

# Nonlinear Wave Interaction in Magnetized Plasma with Two-Temperature Kappa-Distributed Electrons

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## Abstract

Nonlinear head-on collision between finite cylindrical/spherical dispersive solitons, characterized by two-temperature superthermal electrons, is pondered. Nonlinear interactions were found to be modelled by a two coupled Korteweg-de Vries (KdV) system. Both solitons polarities existence domains are identified. Role of the cold and hot electrons superthermality on analytical phase shifts is pointed out. Departing from the Maxwellian limit; the criticality in the phase shift extends to higher values of the equilibrium density. Spherical and cylindrical solitary waves analytical phase shifts give same qualitative behavior. However, the amplitude of the spherical interacting electrostatic waves was higher than the cylindrical one, at the impact point. Plasma parameters accounting for Cassini spacecraft observations in Saturn's magnetosphere are considered.

**Keywords:** nonlinear phenomena, plasma waves, solitons in space plasmas.

## INTRODUCTION

Particles distribution functions according to space observations depart from Maxwellian equilibrium [1-4]. The theory of particles excess superthermality, modeled by the kappa-distribution, witnessed notable developments [5-15]. Kappa distributions are situated to modelling space plasmas observational, where succeed in analyzing and interpreting spacecraft data on the Earth's magnetospheric plasma sheet [16], the solar wind [17], Jupiter [18] and Saturn [11, 19, 20].

Schippers et al. [8] showed that, particle distributions observed by Voyager PLS [1, 2] and Cassini CAPS [3] are modelled by cool and hot  $\mathcal{K}$  - distributed electrons. Balukua and Hellberg [11] investigated the existence domain of ion acoustic (IA) solitons in a plasma featuring two (hot and cool)  $\mathcal{K}$ -distributed electrons. They showed that both soliton polarities exist and the soliton domain for low-  $\mathcal{K}$  is smaller than higher values. Also, positive double layers are found to exist for low-  $\mathcal{K}$  distributions like Maxwellian behavior. Later, Sabry [20] studied properties of IA freak waves in a plasma

with two (hot and cool)  $\mathcal{K}$  - distributed electrons, taking into account geometry effect. Soliton dynamics were found to be modelled through a modified nonlinear Schrödinger equation and a modulation instability period exist that not appearing in planar geometry[20]. Furthermore, spherical IA freak waves grow faster than cylindrical waves.

Motivated by the findings in [11, 20], the wave interaction of cylindrical as well as spherical solitons in Saturn's magnetosphere will be investigated. The following points will be addressed; two-temperature electrons superthermality effect on the process of wave interaction as well as their existence domains. To what extent geometry will affect interactions regimes and wave amplitudes. The effect of the electron species equilibrium density ratio as well as their spectral indices, on the wave behavior nonlinearity and dispersion, and phase shifts of the colliding solitons.

## BASIC EQUATIONS

Taking the plasma composition's in Saturn's magnetosphere, one considers cool adiabatic positively ions fluid with a combination of hot and cool superthermal electrons [11, 20]. To investigate ion acoustic (IA) waves, we consider the normalized fluid system[20];

$$\frac{\partial n_i}{\partial t} + \frac{1}{r^m} \frac{\partial}{\partial r} (r^m n_i u_i) = 0, \quad (1)$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial r} + 3\tau_{ic} n_i \frac{\partial n_i}{\partial r} + \frac{\partial \varphi}{\partial r} = 0, \quad (2)$$

$$\frac{1}{r^m} \frac{\partial}{\partial r} \left( r^m \frac{\partial \varphi}{\partial r} \right) + n_i - n_c - n_h = 0. \quad (3)$$

Where  $(\varphi, n_c, n_h)$  are normalized electrostatic potential; cool and hot electrons densities, respectively.  $n_i(u_i)$  are ion fluid normalized density (velocity) and  $\tau_{ic} = T_i / T_c$ . Where  $T_i$  (ion temperature) and  $T_c$  (cool electrons temperature)[20].

$n_c$  and  $n_h$  are given as[11],

$$n_j = \frac{N_{j0}}{N_{e0}} \left( 1 - \frac{\beta_j \varphi}{\kappa_j - 3/2} \right)^{-(\kappa_j - 1/2)}, \quad (4)$$

with  $j=c$  for cool electron species while  $j=h$  for hot electron species and  $\beta_j = T_c/T_j$ . Where  $N_{j0}$  is the equilibrium density for the electron species and  $\kappa_j > 3/2$  is the spectral index.

