channel is tuned to the fixed value $\gamma=\gamma_j$ from the set sequence γ_j, $j=1,2,\ldots,N$, so that the section $L(\lambda,\gamma_j)$ of the logarithm of FLR (7) is formed as the function of the current value of the time of arrival $\lambda\in[\Lambda_1,\Lambda_2]$. At the channel output the value $L^*(\gamma_j)=\sup L(\lambda,\gamma_j)$ of the absolute maximum of the logarithm of FLR is generated by the variable $\lambda\in[\Lambda_1,\Lambda_2]$ under $\gamma=\gamma_j$. The constant signals $L^*(\gamma_j)$ from the outputs of all the detector channels arrive to the input of the resolver RS which fixes the greatest one of these signals and compares it to the set threshold h. If the threshold is exceeded, then the decision is made in favor of H_1 hypothesis stating the presence of the detected signal (1) in the observed data $x(t)$. Otherwise, the decision is made in favor of H_0 hypothesis stating the signal absence.

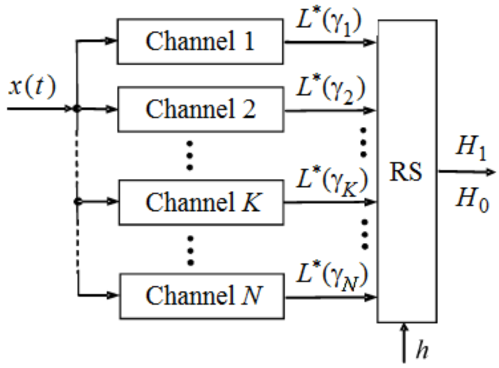

Figure 1: The general block diagram of the maximum likelihood detector

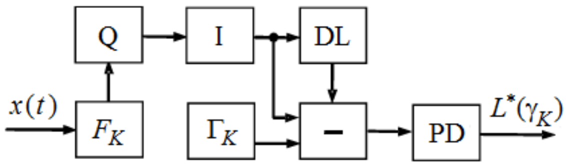

Figure 2: The block diagram of one channel of the maximum likelihood detector

All the channels of the detector presented in Fig. 1 are constructed on the basis of the same structure and differ by their blocks parameters only. The block diagram of the one of the detector channels (K-th channel) is presented in Fig. 2. Here F_K is the linear input filter with the transfer function $H(\omega,q_K)$ (9), where $q_K=\gamma_K/N_0$; Q is the quadrator; I is the integrator operating within the interval from the point T_1 up to the current time t; DL is a delay line for the signal (1) duration τ, « – » is the subtractor; PD is the peak detector and Γ_K is the DC source generating the signal

$$A_K=\mu_0\int_{-\infty}^{\infty}\ln[1+q_K g(x)]\,dx\,. \tag{10}$$

The integrator I, the delay line DL and the substractor (except its input for the signal A_K) cohere the rectangular video pulse $I(t/\tau)$ matched filter [2, 3]. This filter implements the input signal integration within the sliding interval $[t-\tau,t]$, where t is the current time. After subtraction of the constant signal A_K (10), at the PD input the signal $u_K(t)=L(t-\tau/2,\gamma_K)$ is formed representing the section of the logarithm of FLR (7) delayed by $\tau/2$, while the value $\gamma=\gamma_K$ is fixed. Then the peak detector PD in Fig. 2 determines the value of the absolute maximum of the input signal $u_K(t)$ over the interval $[\Lambda_1+\tau/2,\Lambda_2+\tau/2]$.

The channels in Fig. 1 differ by the input filters having the transfer functions $H(\omega, q_j)$ for various q_j and by the values of A_j (10) of the output the signals at the blocks Γ_j.

Practical implementation of the ML detector (Figs. 1, 2) may be rather complex. Firstly, determining the absolute maximum of the logarithm of FLR (7) by the variable γ requires forming a large number of the sections of the logarithm of FLR by this variable. Therefore, detector channel capacity N (fig. 1) required to achieve the acceptable detection quality can be very large that leads to the considerable technical challenges. Secondly, it is often difficult to make an informed choice of the series of values γ_j while designing the detector channels – there is no any universal methods for this. Through the choice of the fixed values γ_j of the channels parameters can indeed be conducted on the basis of the detector performance characteristics analysis, it is not, however, possible to find the signal (1) detection characteristics analytically, with the ML algorithm (6) applied.

