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Matrix Method of Calculation for Simulation of Distribution Electric Networks Of Medium Voltage

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Abstract

The article describes the matrix method for calculation of linearized medium voltage distribution networks of high dimensionality. The scope of the method is limited to the calculation of the steady-state regime of complex linear electric circuits in the presence of inductively coupled elements and ideal independent and controlled sources of voltage and current.

Keywords: distribution electric network, medium voltage network, simulation model, intelligent control, nodal potential method, electric circuit graph, infinite conductivity, inductively coupled elements.

INTRODUCTION

The problem of modeling complex electrical circuits of large dimension is becoming increasingly important and currently has many applied aspects. The article describes one of the stages of solving the problem of increasing the energy efficiency of electric energy transportation in distribution electric networks (DEN) 6-20 kV based on the introduction of regulating devices with intelligent control functions [1, 2]. This task requires the development and refinement of control algorithms for such regulating devices, as well as studies of their mutual influence on the electric network, which is impossible without the creation of simulation models for distribution electric networks with the ability to programmatically control the modeling process and programmatically change the parameters and operating modes of the model. When controlling regulators in real time, the simulation speed is a critical factor, so it is logical to use matrix methods for calculating linearized models of large-scale distribution electric networks. The linearized model of medium-voltage distribution electric networks can be described by air and cable lines of different cross-sections and lengths with the possibility of their parallel connection, by lowering transformers and regulating devices with intelligent control functions of the represented transformers with a complex voltage transmission coefficient for changing the amplitude and phase of the voltage.

Matrix methods for the calculation of linear electric circuits include a node-potential method, a mesh-current method, a method of sections [3], etc. Calculating complex electrical circuits, when the number of nodes is comparable to the number of independent circuits, the node-potential method is preferred [3]. It does not require the use of a sufficiently time-consuming operation to select a system of independent circuits (sections) and does not require the study of the graph of the circuit. However, if there are *z*-branches with zero intrinsic resistances in the electrical circuit, there is a need for equivalent transformations, for example, the transfer of voltage sources, or the introduction of additional current variables, as in the method of extended nodal equations [4].

Equivalent transformations can reduce the order of the system of equations to be solved, but they require the analysis of the entire graph, a circuit may be required to return back to the original chain and large electrical circuits substantially increase the computational cost. The proposed method allows us to calculate the matrix linear electric circuit without converting circuit diagrams, without constructing a matrix or matrices main circuit sections, without using special methods of forming the electrical circuit of the graph at an arbitrary numbering of nodes and its random sequence branches.

METHOD DESCRIPTION

The nodal method, in the presence of degenerate z-branches (with ideal voltage sources), consists in replacing the equilibrium equation for the node-point potentials to which the degenerate z-branch is attached, by another equation where the conductivity of this branch would not participate in the solution [5]. However, the direct application of this approach is difficult when there are branches in the electrical circuit that are controlled by the current of degenerate z-branches.

The essence of the proposed method is to express the currents of degenerate z-branches - z-branches with zero intrinsic resistance, then $z_{\emptyset}$-branches (such branches are not mutually defined with y-branches, their component equations express only voltages), through weighted sums of currents in the remaining branches and the use of the obtained dependences in the node-potential method.

To $z_{\emptyset}$-branches we will refer branches containing an ideal voltage source with zero internal resistance, (k-1) a branch of k branches that are windings of a k-winding ideal transformer, a branch with zero resistance and output circuits of controlled voltage sources. The remaining branches may contain a non-zero resistance (generally complex) or a current source (independent or controllable).

It is known that the currents of the tree branches of the graph of the electrical circuit can be expressed through the currents of the branches of the links [3] on the basis of the matrix of nodal connections:

$$\mathbf{I}_T = \mathbf{A}_T^{-1}\mathbf{A}_B\ \mathbf{I}_B, \qquad (1)$$

where $\mathbf{A}_T$ and $\mathbf{A}_B$ – blocks of the matrix of nodal connections, corresponding to the branches of the tree and the branches of the links. We shall note that the calculation of $\mathbf{A}_T^{-1}\mathbf{A}_B$ is equivalent to the calculation of the sections matrix or the matrix of principal contours. It is also known that the rank of the incidence matrix of an arbitrary graph is equal to the difference between the number of vertices and the number of connected components [6].