Introducing the stretched coordinates (considering nonlinear wave interaction through head-on collision (HOC) process and following the standard extended Poincaré-Lighthill-Kuo (PLK) method) [21-24],

$$\begin{aligned} \xi &= \varepsilon (r - \lambda t - r_A) + \varepsilon^2 P_0(\eta, \tau) \\ &\quad + \varepsilon^3 P_1(\xi, \eta, \tau) + \dots, \\ \eta &= \varepsilon (r + \lambda t - r_B) + \varepsilon^2 Q_0(\xi, \tau) \\ &\quad + \varepsilon^3 Q_1(\xi, \eta, \tau) + \dots, \\ \tau &= \varepsilon^3 r, \end{aligned} \quad (5)$$

Where  $\xi$  and  $\eta$  denote the outward and inward solitons trajectories, respectively, with

$$\begin{aligned} n_i &= 1 + \varepsilon^2 n_1 + \varepsilon^3 n_2 + \dots, \\ u_i &= \varepsilon^2 u_1 + \varepsilon^3 u_2 + \dots, \\ \varphi &= \varepsilon^2 \varphi_1 + \varepsilon^3 \varphi_2 + \dots, \end{aligned} \quad (6)$$

in the plasma system (1)-(4). The coupled system relating first order perturbed components is

$$\lambda \left( -\frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta} \right) n_1 + \left( \frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta} \right) u_1 = 0, \quad (7)$$

$$\begin{aligned} \lambda \left( -\frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta} \right) u_1 + \left( \frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta} \right) \varphi_1 \\ + 3\tau_{ic} \left( \frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta} \right) n_1 = 0, \end{aligned} \quad (8)$$

$$n_1 = \delta_1 \varphi_1, \quad (9)$$

where

$$\delta_1 = \sum_{j=c,h} \frac{N_{j0}}{N_{e0}} \frac{(\kappa_j - 1/2)}{\beta_j (\kappa_j - 3/2)}.$$

Solving system of Eq. (7)-(9), one obtains

$$n_1 = \delta_1 [\Phi_1(\xi, \tau) + \Phi_2(\eta, \tau)], \quad (10)$$

$$u_1 = \lambda \delta_1 [\Phi_1(\xi, \tau) - \Phi_2(\eta, \tau)], \quad (11)$$

where  $\lambda$  equals

$$\lambda^2 = 3\tau_{ic} + \frac{1}{\delta_1}.$$

Proceeding to higher order, results in

$$\begin{aligned} \frac{\partial^2 u_3}{\partial \xi \partial \eta} = & -\frac{\lambda \delta_1}{2} \frac{\partial}{\partial \xi} \left( \frac{\partial \Phi_1}{\partial \tau} + a \Phi_1 \frac{\partial \Phi_1}{\partial \xi} \right. \\ & \left. + b \frac{\partial^3 \Phi_1}{\partial \xi^3} + \frac{m}{2\tau} \Phi_1 \right) \\ & + \frac{\lambda \delta_1}{2} \frac{\partial}{\partial \eta} \left( \frac{\partial \Phi_2}{\partial \tau} + a \Phi_2 \frac{\partial \Phi_2}{\partial \eta} \right. \\ & \left. + b \frac{\partial^3 \Phi_2}{\partial \eta^3} + \frac{m}{2\tau} \Phi_2 \right) \\ & + \lambda \delta_1 \left( -\frac{\partial P_0}{\partial \eta} + c \Phi_2 \right) \frac{\partial^2 \Phi_1}{\partial \xi^2} \\ & - \lambda \delta_1 \left( -\frac{\partial Q_0}{\partial \xi} + c \Phi_1 \right) \frac{\partial^2 \Phi_1}{\partial \eta^2}, \end{aligned} \quad (12)$$

or simply,

$$\begin{aligned} u_3 = & -\frac{\lambda \delta_1}{2} \int \left( \frac{\partial \Phi_1}{\partial \tau} + a \Phi_1 \frac{\partial \Phi_1}{\partial \xi} \right. \\ & \left. + b \frac{\partial^3 \Phi_1}{\partial \xi^3} + \frac{m}{2\tau} \Phi_1 \right) d\eta \\ & + \frac{\lambda \delta_1}{2} \int \left( \frac{\partial \Phi_2}{\partial \tau} + a \Phi_2 \frac{\partial \Phi_2}{\partial \eta} \right. \\ & \left. + b \frac{\partial^3 \Phi_2}{\partial \eta^3} + \frac{m}{2\tau} \Phi_2 \right) d\xi \\ & + \lambda \delta_1 \int \left( -\frac{\partial P_0}{\partial \eta} + c \Phi_2 \right) \frac{\partial^2 \Phi_1}{\partial \xi^2} d\eta d\xi \\ & - \lambda \delta_1 \int \left( -\frac{\partial Q_0}{\partial \xi} + c \Phi_1 \right) \frac{\partial^2 \Phi_1}{\partial \eta^2} d\eta d\xi, \end{aligned} \quad (13)$$