The Case of the Band Signal

The structure of the ML detector of the fluctuating pulse (1) with unknown time of arrival and intensity is simplified considerably, if the shape of the spectral density (2) of the stochastic pulse carrier can be approximated by the rectangle function. In this case – $g(x) = I(x)$, where $I(x)$ is determined the same as in Eq. (1), so, according to Eq. (2), we have

$$G(\omega) = \frac{\gamma_0}{2}\left[I\left(\frac{\nu - \omega}{\Omega}\right) + I\left(\frac{\nu + \omega}{\Omega}\right)\right]. \qquad (11)$$

The fluctuating pulse (1) with rectangular spectral density (11) of the stochastic carrier $\xi(t)$ will be referred to as the band one.

If the condition (11) holds, then the logarithm of FLR (7) can be presented in the form of

$$L(\lambda, \gamma) = qM_B(\lambda)/N_0(1+q) - \Gamma_B(q), \qquad (12)$$

$$M_B(\lambda) = \int_{\lambda-\tau/2}^{\lambda+\tau/2} y_B^2(t)\,dt, \quad \Gamma_B(q) = \mu \ln(1+q), \qquad (13)$$

where $y_B(t) = \int_{-\infty}^{\infty} x(t') h_B(t-t')\,dt'$ is the response of the filter with the pulse response $h_B(t)$ on the observed realization $x(t)$, with its transfer function $H_B(\omega)$ satisfying the condition

$$|H_B(\omega)|^2 = I[(\nu - \omega)/\Omega] + I[(\nu + \omega)/\Omega]. \qquad (14)$$

If the intensity γ_0 is known, then the ML detection algorithm is written down in the form of [4-6]

$$L_0 = \sup_{\lambda \in [\Lambda_1, \Lambda_2]} L(\lambda, \gamma_0) \underset{H_0}{\overset{H_1}{\gtrless}} h, \qquad (15)$$

where L_0 is the value of the absolute maximum of the logarithm of FLR (12) by the variable λ, while $\gamma = \gamma_0$ ($q = q_0$). When we take into account the representation (12) of the logarithm of FLR (7), the ML detection algorithm (15) can be transformed as

$$M_0 = \sup_{\lambda \in [\Lambda_1, \Lambda_2]} M_B(\lambda) \underset{H_0}{\overset{H_1}{\gtrless}} h_0, \qquad (16)$$

where $h_0 = N_0[h + \Gamma_B(q_0)](1+q_0)/q_0$ and the decision statistics is determined from Eq. (13). In practice, it is convenient to choose the detection threshold h_0 in Eq. (16) without referring to the threshold h, but independently from it, in accordance with the specified criterion of the detection optimality[1-4]. In this case, the h_0 threshold value may not depend on the intensity γ_0 of the received signal. This is true, for example, when the Neumann-Pirson criterion is used during the detection [1-4].

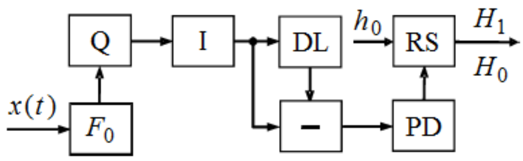

Figure 3: The general block diagram of the maximum likelihood detector of the band signal and the asymptotically optimal detector

The detection algorithm (16) with the threshold h_0 chosen independently of γ_0 does not require a priori information on the signal (1) intensity. Therefore, the ML algorithm (16) can be used to detect the band signal (1) with unknown time of arrival and intensity. It can be implemented by the device the block diagram of which is shown in Fig. 3. Here F_0 is the linear filter with the transfer function $H_B(\omega)$ (14), and the designations of the other blocks are the same as in the circuit in Fig. 2. The resolver RS compares the output signal of the detector PD with the set threshold h_0. If the threshold is exceeded, then the decision is made in favor of H_1 hypothesis stating the presence of the useful signal in the observed realization $x(t)$. Otherwise, the decision is made in favor of H_0 hypothesis declaring the signal absence.

Thus, the algorithm (16) is implemented as a single-channel detector. The practical implementation of this detector is much simpler than in the case of the the multi-channel ML detector in Figs. 1, 2. However, the detection algorithm (16) is synthesized only for the case of the band signal (1) with the rectangular carrier density (11). In practice, the spectral density of the fluctuating pulse (1) carrier may differ from the rectangular one. In this case, the ML detector is implemented as a rather complex multi-channel device (Figs. 1, 2). Therefore, by means of the ML algorithm (6), we synthesize the simpler suboptimal algorithms for detecting the fluctuating pulse (1).