Thus, the square submatrix of the nodal join matrix is not degenerate if its columns correspond to the branches of the trees, and the lines correspond to the vertices incident to these branches (excluding one vertex for each connected component). (*)

Consequently, for an electrical circuit whose $z_Ø$-branches do not form contours (as is true for the vast majority of electrical circuits), an approach analogous to can be used to express the currents of the $z_Ø$-branches (1).

The graph describing the electrical circuit contains q nodes and p branches (and s branches are $z_Ø$-branches). In the proposed method, each $z_Ø$-branch must be associated with one of the nodes incident to it, and then the $z_Ø$-node. Note that the $z_Ø$-node must not be a basic (reference) node. The number of $z_Ø$-nodes coincides with the number of $z_Ø$-branches and is equal to s. There are several options for choosing the set of $z_Ø$-nodes. For this method all variants are equivalent.

The subgraph of the graph of the electric circuit, containing all the $z_Ø$-branches and the nodes incident to them, will be referred to below as the $z_Ø$-subgraph.

In the case when the connected components of the $z_Ø$-subgraph are trees, the number of $z_Ø$ -branches cannot be greater than (q – 1), and the number of nodes incident to $z_Ø$-branches cannot be less than (s + 1) [7]. Therefore, the choice of s nodes is possible even if the base node is incident to one of the $z_Ø$-branches.

Let us introduce the notation for sets of nodes and branches of the graph of an electric circuit:

V – set of all nodes of an electric circuit graph, $V = \{v_1, v_2, \ldots, v_q\}$;

B – set of all branches of the graph of the electrical circuit, $B = \{b_1, b_2, \ldots, b_p\}$;

Θ – set of $z_Ø$-nodes;

Ω – set of $z_Ø$-branches;

R – a set of branches containing nonzero resistance;

I – a set of branches containing independent sources of current;

II – a set of branches containing current controlled current sources **(CCCS);**

IU – The branches containing voltage controlled current sources **(VCCS);**

Ω, R, I, II and IU – - disjoint sets, their union $\Omega \cup R \cup I \cup II \cup IU$ forms a set B of all branches of the graph of the electrical circuit. Consideration of branches containing several elements of different types is beyond the scope of this article.

When describing the method, two rules for forming matrices were used:

Rule 1: let on the sets V and B some matrix is defined $\mathbf{Q}_{B;qxp}^{V}$ of the size (q × p), in which each line corresponds to one element from the set V, and to each column there corresponds one element from the set B. The matrix $\mathbf{Q}_{Y;qxp}^{X}$, where $X \subset V$, and $Y \subset B$, is formed from the matrix $\mathbf{Q}_{B;qxp}^{V}$ by zeroing out lines and columns not included in X and Y.

Rule 2: The matrix $\mathbf{Q}_{Y;mxn}^{X}$ is formed from a larger matrix$\mathbf{Q}_{B;qxp}^{V}$, where m, n, q and p – are cardinalities of sets X, Y, V and B, $X \subset V$ and $Y \subset B$, by exclusion from $\mathbf{Q}_{B;qxp}^{V}$ lines and columns corresponding to elements not in the set X and Y.

Let us introduce the following notations:

$\mathbf{A}_{(q-1)\,x\,p}$ – a nodal matrix [**Error! Reference source not found.**];

$\mathbf{0}_{(q-1)\,x\,1}$ and $\mathbf{1}_{p\,x\,p}$ – respectively, a zero and a unit matrix;

$\mathbf{U_0}^{V;(q-1)x1}$ or $\mathbf{U_0}^{;(q-1)x1}$ – a vector of values of node voltages of each of the nodes;

$\mathbf{U}^{B;px1}$ or $\mathbf{U}_{px1}$ – a branch voltage vector;

$\mathbf{E}^{B;px1}$ or $\mathbf{E}^{;px1}$ – the vector of the electromotive forces (EMF) values of voltage sources;