where

$$a = -\frac{1}{2\delta_1 (1 + 3\tau_{ic} \delta_1)} [2\delta_2 - 3\delta_1^2 (4\delta_1 \tau_{ic} + 1)],$$

$$b = \frac{1}{2\delta_1 (1 + 3\tau_{ic} \delta_1)},$$

and

$$c = \frac{\delta_1^2 + 2\delta_2}{4\delta_1(1 + 3\tau_{ic}\delta_1)},$$

where

$$\delta_2 = \sum_{j=c,h} \frac{N_{j0}}{N_{e0}} \frac{(\kappa_j^2 - 1/4)}{2\beta_j^2(\kappa_j - 3/2)^2}.$$

Spurious resonances (due to secularity) must be avoided. That's results in

$$\frac{\partial \Phi_1}{\partial \tau} + a \Phi_1 \frac{\partial \Phi_1}{\partial \xi} + b \frac{\partial^3 \Phi_1}{\partial \xi^3} + \frac{m}{2\tau} \Phi_1 = 0, \quad (14)$$

$$\frac{\partial \Phi_2}{\partial \tau} + a \Phi_2 \frac{\partial \Phi_2}{\partial \eta} + b \frac{\partial^3 \Phi_2}{\partial \eta^3} + \frac{m}{2\tau} \Phi_2 = 0, \quad (15)$$

with

$$\frac{\partial P_0}{\partial \eta} = c \Phi_2, \quad (16)$$

$$\frac{\partial Q_0}{\partial \xi} = c \Phi_1. \quad (17)$$

Two solitary wave solutions for the system (15)-(18), may be expressed as

$$\Phi_{1,2} = \Phi_{A,B} \left( \frac{\tau_{A,B}}{\tau} \right)^{2m/3} F, \quad (18)$$

where,

$$F = \operatorname{sech}^2 \left\{ \begin{array}{l} \left[ \left( \frac{a \Phi_{A,B}}{12b} \right)^{1/2} \left( \frac{\tau_{A,B}}{\tau} \right)^{m/3} \right. \\ \left. \left[ \zeta, \eta - \frac{1}{3} a \Phi_{A,B} \left( \frac{\tau_{A,B}}{\tau} \right)^{2m/3} \tau \right] \right] \end{array} \right\}$$

$\Phi_A(\Phi_B)$  is soliton  $A(B)$  amplitude at the initial position  $\tau_A(\tau_B)$  [24]. Collision induced phase changes can be calculated from Eqs. (17) and (18):

$$P_0, Q_0 = c \left( \frac{12b \Phi_{B,A}}{a} \right)^{1/2} \left( \frac{\tau_{B,A}}{\tau} \right)^{m/3} (s_{\eta,\zeta} - s_{\eta_0,\zeta_0}), \quad (19)$$

$$s_{\eta,\zeta} = \tanh \left[ \left( \frac{a \Phi_{B,A}}{12b} \right)^{1/2} \left( \frac{\tau_{B,A}}{\tau} \right)^{m/3} \left[ \eta, \zeta \right. \right. \\ \left. \left. - \frac{1}{3} a \Phi_{B,A} \left( \frac{\tau_{B,A}}{\tau} \right)^{2m/3} \tau \right] \right]$$

$$s_{\eta_0,\zeta_0} = \tanh \left[ \left( \frac{a \Phi_{B,A}}{12b} \right)^{1/2} \left( \frac{\tau_{B,A}}{\tau} \right)^{m/3} \left[ \eta_0, \zeta_0 \right. \right. \\ \left. \left. - \frac{1}{3} a \Phi_{B,A} \left( \frac{\tau_{B,A}}{\tau} \right)^{2m/3} \tau \right] \right]$$

where at  $t=0$ ,  $\zeta_0 = -\eta_0 = \varepsilon(r_B - r_A)$ . The estimated phase shifts  $\Delta_A$  and  $\Delta_B$  are [23, 24]:

$$\Delta_{A,B} = -\varepsilon^2 c \left( \frac{12b \Phi_{B,A}}{a} \right)^{1/2} \left( \frac{r_{B,A}}{r} \right)^{m/3} \\ \left[ \tanh \Gamma_{A,B} - \tanh \Theta_{A,B} \right], \quad (20)$$