THE ASYMPTOTICALLY OPTIMAL ALGORITHM FOR DETECTING THE WEAK SIGNAL

The Synthesis of the Detection Algorithm

The structure of the ML detector of the fluctuating signal (1) with unknown time of arrival and intensity is considerably simplified, if the signal intensity γ_0 is much less than the spectral density N_0 of the additive noise. The fluctuating pulse (1) with intensity $\gamma_0 << N_0$ we call the "weak" signal. The case $q_0 = \gamma_0/N_0 << 1$ of the "weak" signal is of great interest for practical applications, since it is more difficult to detect such a signal.

We write down the expression for the logarithm of FLR (7) for the case of receiving the "weak" signal. Then the form $g(x)$ of spectral density (2) is considered as arbitrary. We take into account that for $q \to 0$ the transfer function $H(\omega, q)$ (9) transforms into the transfer function $\sqrt{q} H_C(\omega)$, where

$$|H_C(\omega)|^2 = g\left(\frac{\nu - \omega}{\Omega}\right) + g\left(\frac{\nu + \omega}{\Omega}\right). \qquad (17)$$

In turn, the function $\Gamma(q)$ (8) under $q \to 0$ goes over to the product $q\Gamma_C$, where $\Gamma_C = \mu \int_{-\infty}^{\infty} g(x)\,dx$. Then the asymptotic approximation of the logarithm of FLR (7) for the "weak" signal ($q << 1$) can be represented in the form of

$$L(\lambda, \gamma) = q M_C(\lambda)/N_0 - q\Gamma_C, \qquad (18)$$

$$M_C(\lambda) = \int_{\lambda - \tau/2}^{\lambda + \tau/2} y_C^2(t, q)\,dt, \qquad (19)$$

where $y_C(t) = \int_{-\infty}^{\infty} x(t') h_C(t - t', q)\,dt'$ is the response of the filter with the transfer function $H_C(\omega)$ satisfying the condition (17) concerning the observed realization $x(t)$. The accuracy of the representation (18) increases with the ratio q decreasing.

We presuppose that the signal (1) time of arrival λ_0 is unknown a priori, but the interval of its possible values $[\Lambda_1, \Lambda_2]$ can be specified. If the intensity γ_0 of the pulse (1) is known, then the algorithm for detecting the "weak" signal (1) can be written in the form of Eq. (15), where the logarithm of FLR is determined from Eq. (18). This algorithm, similarly to Eq. (16), can be represented as

$$M_0 = \sup_{\lambda \in [\Lambda_1, \Lambda_2]} M_C(\lambda) \underset{H_0}{\overset{H_1}{\gtrless}} h_0, \qquad (20)$$

where the detection threshold h_0 is chosen according to the accepted optimality criterion [1-4]. We choose the value of the threshold h_0 independently from the intensity γ_0 of the received signal. It is possible, for example, when carrying out detection by the Neumann-Pearson criterion [1-4].

The detection algorithm (20) does not require a priori information on the signal (1) intensity. Therefore, we use the algorithm (20) for the case when the signal (1) time of arrival λ_0 and intensity γ_0 are unknown.

The detection algorithm (20) can be implemented as a single-channel device, its block diagram coincides with the circuit of the ML detector of the band signal shown in Fig. 3. But, according to Eq. (19), the linear filter with the transfer function $H_C(\omega)$ (17) should be used as the input filter F_0. The characteristics and designations of other detector blocks are the same as for the ML band signal detector. Practical implementation of the single-channel detector presented in Fig. 3 is rather simpler than the one for the multi-channel ML detector presented in Figs. 1, 2.

It must be emphasized that the synthesis of the detection algorithm (20) is conducted for the case of the "weak" signal, when the intensity of the pulse (1) is much lower than the spectral density of the additive noise. Therefore, the detection algorithm (20) and the corresponding detector shown in Fig. 3 are asymptotically optimal ones under $q_0 \to 0$. If the ratio q_0 is large enough, then the characteristics of the detection algorithm (20) can be worse in comparison with the ML algorithm (6). The valid conclusion on the usefulness of the asymptotically optimal algorithm (20) can be made by the analysis of the characteristics of its efficiency only.

It is noteworthy that while detecting the band signal, when $g(x) = I(x)$ and the spectral density of the stochastic signal carrier satisfies the condition (11), the algorithm (20) transforms into the ML algorithm (16). Therefore, in the case of the band signal (1), the algorithm (20) is optimal for all the values of γ_0.

