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Minimizing Weighted Earliness and Tardiness under Fuzziness using Firefly Algorithm

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Abstract

In this article we consider a single machine scheduling problem with processing times and due dates to be uncertain in nature. Trapezoidal and triangular fuzzy numbers are considered as fuzzy processing times and fuzzy due dates respectively. Asymmetric earliness and tardiness costs are specified for each job. A nature inspired meta-heuristic Firefly Algorithm is used to optimize the total weighted earliness and tardiness costs and few near optimal job sequences are obtained. Computational results show that the fitness values obtained by the Firefly Algorithm are more optimal when compared with Particle Swarm Optimization.

Keywords: Single Machine Scheduling, Early/Tardy, Fuzzy processing times, Fuzzy due dates, Firefly Algorithm, Particle Swarm Optimization.

INTRODUCTION

Production scheduling allocates resources to tasks over time and determines the sequence of operations so that the constraints of the system are met and the performance criteria is optimized. The *just-in-time* (JIT) environment is the most significant problem in production scheduling. Jobs are to be completed within the stipulated time period, jobs which are not completed within the specified time may disrupt the customer's operations and jobs that are finished early must be held in inventory. Therefore, an ideal schedule is one in which all the jobs finish exactly on their assigned due dates. There are various factors involved in real-time scheduling problems that are often imprecise or uncertain in nature so that the jobs are either early or tardy. This impreciseness can be dealt with fuzzy parameters. Zadeh [1] pioneers basics of fuzzy set theory. Procedure on ranking fuzzy sets is detailed by Yager [2]. The first case study in production scheduling using fuzzy set theory was done by Prade [3]. This article is structured in the following sections: In section 2 literature review on fuzzy single machine scheduling problems using bio-inspired algorithms are explored. In section 3 the basic definitions of fuzzy set theory are discussed. The representation of the proposed problem with fuzzy parameters and the objective of the problem is detailed in section 4. In section 5 Particle Swarm Optimization (PSO) algorithm is specified. Firefly Algorithm (FA) and the Framework of FA for the problem considered are detailed in section 6. Comparison of the results obtained from FA and PSO is discussed in section 7. We conclude and discuss about future research work in section 8.

LITERATURE REVIEW

Scheduling with performance criteria involving due dates, such as total weighted earliness and tardiness (E/T), with uncertain parameters, is a reference problem in many real industrial contexts. A detailed literature survey on early/tardy problems is done by Baker and Scudder [4]. Hall and Posner [5] consider a single processor with a common due date and optimizes the weighted sum of earliness and tardiness costs and proves that a special case of early/tardy model is NP-Complete. The randomness occurring in the scheduling problems are categorized as stochastic scheduling problems and are cited in Pinedo [6]. Cahon and Lee [7] consider processing times as triangular and trapezoidal fuzzy numbers and uses modified Johnson's algorithm to optimize fuzzy makespan and fuzzy mean flow times. Korner [8] works on generating fuzzy random variables. A collection of articles in fuzzy scheduling can be referred in Słowinski and Hapke [9]. Cheng Yao *et al* [10] works on ready time scheduling problem with imprecise processing times, chance constrained programming with likelihood profile is applied to obtain the objective. Stefan Chanas [11] considers one machine problem with uncertain processing times and due dates and optimizes fuzzy tardiness using Zadeh's extension principle. Chen *et al* [12] considers an EPQ model with trapezoidal fuzzy costs and uses graded mean integration representation method to defuzzify the fuzzy costs and optimizes the model considered. Jayanthi and Karthigeyan [13] present a brief review on fuzzy single machine and parallel machine scheduling. Kennedy and Eberhard [14] propose a new algorithm namely particle swarm optimization for optimizing nonlinear functions. Van den Bergh [15] explores the trajectories of the particles in particle swarm optimization algorithm. Davide Anghinolfi [16] constructs a new discrete particle swarm optimization method on one machine problem to minimize total weighted tardiness with set up times and tests the algorithm for benchmark problems specified by Cicirello and ORLIB. Torabia *et al* [17] works on parallel machine scheduling problem with fuzzy processing times and fuzzy due dates with sequence dependent set up times to optimize total weighted flow time, total weighted tardiness and total machine load variation using MOPSO. Assem *et al* [18] considers machine time scheduling problem in which starting time is taken as trapezoidal fuzzy number and optimizes the starting time using PSO. Nader *et al* [19] works on one machine scheduling with sequence dependent set up times and set up costs to minimize weighted tardiness and using discrete particle swarm optimization. Jayanthi and Anusuya [20] consider a single machine scheduling problem to optimize total weighted earliness and tardiness using PSO. Xin-She Yang *et al* [21] writes an elaborate description of Firefly Algorithm (FA) and its application in multimodal optimization problems and proves that Firefly Algorithm is far superior to the existing bio-inspired algorithms. Haomiao [22] applies firefly algorithm for combinatorial optimization problems in production scheduling. Mohammad *et al* [23] considers a permutation flow shop scheduling problem and minimizes the makespan using a discrete firefly algorithm. Xin-SheYang [24] reviews the fundamentals of firefly algorithm and concludes that firefly algorithm produce better optimal solutions for higher dimensional optimization problems. Iztok Fister *et al* [25] review applications of firefly algorithm in the field of engineering and optimization. Udaiyakumar [26] uses firefly algorithm to minimize makespan for 1-25 benchmark Lawrence problems from OR library. Karthikeyan *et al* [27] works on flexible job shop scheduling problem and uses a hybrid discrete firefly algorithm to obtain the three objectives namely makespan, workload of a machine and total workload of all machines. Amit Kumar [28] considers a flexible job shop scheduling problem for allocation of machines and sequencing the jobs. An Enhanced firefly algorithm is used to obtain the objective.