where

$$\Gamma_{A,B} = \left( \frac{a \Phi_{B,A}}{12b} \right)^{1/2} \left( \frac{r_{B,A}}{r} \right)^{m/3} \left[ \begin{array}{l} \varepsilon(\pm 2\lambda t \pm r_A - r_B) \\ -\frac{1}{3} \varepsilon^3 a \Phi_{B,A} \left( \frac{r_{B,A}}{r} \right)^{2m/3} \end{array} r \right],$$

$$\Theta_{A,B} = \left( \frac{a \Phi_{B,A}}{12b} \right)^{1/2} \left( \frac{r_{B,A}}{r} \right)^{m/3} \left[ \begin{array}{l} \varepsilon(r_{A,B} - r_{B,A}) \\ -\frac{1}{3} \varepsilon^3 a \Phi_{B,A} \left( \frac{r_{B,A}}{r} \right)^{2m/3} \end{array} r \right],$$

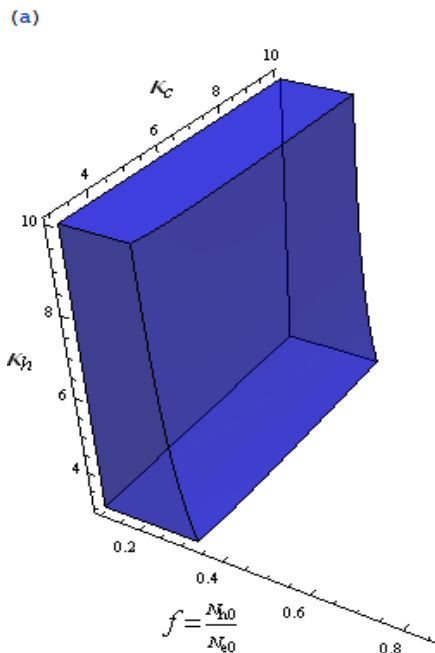
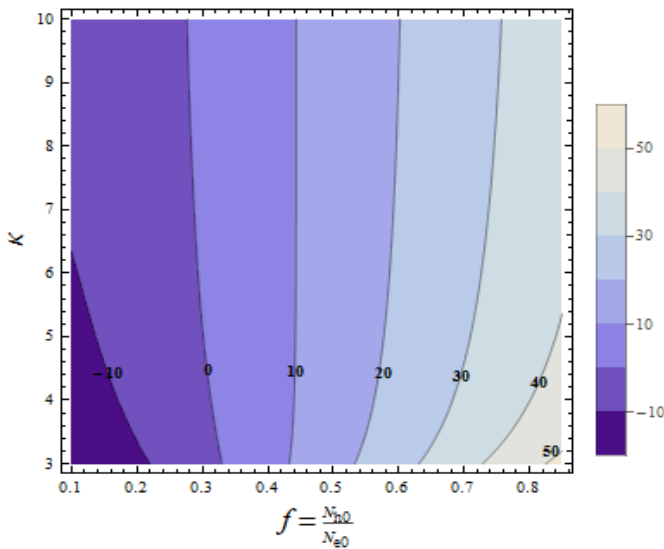
For  $r_B \gg r_A$  (solitons initial separation taking to be large), and  $t(\text{observation time}) \gg t_c(\text{collision time})$

$$\Delta_{A,B} = \mp 2\varepsilon^2 c \left( \frac{12b \Phi_{B,A}}{a} \right)^{1/2} \left( \frac{r_{B,A}}{r} \right)^{m/3}, \quad (21)$$