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The Detection Characteristics

The performance of the signal detection algorithms is characterized by the detection error probabilities: the false alarm probability α and the missing probability β [1-4, 13]. The false alarm probability α is a probability of making a decision on the presence of useful signal in the observed data, while there is no signal (H_0 hypothesis is true). The missing probability β is a probability of making a decision on the signal absence, while the useful signal is present in the observed data (H_1 hypothesis is true).

The error probabilities for the detection algorithm (20) are determined by the probabilistic properties of its decision statistics (19). We now consider the characteristics of the functional (19). Let us designate $S_R(\lambda)=\langle M_C(\lambda)\rangle$ as the regular component and $N_R(\lambda)=M_C(\lambda)-\langle M_C(\lambda)\rangle$ as the fluctuating component of the functional (19) while the hypothesis H_R , $R=0{,}1$ is realized. Here $\langle\cdot\rangle$ means averaging on all the realizations of the observed data $x(t)$ under the fixed values λ_0 and γ_0 (q_0) and the corresponding H_R hypothesis validity. From [4], it follows that the functional $M_C(\lambda)$ (19) is asymptotically, under $\mu\to\infty$, Gaussian random process. The statistical characteristics of the Gaussian random process are determined by its first two moments: mathematical expectation and covariance function [3]. Therefore, for the condition (3) fulfilling, analysis of only the first two moments of the functional (19) is sufficient: the regular components $S_R(\lambda)$ and the covariance functions $K_R(\lambda_1,\lambda_2)=\langle N_R(\lambda_1)N_R(\lambda_2)\rangle$ of the fluctuating components $N_R(\lambda)$ for both hypotheses H_R, $R=0{,}1$.

If the useful signal is absent in the observed data (H_0 hypothesis is true) and the condition (3) is satisfied, then the first two moments of the functional (19), similarly to [4-7], are written down as

$$S_0(\lambda)=A_0\,,\;K_0(\lambda_1,\lambda_2)=D_0C(\lambda_2-\lambda_1),$$

$$A_0=\mu N_0\int_{-\infty}^{\infty}g(x)\,dx\,,\quad D_0=\mu N_0^2\,,\quad C(t)=\max(0{,}1-|t|/\tau). \tag{21}$$

From Eq. (21), it follows that in the signal absence the decision statistics (19) is a stationary random process with the constant mathematical expectation A_0 and the dispersion $\sigma_0^2=D_0$.

If the useful signal is present in the observed data (H_1 hypothesis is true) and the condition (3) holds, then the first two moments of the functional (19), similarly to [4-7], can be represented in the form of

$$S_1(\lambda)=A_1C(\lambda-\lambda_0)+A_0\,,\qquad A_1=\mu q_0N_0\,,$$

$$K_1(\lambda_1,\lambda_2)=D_1R(\lambda_1-\lambda_0,\lambda_2-\lambda_0)+D_0C(\lambda_2-\lambda_1), \tag{22}$$

$$D_1=\mu q_0N_0^2\int_{-\infty}^{\infty}g^3(x)[2+q_0g(x)]\,dx\,,$$

$$R(t_1,t_2)=\max[0{,}1-\max(|t_1|,|t_2|,|t_2-t_1|)/\tau],$$

where $C(t)$, A_0 and D_0 are determined from Eqs. (21). From Eq. (22), it follows that in the presence of the useful signal the decision statistics (19) is the nonstationary random process. The regular component $S_1(\lambda)$ (22) reaches the absolute maximum at the point $\lambda=\lambda_0$ of the true value of the time of arrival of the signal (1). In addition, the dispersion $\sigma_1^2(\lambda)=K_1(\lambda,\lambda)$ of the fluctuating component $N_1(\lambda)$ also tops under $\lambda=\lambda_0$. Then, the power signal-to-noise ratio (SNR) at the detector output is determined as [4-7]

$$z^2=A_s^2/\sigma_s^2=A_1^2/(D_0+D_1)= \\ =\mu q_0^2\Big/\int_{-\infty}^{\infty}g^2(x)[1+q_0g(x)]^2\,dx\,, \tag{23}$$

where $A_s=S_1(\lambda_0)-A_0=A_1$ is the amplitude of the regular component $S_1(\lambda)$ at the point of the maximum $\lambda=\lambda_0$; and $\sigma_s^2=\sigma_1^2(\lambda_0)=D_0+D_1$ is the dispersion of the fluctuating component $N_1(\lambda)$ under $\lambda=\lambda_0$.

The exact expressions for the error probabilities that characterize detection of the fluctuating pulse (1) can not be found [1, 4, 11, 13]. Still, the expressions (21) and (22) for the moments of the functional (19) make it possible to use the results of the studies [4-7] and write down the asymptotically exact expressions for the detection error probabilities arising when the asymptotically optimal algorithm (20) is applied.