FUZZY PRELIMINARIES

In this section basic definitions of fuzzy set theory proposed by Zadeh [1] is reviewed.

Definition:1 (*fuzzy set*)

Let X be a nonempty set. A *fuzzy set* $\tilde{A}=\left\{(x,\zeta_A(x))\,/\,x\,\varepsilon\,X\right\}$ in X is characterized by its membership function $\zeta_A(x): X\rightarrow[0,1]$ and $\zeta_A(x)$ is interpreted as the degree of membership of element x in fuzzy set $\tilde{A}$ for each $x\,\varepsilon\,X$.

Definition:2 (*support*)

The *support* of a fuzzy set $\tilde{A}$ is the set of all points $x\,\varepsilon\,X$ such that $\zeta_A(x)>0$. $supp(\tilde{A})=\left\{x\,\varepsilon\,X\,/\,\zeta_A(x)>0\right\}$.

Definition:3 (*core)*

The *core* of a fuzzy set $\tilde{A}$ is the set of all points $x\,\varepsilon\,X$ such that $\zeta_A(x)=1$. $core(\tilde{A})=\left\{x\,\varepsilon\,X\,/\,\zeta_A(x)=1\right\}$.

Definition: 4 (*normal)*

A fuzzy set $\tilde{A}$ is said to be *normal* if its core is non empty. In other words we can always find a $x\,\varepsilon\,X$ such that $\zeta_A(x)=1$.

Definition : 5 ($\alpha-cut$)

The $\alpha-cut$ or $\alpha-level\,set$ of a fuzzy set $\tilde{A}$ is a crisp set defined by $A_\alpha=\left\{x\,\varepsilon\,X\,/\,\zeta_A(x)\geq\alpha\right\}$ where $\alpha\,\varepsilon[0,1]$.

Definition : 6 (*convex fuzzy set)*

A fuzzy set $\tilde{A}$ is called *convex fuzzy set* if all A_α are convex fuzzy sets. i.e. for every element $x_1, x_2\,\varepsilon\,A_\alpha$ and for every $\alpha\,\varepsilon[0,1]$, $\lambda x_1+(1-\lambda)x_2\,\varepsilon\,A_\alpha\ \forall\lambda\,\varepsilon[0,1]$.Otherwise the fuzzy set is said to be non convex.