$\mathbf{I}^{B;px1}$or $\mathbf{I}^{;px1}$ –- a vector of branch currents;

$\mathbf{I}^{\Omega;px1}$, $\mathbf{I}^{R;px1}$, $\mathbf{I}^{I;px1}$, $\mathbf{I}^{II;px1}$, $\mathbf{I}^{IU;px1}$ – the vectors formed by rule 1, containing the values of the currents in Ω, R, I, II and IU branches: $I^{;px1} = I^{\Omega;px1} + I^{R;px1} + I^{I;px1} + I^{II;px1} + I^{IU;px1}$;

$\mathbf{Y}_{p\times p}$ – matrix of conductivity of branches, containing values of conductances of resistive branches and current coupling coefficients VCCS with $\mathbf{U}_{p\times 1}$;

$\mathbf{K}_{p\times p}$ – matrix of current coupling coefficients CCCS with control branch currents.

CALCULATION OF CURRENT DISTRIBUTION MATRIX

Let us construct the current distribution matrix, which allows us to represent the current of any branch in the form of a linear combination of branch currents from the set $R \cup I \cup IU$, i.e. the branches containing non-zero resistance, an independent current source or VCCS. To this end, we express the currents in the branches with CCCS and $z_{\varnothing}$-branches through the currents of the remaining branches.

Expression of current sources controlled by current. Let us denote the transformation matrix of the currents of branches that do not contain CCCS, $\mathbf{I}^{\Omega\cup R\cup I\cup IU}_{p\times 1}$, into currents $\mathbf{I}_{p\times 1}$, like $\tilde{\mathbf{K}}_{p\times p}$:

$$\mathbf{I}_{p\times 1} = \tilde{\mathbf{K}}_{p\times p}\mathbf{I}^{\Omega\cup R\cup I\cup IU}_{p\times 1}. \quad (2)$$

If branches with CCCS are not controlling for others CCCS, i.e. $K^{;pxp} = K^{B}_{\Omega\cup R\cup I\cup IU;pxp}$, and it's true $I^{II;pxp} = K^{;pxp} I^{\Omega\cup R\cup I\cup IU;px1}$, then $I^{;px1} = (1^{\Omega\cup R\cup I\cup IU;pxp} + K^{;pxp}) I^{\Omega\cup R\cup I\cup IU;px1}$, a $\tilde{\mathbf{K}}_{p\times p} = \left(\mathbf{1}^{\Omega\cup R\cup I\cup IU}_{p\times p} + \mathbf{K}_{p\times p}\right)$, where $1^{\Omega\cup R\cup I\cup IU;pxp}$, matrix obtained from the unit matrix $1^{;pxp}$ by zeroing (by rule 1) the lines and columns corresponding to the branches with CCCS.

If in the considered electric circuit containing w branches with CCCS, some branches with CCCS are controlling for other CCCS, the initial submatrix $K^{II}_{II\cup\Omega\cup R\cup I\cup IU;wxp}$ of matrix $K_{p\times p}$ It must be replaced with an equivalent, but not containing bonds between CCCS, i.e. **Error!**.

Expression of currents of $z_{\varnothing}$-branches through currents of branches containing non-zero resistance, independent current source or CCCS. Based on the 1st law of Kirchhoff

$$A_{(q-1)\times p} I^{I;px1} = 0_{(q-1)\times 1} \quad (3)$$

and expression (2) we get an expression $\mathbf{A}_{(q-1)\times p}\tilde{\mathbf{K}}_{p\times p}\mathbf{I}^{\Omega\cup R\cup I\cup IU}_{p\times 1} = \mathbf{0}_{(q-1)\times 1}$.