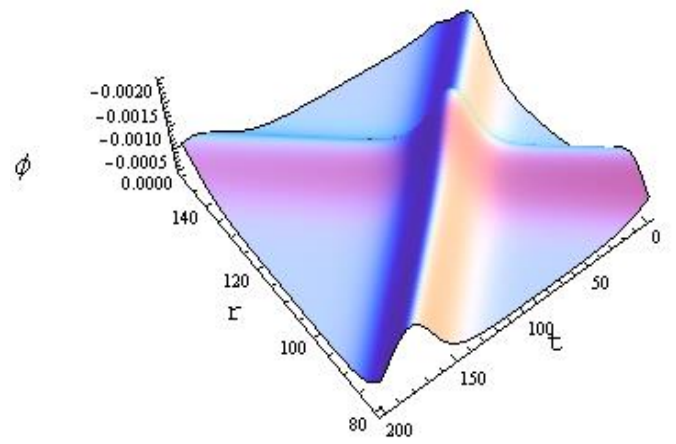
## RESULTS AND DISCUSSION

Choosing plasma parameters in Saturn's magnetosphere [8, 11, 20],  $\kappa_c \approx 1.8-3$ , modifies to 8-10 in the inner magnetosphere. Also,  $\kappa_h \approx 3-7$ . The nonlinear coefficient  $a$  of the KdV Eqs. (15) & (16), is investigated against the equilibrium density ratio  $f = N_{h0}/N_{c0}$  and the spectral index  $\kappa_j$ , where it shown that it will have both polarities and vanishes for certain values of the parameters  $f$  and  $\kappa_j$ , as depicted in Fig. 1. Furthermore, it is found the existence domain for positive solitary potential structures are greater than the negative one. For the case  $a=0$ , Eqs. (15) and (16) are not valid for describing the HOC process and one has to modify the expansions in (6), to account for such criticality, and derive a mKdV equations. For cylindrical IA waves, compressive ( $a$  is positive) and rarefactive ( $a$  is negative) IA solitary waves are identified, as depicted in Fig. 2. Also,

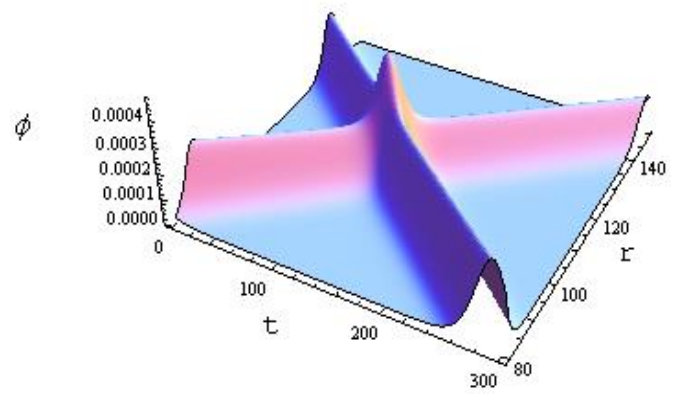
increasing equilibrium density ratio  $f$  decreases (increases) the amplitude of the compressive (rarefactive) potential structures at the impact point (such behavior is not shown for brevity). For rarefactive solitons  $|\Delta_A|$  increase while it decreases for compressive solitons as  $f$  is increased, as illustrated in Fig. 3-a. Also, it is found that as we move away from the Maxwellian limit ( $\kappa_j \rightarrow \infty$ ), the criticality in  $|\Delta_A|$



**Figure 1:** (Color Online) The nonlinearity coefficient,  $a$ ; (a) in the  $f - \kappa$  plane setting  $\kappa_c = \kappa_h = \kappa$ , and (b) in the parametric space  $(f, \kappa_c, \kappa_h)$ , for the case  $a \leq 0$ . Here,  $(T_c, T_h, T_i) \equiv (30, 10^3, 0.1)$ .