Definition : 7 (*fuzzy Number)*

A fuzzy subset $\tilde{A}$ of $\mathbf{R}$ is said to be a fuzzy number if the following conditions are satisfied:

a) $\tilde{A}$ is normal i.e. there exists an $x\,\varepsilon\,\mathbf{R}$ such that $\zeta_A(x)=1$;
b) ζ_A is quasi concave, for $t\,\varepsilon[0,1]$ i.e. $\zeta_A(tx+(1-t)y)\geq\min\left\{\zeta_A(x),\zeta_A(y)\right\}$.
c) ζ_A is upper semi continuous, i.e $\left\{x\,\varepsilon\,\mathbf{R}/\zeta_A(x)\geq\alpha\right\}$ is a closed subset of $\mathbf{R}$ for each $\alpha\,\varepsilon[0,1]$.

d) The 0-level set A_0 is a closed and bounded subset of $\mathbf{R}$ or equivalently, A convex and normalized fuzzy set whose membership function is piecewise continuous is called *fuzzy number*.

Definition:8 (*Triangular fuzzy number)*

A fuzzy set $\tilde{A}=(a_1,a_2,a_3)$ is called a *triangular fuzzy number* whose membership function is defined as

$$\zeta_{\tilde{A}}(x)=\begin{cases}0, & x<a_1\\ \dfrac{x-a_1}{a_2-a_1}, & a_1\le x\le a_2\\ \dfrac{a_3-x}{a_3-a_2}, & a_2\le x\le a_3\\ 0, & x>a_3\end{cases}$$

Definition:9 (*Trapezoidal fuzzy number)*

A fuzzy set $\tilde{A}=(a_1,a_2,a_3,a_4)$ is called a *trapezoidal fuzzy number* whose membership function is defined as

$$\zeta_{\tilde{A}}(x)=\begin{cases}0, & x<a_1\\ \dfrac{x-a_1}{a_2-a_1}, & a_1\le x\le a_2\\ 1, & a_2\le x\le a_3\\ \dfrac{a_4-x}{a_4-a_3}, & a_3\le x\le a_4\\ 0, & x>a_4\end{cases}$$

Defuzzification Methods Adopted:

Average High Ranking Method

Let $\tilde{A}=(a_1,a_2,a_3)$ be a triangular fuzzy number then

the average high ranking method of $\tilde{A}$ is given by

$$hr(\tilde{A})=\frac{3a_2+a_3-a_1}{3}$$

Graded Mean Integration Representation Method

Let $\tilde{A}=(a_1,a_2,a_3,a_4)$ be a trapezoidal fuzzy number then the graded mean representation method of $\tilde{A}$ is

$$gm(\tilde{A})=\frac{a_1+2a_2+2a_3+a_4}{6}$$

PROBLEM REPRESENTATION

$$1||Min\sum_{j=1}^{n}(\alpha_j E_j+\beta_j T_j)$$

Consider a set of jobs j = 1,2,…,n; to be processed on a single machine. The processing times for the jobs are considered to be trapezoidal fuzzy numbers $\tilde{p}_j=(p_1,p_2,p_3,p_4)$ and due dates for the jobs are considered as triangular fuzzy numbers $\tilde{d}_j=(d_1,d_2,d_3)$. The early and tardy costs the jobs are assigned to be uniform random numbers in the interval (1, 10). The Mathematical model is stated as follows:

$$f = Min\sum_{j=1}^{n}(\alpha_j E_j+\beta_j T_j)$$

s.t

$$E_j\ge 0 \qquad j=1,2,...,n \qquad (1)$$

$$E_j\ge d_j-C_j \qquad j=1,2,...,n \qquad (2)$$

$$T_j\ge 0 \qquad j=1,2,...,n \qquad (3)$$

$$T_j\ge C_j-d_j \qquad j=1,2,...,n \qquad (4)$$

Where E_j is earliness of job j , T_j tardiness of job *j*, α_j early penalty of job *j*, tardy penalty β_j of job *j*. If the completion time of a job is lesser than its due date then the job as early and if the completion time of a job is greater than its due date then the job as tardy. Equations (2) and (4) are used to calculate earliness and tardiness of all jobs *j=1,2,…,n*. The objective is to minimize total weighted early and tardy costs and to find an optimal schedule. A novel coding scheme is adopted to optimize the total weighted earliness and tardiness costs. The following are the assumptions and notations considered for the problem.