Following [4-7], we presuppose that the output SNR (23) is great enough, i.e. $z^2\gg 1$, while the q_0 value, which can be considered as the input SNR, can be small. Thus, let us assume, according to [4-7], that the uncertainty $\Lambda_2-\Lambda_1$ of the time of arrival of the random pulse (1) is considerably greater than its duration τ, that is, $m=(\Lambda_2-\Lambda_1)/\tau\gg 1$. We also propose that the detection threshold h_0 is sufficiently large and ensures a small value of the false alarm probability α [4]. Then, by applying Eqs. (21), (22) and the results from the studies [4-7], we obtain the asymptotically exact expressions for the false alarm α and missing β probabilities during the asymptotically optimal detection of the pulse (1) with unknown time of arrival and intensity:

$$\alpha = \begin{cases} 1 - \exp\left[- mu \exp\left(- u^2/2\right)/\sqrt{2\pi}\right], & u \ge 1, \\ 1, & u < 1, \end{cases} \tag{24}$$

$$\beta = (1-\alpha)\left\{\Phi\left(\frac{u}{\eta} - z\right) - \right.$$
$$- 2\exp\left[\frac{\psi^2 z^2}{2} + \psi z\left(z - \frac{u}{\eta}\right)\right]\Phi\left[\frac{u}{\eta} - z(1+\psi)\right] + \tag{25}$$
$$\left. + \exp\left[2\psi^2 z^2 + 2\psi z\left(z - \frac{u}{\eta}\right)\right]\Phi\left[\frac{u}{\eta} - z(1+2\psi)\right]\right\}.$$

Here $u = (h_0 - A_0)/\sqrt{D_0}$ is the normalized detection threshold; SNR z^2 is determined from Eq. (23); $\eta^2 = (D_0 + D_1)/D_0$; $\psi = 2\eta^2/(1+\eta^2)$; D_0 and D_1 are determined from Eqs. (21), (22), correspondingly, and $\Phi(x) = \int_{-\infty}^{x} \exp\left(-t^2/2\right)dt/\sqrt{2\pi}$ is the probability integral.

The accuracy of the expression (24) increases with the parameters μ, *m* and *u*, while the accuracy of the expression (25) – with μ, *m*, z^2 increasing and the false alarm probability α decreasing [4-7]. The analysis of the limits of the applicability of the asymptotic formulas (24), (25) is carried out by the statistical computer simulation of the detection errors. Certain simulation results obtained for the case of the band signal are presented in [4, 6, 7]. From the simulation results, it follows that the expression (24) for the false alarm probability α already possesses a satisfactory accuracy under $m > 5\ldots7$, $\mu > 30\ldots40$ and under such values of *u*, when $\alpha < 0.1$. The expression (25) for the missing probability β possesses a satisfactory accuracy when $\mu > 30\ldots40$, $z > 1.5\ldots2$, $m > 5\ldots7$ and $\alpha < 0.1$ already.

If the signal (1) time of arrival λ_0 is known a priori, then in Eq. (20) we should set $\Lambda_2 = \Lambda_1 = \lambda_0$. Then $m = 0$ and the formulas (24), (25) are inapplicable, since they are obtained for the case of $m \gg 1$. Let us write down the characteristics of the detection algorithm (20) for the known time of arrival of the signal (1). According to Eqs. (21), (22), under $\Lambda_2 = \Lambda_1 = \lambda_0$ the mathematical expectation of the random variable M_0 is equal to $S_1(\lambda_0) = A_0 + A_1$ (if there is a signal in the observed data), or to $S_{01}(\lambda_0) = A_0$ (if there is no signal in the observed data). In this case, the dispersion of the random variable M_0 is equal either to $\sigma_0^2 = D_0$ (in the signal absence), or to $\sigma_s^2 = \sigma_1^2(\lambda_0) = D_0 + D_1$ (in the signal presence). We take into account that under $\Lambda_2 = \Lambda_1$ the random variable M_0 (20) is the asymptotically Gaussian one with $\mu \to \infty$ [4]. Then, similarly to [4], we get the asymptotic expressions for the false alarm α and missing β probabilities when asymptotically optimal detecting the pulse (1) with a priori known time of arrival (under $m = 0$):

$$\alpha = 1 - \Phi(u), \qquad \beta = \Phi(u\zeta - z). \tag{26}$$

Here $\zeta = \sigma_0/\sigma_1 = \sqrt{D_0/(D_0 + D_1)}$, $u = (h_0 - A_0)/\sqrt{D_0}$ is the normalized detection threshold and SNR $z^2 = A_1^2/(D_0 + D_1)$ is determined from Eq. (23).