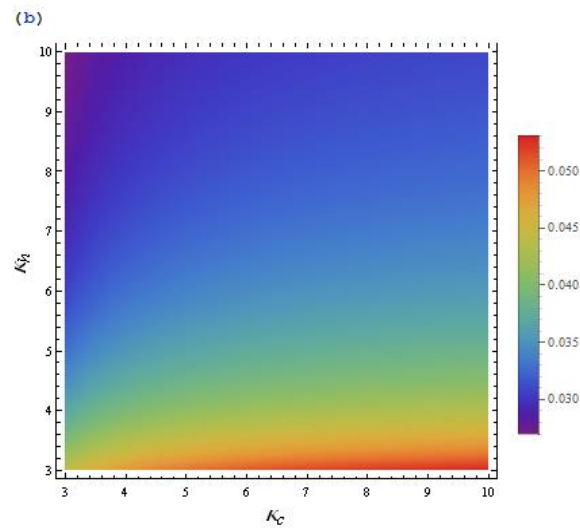
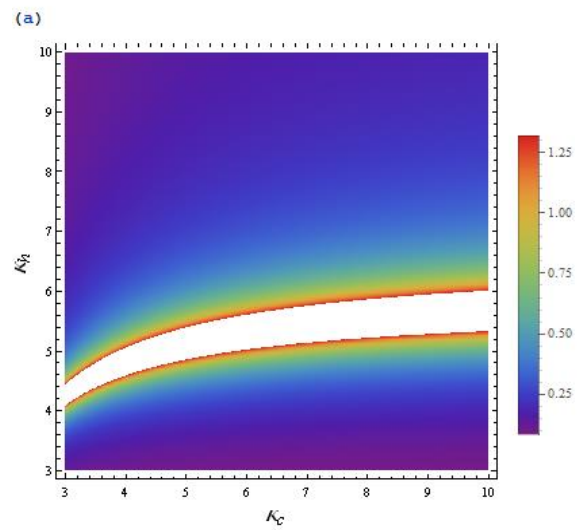
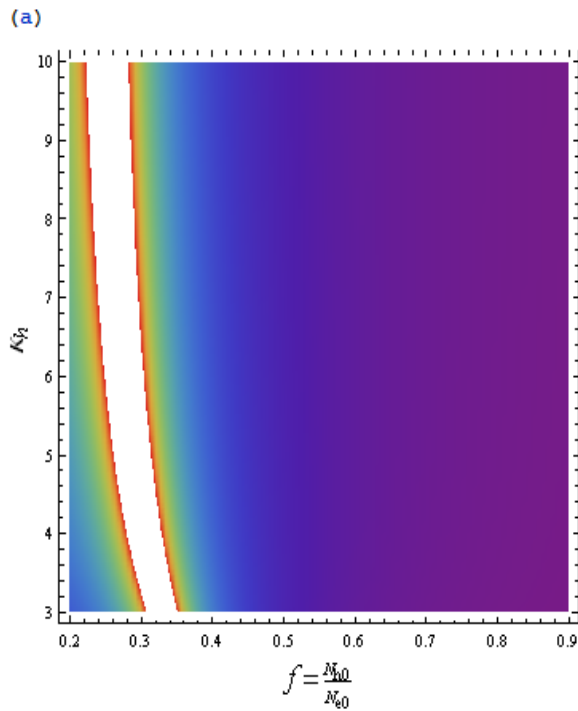
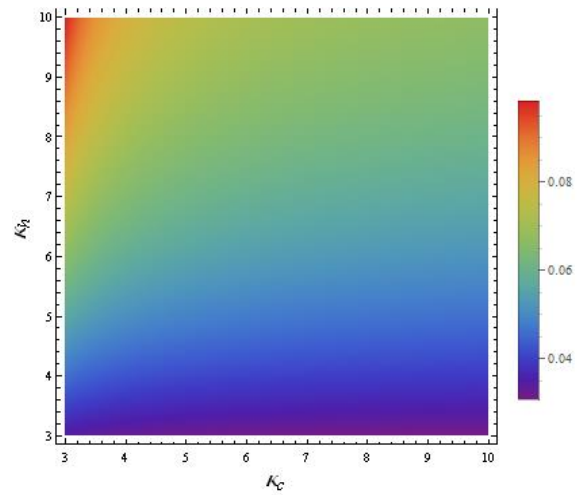
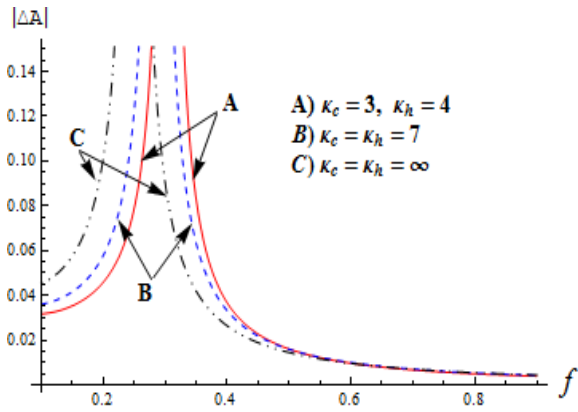
(a)



(b)

**Figure 2:** (Color Online) Cylindrical solitary waves collision in the  $r - t$  plane; (a)  $f = 0.25$  and (b)  $f = 0.7$  Here  $(T_c, T_h, T_i, k_c, k_h) \equiv (30, 10^3, 0.1, 3, 4)$ .

extended to higher ratios of the equilibrium density  $f$ , as shown in Fig. 3-a. Furthermore, it is found that the variations in the phase shift  $|\Delta_A|$  will be higher near the criticality domain, as depicted in Fig. (3-b). In the domain of negative potential structures, it is found that higher values of the phase shift corresponds to lower values of  $\kappa_c$  and higher values of  $\kappa_h$ , as shown in Fig. 4. While, for the domain of positive potential structure,  $|\Delta_A|$  will have a reverse effect. It should be pointed out that most of the variation in the phase shift will be concentrated at the criticality boarder lines, which shades doubt about the stability of the HOC dynamical process near the criticality domain.