Assumptions:

- Single machine.
- One job is processed at a time.
- Pre-emption is not allowed.
- Jobs are not dependent on each another.
- Release times of all jobs are taken to be zero.
- Early and tardy penalties are assigned to each job.
- Processing times are considered as trapezoidal fuzzy numbers.
- Due dates are taken are considered as triangular fuzzy numbers.

Table I: Notations

j	number of jobs $j=1,2,\dots n$
k	number of iterations
i	Particle / Firefly
$\tilde{p}_j = (p_1, p_2, p_3, p_4)$	fuzzy processing time of job j
$\tilde{d}_j = (d_1, d_2, d_3)$	fuzzy due date of job j
$hr(\tilde{p}_j)$	average high ranking of the processing time of job j
$gm(\tilde{d}_j)$	graded mean value of the due date of job j
α_j	the early penalty of job j
β_j	the tardy penalty of job j
E_j	earliness of job j
T_j	tardiness of job j

PARTICLE SWARM OPTIMIZATION

Particle Swarm Optimization (PSO) was introduced by Kennedy and Eberhart [14] in the year 1995, inspired by birds flocking and fish schooling. PSO is applied in various fields like production scheduling, artificial neural network, function optimization, bio-medicine, Vehicle routing, pattern recognition, fuzzy control, multi commodity network design, etc. In PSO, each particle is represented as a potential solution to a problem in an n-dimensional space and its position at the k^{th} iteration is denoted as $X_i^k = [X_{i1}^k, X_{i2}^k, \dots, X_{in}^k]$. Each particle remembers its own previous best position and its velocity at each dimension as $V_i^k = [V_{i1}^k, V_{i2}^k, \dots, V_{in}^k]$.The velocity and position of the particle i at $(k+1)^{th}$ iteration are updated by the following equations

$$V_{ij}^{k+1} = w\,V_{ij}^k + c_1 r_1 (pb_{ij}^k - X_{ij}^k) + c_2 r_2 (gb_j^k - X_{ij}^k) \qquad (5)$$

$$X_{ij}^{k+1} = X_{ij}^k + V_{ij}^{k+1} \qquad (6)$$

where c_1 and c_2 are two positive acceleration coefficients, r_1 and r_2 are random numbers uniformly distributed in the interval (0,1) for the j^{th} dimension of the i^{th} particle, $pb_{ij}^k = [p_{i1}^k, p_{i2}^k, \dots, p_{in}^k]$ is the personal best (pbest) position of the i^{th} particle and $gb_j^k = [g_1^k, g_2^k, \dots, g_n^k]$ is the global best (gbest) position of the swarm. w is the inertia weight which balances between local and global search abilities.

FIREFLY ALGORITHM

Firefly algorithm (FA) was developed by Xin-She Yang [21] in 2008, inspired by the natural behaviour of tropical fireflies. Firefly Algorithm is a population based technique which is efficient in solving optimization problems.

Firefly Algorithm is based on three aphorisms

- Fireflies are unisex so that one firefly will be attracted to other fireflies regardless of their sex.
- The attractiveness is proportional to the brightness, and they both decrease as their distance increases. Thus for any two flashing fireflies, the less bright one will move towards the brighter one. If there is no brighter one than a particular firefly, it will move randomly.
- The brightness of a firefly is determined by the value of the objective function.

Attractiveness:

The attractiveness of firefly β is a factor that determines the power of attraction of fireflies. It depends on the mutual distance r between the fireflies. Attractiveness is given by

$$\beta(r) = \beta_0 e^{-\gamma r^m} \quad m \geq 1 \qquad (7)$$

Where β_0 denotes the maximum attractiveness and γ is the light absorption coefficient which controls the intensity of light.

Distance:

The distance between two fireflies i and l at positions x_i and x_l can be represented as

$$r_{il} = \|x_i - x_l\| = \sqrt{\sum_{j=1}^{d} (x_{i,j} - x_{l,j})^2} \qquad (8)$$

Where $x_{i,j}$ is the j^{th} component of the spatial coordinate x_i of the i^{th} firefly and d denotes the number of dimensions.