Let us denote $\mathbf{S}_{(q-1)\times p} = \mathbf{A}_{(q-1)\times p}\tilde{\mathbf{K}}_{p\times p}$ and write the equality (3) only for $z_{\varnothing}$-nodes, i.e.

$$S^{\Theta}_{\Omega;sxs} I^{\Omega;sx1} + S^{\Theta}_{R\cup I\cup IU;sxp} I^{R\cup I\cup IU;px1} = 0^{;sx1},$$

where $S^{\Theta}_{\Omega;sxs}$ and $S^{\Theta}_{R\cup I\cup IU;sxp}$ – matrices derived from the matrix $S_{(q-1)\times p}$ by rule 2. Since $S^{\Theta}_{\Omega;sxs} I^{\Omega;sx1} = -S^{\Theta}_{R\cup I\cup IU;sxp} I^{R\cup I\cup IU;px1}$, so $I^{\Omega;sx1} = -(S^{\Theta}_{\Omega;sxs})^{-1};\ S^{\Theta}_{R\cup I\cup IU;sxp} I^{R\cup I\cup IU;px1}$. Let us denote $T^{;sxp} = -(S^{\Theta}_{\Omega;sxs})^{-1};\ S^{\Theta}_{R\cup I\cup IU;sxp}$ – the matrix of the substitution of the currents of the $z_{\varnothing}$-branches by the currents of branches from the sets R, I, IU, as the result we'll get

$$\begin{aligned}\mathbf{I}^{\Omega\cup R\cup I\cup IU}_{p\times 1} &= \mathbf{I}^{\Omega}_{p\times 1} + \mathbf{I}^{R\cup I\cup IU}_{p\times 1}\\ &= \mathbf{T}_{p\times p}\mathbf{I}^{R\cup I\cup IU}_{p\times 1} + \mathbf{I}^{R\cup I\cup IU}_{p\times 1}\\ &= \left(\mathbf{T}_{p\times p} + \mathbf{1}^{R\cup I\cup IU}_{p\times p}\right)\mathbf{I}^{R\cup I\cup IU}_{p\times 1}\end{aligned} \quad (4)$$

where $T^{;pxp}$ – a matrix obtained from the matrix $T^{;sxp}$ by adding of zero lines corresponding to branches from the sets R, I, IU, a $1^{R\cup I\cup IU;pxp}$ – a matrix obtained from the identity matrix $1^{;pxp}$ by zeroing of lines and columns corresponding to $z_{\varnothing}$-branches and branches with (CCCS).

From (2) and (4) follows, that $\mathbf{I}_{p\times 1} = \tilde{\mathbf{K}}_{p\times p}\left(\mathbf{T}_{p\times p} + \mathbf{1}^{R\cup I\cup IU}_{p\times p}\right)\mathbf{I}^{R\cup I\cup IU}_{p\times 1}$. Let us denote matrix of the distribution of branch currents from sets R, I, IU as $\mathbf{D}_{p\times p} = \tilde{\mathbf{K}}_{p\times p}\left(\mathbf{T}_{p\times p} + \mathbf{1}^{R\cup I\cup IU}_{p\times p}\right)$, and write the final expression for representing the current in any branch through the currents in the branches from the sets R, I, IU:

$$\mathbf{I}_{p\times 1} = \mathbf{D}_{p\times p}\mathbf{I}^{R\cup I\cup IU}_{p\times 1}. \quad (5)$$

The resulting current distribution matrix D_{pxp} will be used in the implementation of the a node-potential method.

ELECTRIC CIRCUIT CALCULATION

On the basis of (5) we write the first law of Kirchhoff (3), as

$$\mathbf{A}_{(q-1)\times p}\mathbf{D}_{p\times p}\mathbf{I}^{R\cup I\cup IU}_{p\times 1} = \mathbf{0}_{(q-1)\times 1}. \quad (6)$$

Let's express currents in branches with resistances $\mathbf{I}^{R;p\text{x}1}$ and branches with VCCS $\mathbf{I}^{IU;p\text{x}1}$ through the potential differences at their ends $\mathbf{U}^{;p\text{x}1} = (\mathbf{A}^{;(q-1)\text{x}p})^{T;}\ \mathbf{U_0}^{;(q-1)\text{x}1}$, and conductivity values:

$$\mathbf{I}^{R\cup IU}_{p\times 1} = \mathbf{Y}_{p\times p}\left(\mathbf{A}_{(q-1)\times p}\right)^T \mathbf{U}_{0(q-1)\times 1}. \quad (7)$$