The accuracy of the expressions (26) increases with μ (3). The analysis of the limits of applicability of the formulas (26) carried out by the statistical computer simulation shows that these formulas possess a satisfactory accuracy under $\mu > 25\ldots30$ already.

The Analysis of the Detection Algorithm Performance

Calculations by the formulas (24)-(26) show that the efficiency of the asymptotically optimal algorithm (20) increases with the parameters q_0 and μ (*z*). As an example, in Fig. 4 the values of the missing probability β (24), (26) are presented when detecting the pulse (1) with the spectral density (2) of the Lorentz form $g(x) = 1/\left[1 + (\pi x/2)^2\right]$. It is assumed that the false alarm probability is fixed and equal to $\alpha = 0.001$, while the normalized threshold *u* is calculated for the set false alarm probability by the formulas (24), (26). The solid lines correspond to $m = 100$, dashed lines – to $m = 10$ and dash-dotted lines – to $m = 0$. Curves 1 are plotted for the case of $\mu = 100$, curves 2 – $\mu = 200$, curves 3 – $\mu = 400$. From Fig. 4, it can be seen that, under the fixed false alarm probability α, the missing probability β is rapidly decreasing with the parameters q_0 , μ (*z*) increasing and reaches sufficiently small values when $q_0 > 1$.

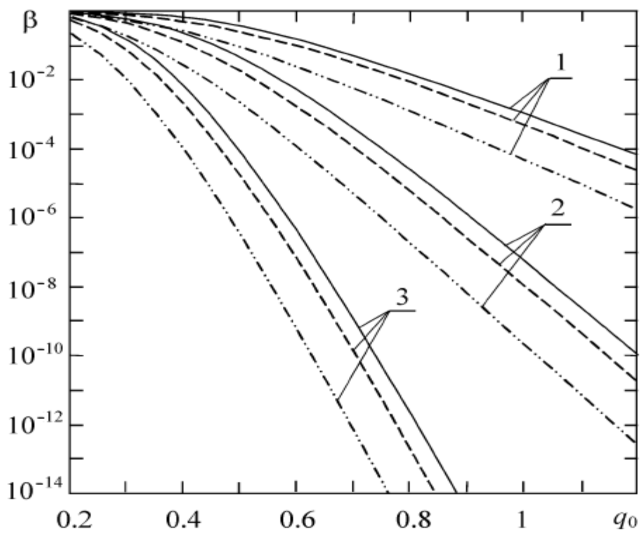

Figure 4: The missing probability for the asymptotically optimal detector under the fixed false alarm probability

The detection algorithm (20) is asymptotically optimal and transforms into the ML algorithm (6) under $q_0 \to 0$. If the ratio q_0 is large enough, then the characteristics of the algorithm (20) can be worse than the corresponding ones of the ML algorithm (6). The characteristics of the ML algorithm (6) can not be analytically found. Therefore, we compare the characteristics of the asymptotically optimal algorithm (20) with the characteristics of the ML algorithm [4-6] when the signal (1) intensity is known. From general considerations, it is clear that the efficiency of the ML algorithm (6) can not be higher than the efficiency of the ML algorithm from [4-6]. Thus, the loss in the efficiency of the asymptotically optimal algorithm (20) in comparison with the ML algorithm [4-6], when the signal intensity is known, can be considered as the upper loss boundary in the efficiency of the asymptotically optimal algorithm (20) in comparison with the ML algorithm (6).

The characteristics of the detection algorithm [4-6] under the known intensity of the pulse (1) can be calculated from the formulas (24)-(26), where, according to [4, 5, 7], we have

$$A_0 = \mu q_0 \int_{-\infty}^{\infty} \frac{g(x)\,dx}{1+q_0 g(x)}, \qquad A_1 = \mu q_0^2 \int_{-\infty}^{\infty} \frac{g^2(x)\,dx}{1+q_0 g(x)},$$

$$D_0 = \mu q_0^2 \int_{-\infty}^{\infty} \frac{g^2(x)\,dx}{[1+q_0 g(x)]^2}, \; D_1 = \mu q_0^3 \int_{-\infty}^{\infty} \frac{g^3(x)[2+q_0 g(x)]\,dx}{[1+q_0 g(x)]^2}.$$