Movement:

The movement of a firefly i is determined by the following equation

$$x_i = x_i + \beta_0\, e^{-\gamma r_{il}^2} (x_l - x_i) + \alpha \left(rand - \frac{1}{2} \right) \qquad (9)$$

Where the first factor is the current position of a firefly i, and the second one denotes a firefly's attractiveness and the last factor is used for the random movement if there is not any brighter firefly, *rand* is a uniform random number generated in the range (0,1). Depending on the choice of the parameter we can control the influence of randomness on the total displacement. For most of the cases $\alpha \in (0,1)$, $\beta_0 = 1$, $\gamma = (0.1, 10)$ .Parameter γ is responsible for the rate of convergence of Firefly Algorithm.

Framework of Firefly Algorithm:

Step 1 : Initialization

- Initialize pop size using $x = x_{\min} + (x_{\max} - x_{\min})*rand$, where $x_{\min} = -0.5$ and $x_{\max} = 0.5$,*i.e.* generate randomly n fireflies where in each firefly represents a job schedule.
- Generate processing times $hr(\tilde{p}_j)$, due dates $gm(\tilde{d}_j)$, early/tardy penalties.
- Set FA parameters γ =0.1 , β_0 = 1, α = 0.5
- Define the objective function $f(x)$, $x = (x_1, x_2, ..., x_d)^T$

Step 2 : Calculate light intensities, light intensity of firefly I_i at x_i is determined by the value of objective function $f(x_i)$.

Step 3 : for each iteration (1, ...,Max It) do:

for $i = 1 : n$

for $l = 1 : n$

If $I_l < I_i$ move firefly *i* towards firefly *l* using Eq (9);

end if ,obtain attractiveness ,which varies with distance *r*

Find new solutions and update light intensity.

end for *l*

end for *i*

Rank the fireflies , find the optimal solution.

Step 4 : If the stopping criteria is met then output the results, otherwise go to step 3.

COMPUTATIONAL EXPERIMENTS

The algorithm is coded in MATLAB. For firefly algorithm, the parameter light absorption coefficient γ is taken as 0.1, β_0 is considered as 1 and the randomized parameter α is taken as 0.5. Regarding the PSO parameters, social and cognitive parameters are taken as c_1 = 2 , c_2 = 2.1 respectively, initial inertia weight is set as 1 and decrement factor is taken as 0.729. The number of instances for each trial is taken as 10 and maximum number of iterations is considered as 100. Comparison of the results proves that the optimal values obtained from the proposed Firefly Algorithm (FA) are far superior than the values of Particle Swarm Optimization (PSO). Table II represents the comparison of fitness values, mean and standard deviation for various job sizes ranging from 3 to 100, where ET_{FA} and ET_{PSO} are best/worst fitness values of the objective using FA and PSO respectively. ET_{opt} is the optimal value of the objective namely total weighted earliness and tardiness. Table III represents few near optimal job schedules.

Table II: Comparison of fitness values with FA and PSO

Jobs	ET_{opt}	**Firefly Algorithm**				**Particle Swarm Optimization**			
		ET_{FA}		**Mean**	**SD**	ET_{PSO}		**Mean**	**SD**
		Best	**Worst**			**Best**	**Worst**		
3	**67**	**67**	75	69.466	2.55	**67**	67	67.00	0.00
5	**225**	**225**	280	238.33	21.24	**225**	225	225.33	0.00
7	**418**	**418**	469	431.66	20.78	**418**	476	430.23	18.62
10	**1046**	**1046**	1046	1046.00	0.000	**1046**	1202	1098.23	54.73
15	**2612**	**2612**	2861	2706.46	73.69	2716	3057	2844.83	114.57
20	**4701**	**4701**	4981	4822.20	107.09	4879	5330	5064.90	160.61
25	**5699**	**5699**	6073	5796.63	109.76	6004	6969	6433.26	368.12
30	**9847**	**9847**	10539	9982.70	208.32	10959	12141	11393.06	398.41
35	**12950**	**12950**	13338	13093.46	137.05	14539	15761	15141.00	403.38
40	**18874**	**18874**	19484	19060.53	188.97	20790	24045	21874.46	950.09
45	**22600**	**22600**	23248	22794.53	215.79	25193	27320	26370.30	686.38