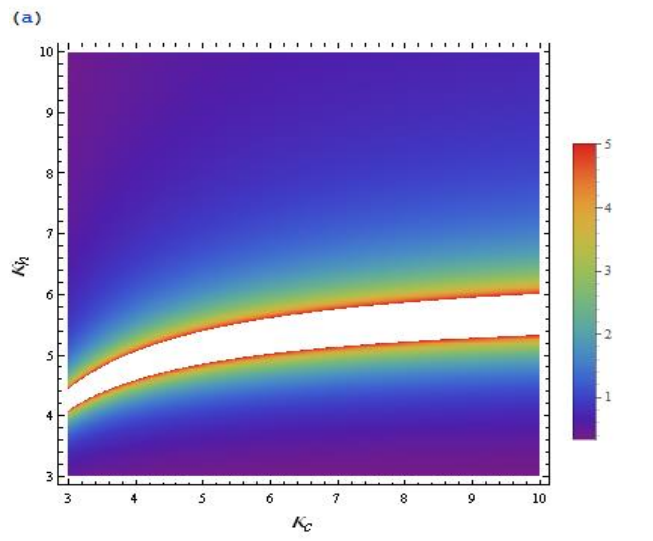
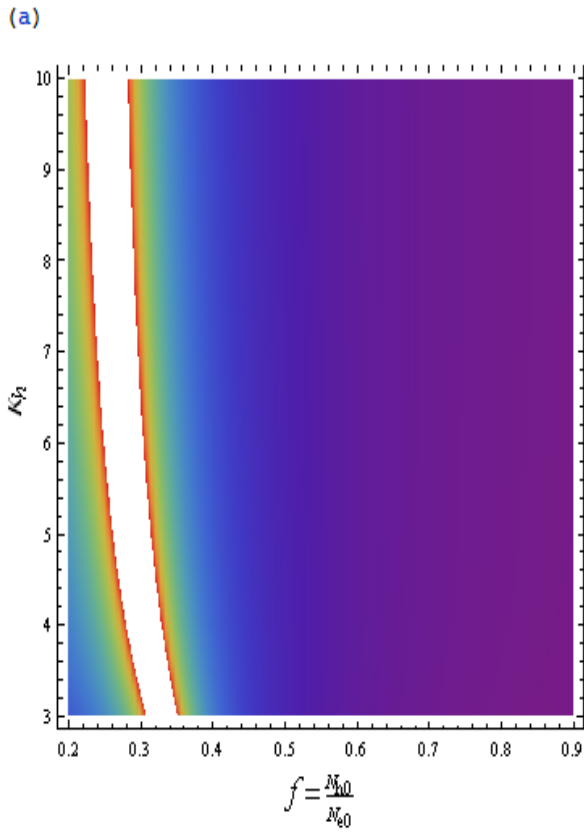
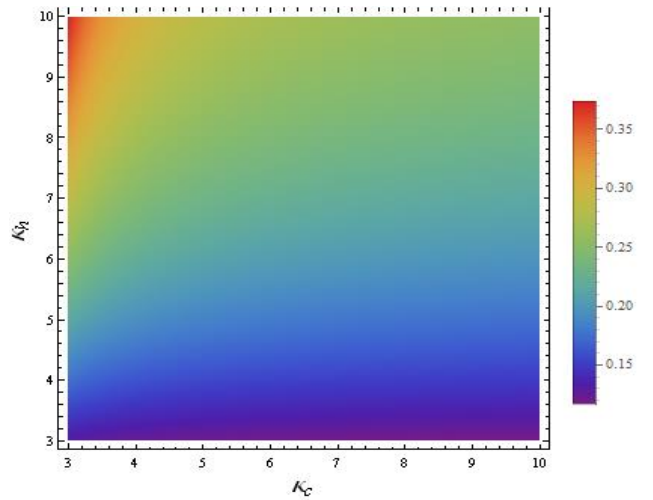
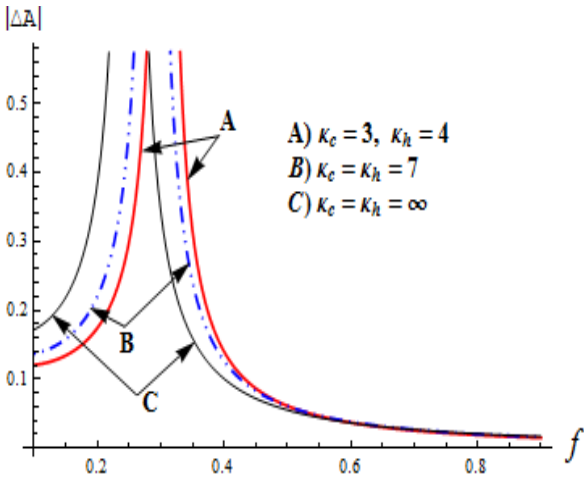
(b)