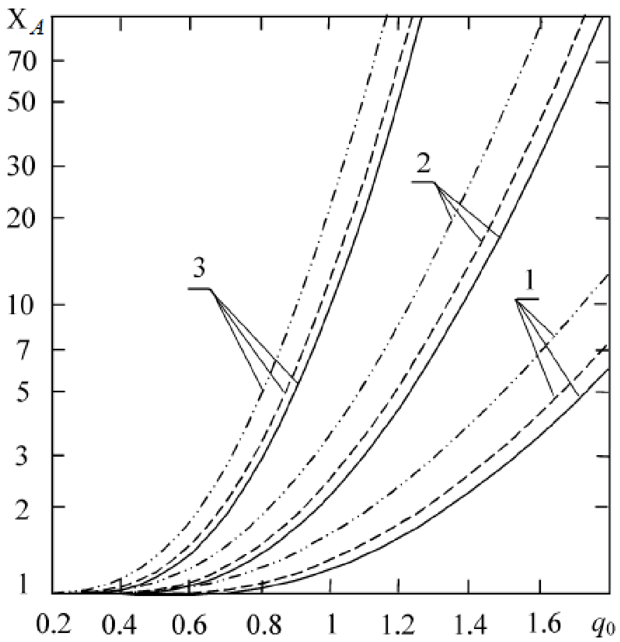

Figure 5: The loss in the efficiency of the asymptotically optimal detector

In Fig. 5, the dependences of the loss X_A in the efficiency of the asymptotically optimal algorithm (20) in comparison with the ML algorithm [4-6] are shown. The loss is calculated as the ratio of the missing probabilities β for the asymptotically optimal and ML algorithms under $g(x) = 1/\left[1+(\pi x/2)^2\right]$ and the fixed false alarm probability $\alpha = 0.001$. The solid lines correspond to $m = 100$, the dashed lines – to $m = 10$ and the dash-dotted lines – to $m = 0$. Curves 1 are plotted for $\mu = 100$, curves 2 – for $\mu = 200$, curves 3 – for $\mu = 400$. From Fig. 5, it follows that the loss in efficiency of the asymptotically optimal algorithm (20) increases with q_0 (γ_0). However, according to Fig. 4, with the ratio q_0 increasing the efficiency of the detection algorithm (20) increases also, so that the characteristics of the algorithm for the large values of q_0 may be acceptable. Under small values of q_0 (γ_0), the efficiency of the asymptotically optimal and ML detectors practically coincide.

Thus, if the "weak" signal is detected, then the simpler asymptotically optimal detection algorithm (20) can be used, instead of a much more complex ML algorithm (6).

OPTIMIZATION OF THE ASYMPTOTICALLY OPTIMAL DETECTION ALGORITHM FOR THE STRONG SIGNAL

The Modified Asymptotically Optimal Detection Algorithm

The loss in the efficiency of the asymptotically optimal algorithm (20) under large ratios q_0 is caused by the difference between the transfer function $H_C(\omega)$ (17) and the optimal transfer function $H(\omega, q)$ (9). This difference increases, if the value of ratio *q* increases. Particularly, the bandwidth of the optimal transfer function $H(\omega, q)$ (9) increases with *q*, while the bandwidth of the transfer function $H_C(\omega)$ (17) does not depend upon *q*. Therefore, in order to increase the efficiency of the asymptotically optimal detector (Fig. 3), under the large ratios q_0, we can expand the input filter F_0 bandwidth. Then, instead of the input filter with the transfer function $H_C(\omega)$ (17), in the asymptotically optimal detector (Fig. 3) we should use the filter with the transfer function satisfying the condition

$$\left|H_C(\omega)\right|^2 = g\left(\frac{\nu-\omega}{\Omega_M}\right) + g\left(\frac{\nu+\omega}{\Omega_M}\right), \qquad \Omega_M > \Omega. \quad (27)$$

The detection algorithm (20) when we apply the transfer function (27) instead of the transfer function (17), we call the modified asymptotically optimal algorithm. The block diagram of the corresponding modified asymptotically

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optimal detector is the same as the one shown in Fig. 3, where the linear filter with the transfer function $H_C(\omega)$ (27) serves as the input filter F_0. The value $\Omega_M > \Omega$ characterizes the input filter bandwidth. The informed choice of the Ω_M value can be made on the basis of the analysis of the characteristics of the modified asymptotically optimal algorithm for various values of Ω_M.