50	**33852**	**33852**	34105	34007.60	82.511	36613	39154	37856.43	1023.62
60	**45680**	**45680**	46749	46269.90	326.33	52004	58676	54403.50	2008.95
70	**63197**	**63197**	63944	63500.96	238.11	73230	77750	75181.50	1704.84
80	**89101**	**89101**	89901	89498.86	260.32	100217	111272	105411.90	3386.73
90	**105513**	**105513**	106861	106115.86	459.41	120562	129870	125667.86	2630.60
100	**109763**	**109763**	111483	110382.30	607.54	131256	140319	135361.73	3171.77

Table III: Few near optimal job schedules

Jobs	Optimal job sequences
3	3 -1 - 2
5	4 - 5 - 1- 2 - 3
7	6 - 7 - 4 - 5 -1 - 3 - 2
10	9 - 4 - 1 - 6 -10 - 2 – 8 - 5 - 7 - 3
15	2 -15 - 8 - 5 - 4 - 14 - 9 -1 - 6 -13 -11- 10 -12 - 7 - 3
20	11-12 -13 -16 -2 -1-18 - 9 -19 - 4 - 6 - 8 -7 - 4 -10 -17 -15 -20 -5 -3
25	14 -1- 22 -12 -17- 20 -13 -2 -15 - 6 - 23 - 4 -5 -10 -16 - 9 - 7-18 -25-24 - 8 -19 -11- 21- 3
30	26 - 23 -12-17- 28 - 3 -5 - 30 -10 -18 - 2 -16 - 21- 22 - 9 - 25 -11-13 - 27-19 -1-29 - 20 -15 - 24 -14 - 4 -7 - -8 - 6
35	5 -1-16 -17 -15 - 32 - 25 - 24 - 20 -19 -27 - 28 - 26 -7 - 6 -11- 33 -31 -22 - 21-10 - 2 - 4 - 35 - 3 -18 - 8 -14 -13 - 34 -12 - 30 - 29 - 23 - 9
40	37 - 13- 22 -7- 9 - 8 - 2 -16 - 29 -32 - 26 - 30-5 -39 -19 -18 - 40 -17 -21 - 6 - 36 - 14 -15 -10 - 34 - 25 - 3 - -31- 38 - 35 - 12 - 28 - 27 -33-23- 24 - 11 -20 - 4 - 1
45	28 -24 -10 -3 -12 - 35 -36 -15 - 29 -33 - 32 - 7- 22 - 39 -17- 38 -23-45 - 9 - 25 - 27 - 40 - 2 - 21 - 20 - 11- 41- 43 - 31 -37 - 6 - 4 – 26 -19 - 8 - 34 - 30 -16 - 5 - 14 -13 - 42 - 44 - 1-18
50	25 - 26 - 31- 24 -13 -17 -19 - 49 - 28 - 37 - 2 - 16 - 46 - 23 - 33- 39 - 5 - 41- 10 - 40 - 35 - 29 - 45 - 47 -14 -27 -50 - 43 -30 – 8-3 - 1- 9 - 4 - 32 - 22 - 44 - 20 - 21 -15 - 36 - 11- 12 - 6 -18 – 34- 48 - 42 - 7 - 38

CONCLUSION

This study highlights the advantage of using fuzzy set theory along with nature inspired algorithms for modelling realistic scheduling problems in the industrial field. In this article we tested the firefly and particle swarm optimization algorithms for minimizing the total weighted earliness and tardiness with asymmetric earliness and tardiness costs where the processing times, due dates are considered as trapezoidal and triangular fuzzy numbers. The computational results show that the firefly algorithm produces optimal fitness. To carry forward this work instead of triangular and trapezoidal fuzzy numbers, bell shaped fuzzy numbers or type-2 fuzzy sets can be considered.

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