**Figure 3:** (Color Online) The phase shift  $|\Delta_A|$  of cylindrical IA solitons; (a) versus  $f$  and (b) in the  $f - \kappa_h$  plane, with  $\kappa_c = 3$ . Here,  $(T_c, T_h, T_i) \equiv (30, 10^3, 0.1)$ .

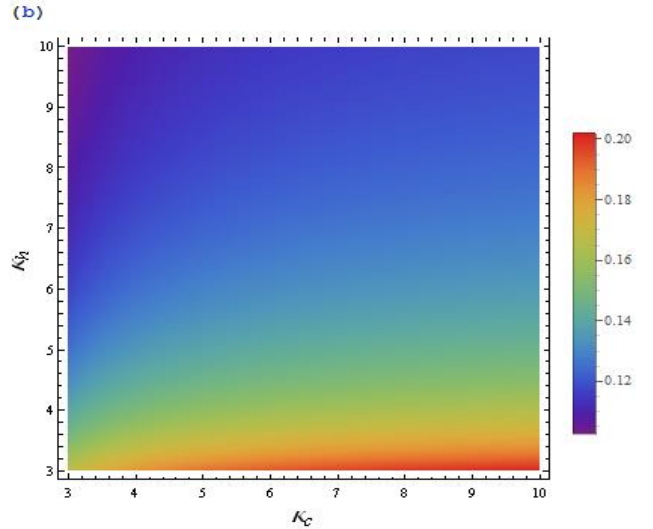
Spherical and cylindrical IA solitary waves HOC process, shown same qualitative behavior, as illustrated in Figs. 5 and 6. However, the values of  $|\Delta_A|$  for the case of spherical waves will be much higher than those for the cylindrical one. Also, the amplitude of the interacting electrostatic potentials at the impact point, will be higher for the spherical solitary waves than the cylindrical waves.

(c)

**Figure 4:** (Color Online) Variation of the cylindrical IA solitons phase shift  $|\Delta_A|$  in the  $\kappa_c - \kappa_h$  plane; (a)  $f = 0.2$ , (b)  $f = 0.3$ , and (c)  $f = 0.4$ . Here,  $(T_c, T_h, T_i) \equiv (30, 10^3, 0.1)$ .



(b)  
**Figure 5:** The phase shift  $|\Delta_A|$  of spherical IA solitons; (a) versus  $f$  and (b) in the  $f - \kappa_h$  plane, with  $\kappa_c = 3$ . Here,  $(T_c, T_h, T_i) \equiv (30, 10^3, 0.1)$ .



(c)  
**Figure 6:** (Color Online) Variation of the spherical IA solitons phase shift  $|\Delta_A|$  in the  $\kappa_c - \kappa_h$  plane; (a)  $f = 0.2$ , (b)  $f = 0.3$ , and (c)  $f = 0.4$ . Here,  $(T_c, T_h, T_i) \equiv (30, 10^3, 0.1)$ .

## CONCLUSIONS

To summarize, the dynamics of the HOC process of IA solitary waves in the considered plasma system are examined through the extended PLK method. Compressive and rarefactive IA waves analytical phase shifts due to HOC are derived. Role of ratio  $f$ , the superthermality spectral index  $\kappa_j$  (i.e.,  $j = c$  for cool and  $h$  for hot electrons) and  $m$  on the phase shifts are investigated. Both soliton polarities (compressive and rarefactive) for cylindrical/spherical IA waves exist in the current plasma model. Increasing equilibrium density ratio  $f$  decreases (increases) the amplitude of the compressive (rarefactive) potential structures at the impact point for non-planar IA solitary waves. Also, as we move away from the Maxwellian limit ( $\kappa_j \rightarrow \infty$ ), the criticality in the phase shift  $|\Delta_A|$  shifts to higher values of the equilibrium density  $f$ . Spherical and cylindrical solitary waves analytical phase shifts give same qualitative behavior. However, the amplitude of the spherical interacting electrostatic waves was higher than the cylindrical one, at the impact point.

The finding of the current investigation may be extended to plasma laboratory experiments investigating HOC dynamics of IA solitary waves where two independent groups of cold and hot superthermal electrons are present.

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