The Detection Characteristics and Their Analysis

Let us write down the detection error probabilities for the asymptotically optimal algorithm (20) with the modified transfer function (27). When applying the transfer function (27) with the condition (3) is satisfied, for the regular components and the covariance functions of the fluctuation components of the functional (19), we obtain the expressions (21), (22), where the coefficients

$$A_0 = \mu N_0 \kappa \int_{-\infty}^{\infty} g(x)\,dx, \quad A_1 = \mu q_0 N_0 \int_{-\infty}^{\infty} g(x) g(x/\kappa)\,dx, \qquad (28)$$

$$D_0 = \mu N_0^2 \kappa, \quad D_1 = \mu q_0 N_0^2 \int_{-\infty}^{\infty} g^2(x/\kappa) g(x)\left[2 + q_0 g(x)\right] dx$$

should be used. In Eqs. (28), $\kappa = \Omega_M / \Omega_0$ is the coefficient of input filter passband expansion. Then, the false alarm α and missing β probabilities, while the modified asymptotically optimal detection algorithm is used, are determined from Eqs. (24)-(26), the coefficients (28) applied.

The expressions obtained for the false alarm and missing probabilities allow us to estimate the efficiency of the modified asymptotically optimal detection algorithm for the various values of κ and q_0. In Fig. 6, the values of the loss X_M are shown that the efficiency of the modified asymptotically optimal algorithm (20), (27) allows them to take, and we can compare these values with the ones the ML algorithm implementation [4-6] demonstrate. The loss is calculated as the ratio of the missing probabilities β for these detection algorithms under the fixed false alarm probability $\alpha = 0.001$. It is assumed that $\mu = 200$ and $g(x) = 1/\left[1 + (\pi x/2)^2\right]$. The solid lines correspond to $m = 100$, the dashed lines – to $m = 10$, and the dash-dotted lines – to $m = 0$. Curves 1 are plotted for $\kappa = 1.3$, curves 2 – for $\kappa = 1.6$, and curves 3 – for $\kappa = 2.2$. For comparison, curves 4 in Fig. 6 show the corresponding dependencies of the loss X_M for the asymptotically optimal detection algorithm (20) with the transfer function (17) ($\kappa = 1$).

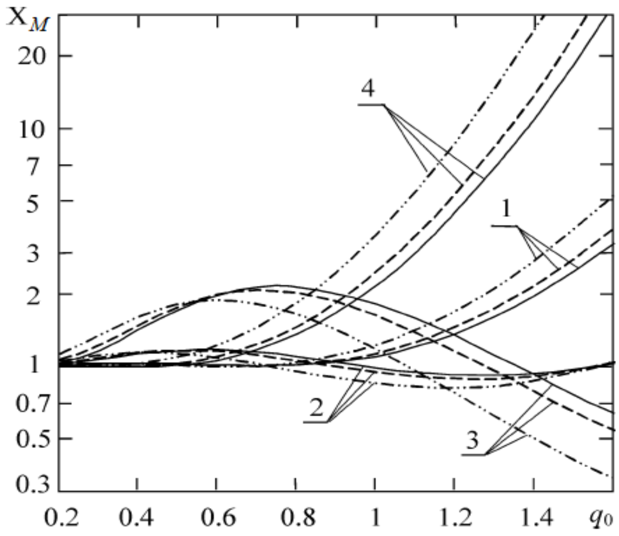

Figure 6: The loss in the efficiency of the modified asymptotically optimal detector

From Fig. 6, we can see that the passband expansion of the input filter of the modified asymptotically optimal detector leads to the significant decrease in the loss in detection efficiency under the large ratios q_0. However, the loss for the small ratios q_0 is slightly increasing. The interval of the parameter q_0 values, when the loss decreasing occurs, shifts to the region of large q_0 with the ratio κ increasing. The gain in the efficiency of the modified asymptotically optimal algorithm ($\mathrm{X}_M < 1$) for the large values of q_0 and κ (curve 3 in Fig. 6) indicates that one can optimize the ML detection algorithm [4-6] for the large ratios q_0.

CONCLUSION

The introduced asymptotically optimal detector (Fig. 3) can be recommended, instead of the more complex maximum likelihood detector (Figs. 1, 2) when the "weak" stochastic signal is received and the ratio of the signal intensity to the noise spectral density is small. In case of the useful signal of high intensity, it is recommended to extend the input filter bandwidth of the asymptotically optimal detector optimizing it by minimizing the detection error probabilities in the specified turndown of the signal intensity. The results we have obtained allow us to make the informed choice of the detection algorithm, depending on the requirements concerning the efficiency of the algorithm and the simplicity of its hardware or software implementation.

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