On the basis of (6) and (7) we will form a system of linear equations with respect to unknowns $\mathbf{U_0}^{;(q-1)\text{x}1}$:

$$\mathbf{A}_{(q-1)\times p}\mathbf{D}_{p\times p}\mathbf{Y}_{p\times p}\left(\mathbf{A}_{(q-1)\times p}\right)^T \mathbf{U}_{0(q-1)\times 1} =$$
$$= -\mathbf{A}_{(q-1)\times p}\mathbf{D}_{p\times p}\mathbf{I}^{I}_{p\times 1}$$

or

$$\mathbf{W}_{(q-1)\times(q-1)}\mathbf{U}_{0(q-1)\times 1} = \mathbf{Q}_{(q-1)\times 1}, \quad (8)$$

Where
$\mathbf{W}^{;(q-1)\text{x}(q-1)} = \mathbf{A}^{;(q-1)\text{x}p}\ \mathbf{D}^{;p\text{x}p}\ \mathbf{Y}^{;p\text{ x }p}\ (\mathbf{A}^{;(q-1)\text{x}p})^{T;}$ – the matrix of coefficients of a system of linear equations, but $\mathbf{Q}^{;(q-1)\text{x}1} = -\mathbf{A}^{;(q-1)\text{x}p}\ \mathbf{D}^{;p\text{x}p}\mathbf{I}^{I;p\text{x}1}$ – is the column vector of its right-hand side.

It can be shown that in the system of equations (8) the lines corresponding to $z_Ø$-nodes will be zero. Moreover, for each connected component of the $z_Ø$-subgraph in (8) there will be one equation based on the first Kirchhoff law for the cross-section that separates the given component in an isolated subgraph. After replacing the zero lines by equations constructed on the basis of the component equations of the corresponding $z_Ø$-branches, the system (8) can be solved by any known method. After solving the system of equations (8) and computing $\mathbf{U_0}^{;q\text{x}1}$ currents $\mathbf{I}^{;m\text{x}1}$ in the branches of the electrical circuit can be found on the basis of (5) and (7): $\mathbf{I}_{p\text{x}1} = \mathbf{D}^{;\,p\text{x}p}\ (\mathbf{Y}^{;p\text{ x }p}\ (\mathbf{A}^{;(q-1)\text{x}p})^{T;}\ \mathbf{U_0}^{;(q-1)\text{x}1} + \mathbf{I}^{I;p\text{x}1})$

DESCRIPTION OF MAIN ELEMENTS OF ELECTRIC CHAIN

Ideal voltage source. If $z_Ø$-branch b_i, incidental to nodes v_k and v_f, contains a voltage source with zero internal resistance and EMF E_i (directed from the node v_k to the node v_f), then the potential difference between the nodes v_k and v_f, is known for it, then $U_{0k} - U_{0f} = -E_i$. If the node v_f is selected as the $z_Ø$-node, then replacing by this expression of a line f in the system of equations (8) is equivalent to replacement in the matrix $\mathbf{W}_{(q-1)\text{x}(q-1)}$ line with number f on the zero line containing (+1) in the k- line and (–1) in the line f ($\mathbf{W}_{f,\,j=1\div(q-1)}$: $W_{f,k} = +1$, $W_{f,f} = -1$) and replacing of the element with the index f in the matrix $\mathbf{Q}_{(q-1)\text{x}1}$ and the value EMF of an ideal voltage source: $Q_f = -E_i$. The description of the branch with zero resistance is analogous to the description of an ideal voltage source with zero EMF.

The point of application of the potential. We will denote the set of points of application of the potential - nodes of the electrical circuit, connected to the basic node by an ideal independent voltage source or branch with zero resistance, as P, and the set of all other nodes of the electric circuit graph, as $\overline{P} = V \setminus P$. Potentials of nodes $\mathbf{U_0}^{P;u\text{x}1}$ with respect to the basis node are known, therefore the number of equations in (8) can be reduced to ($q - 1 - u$), where u – number of points of application potential. The abbreviated system of equations will look like:

$$\mathbf{W}^{\overline{P},\overline{P}}_{(q-1-u)\times(q-1-u)}\mathbf{U}^{\overline{P},}_{0(q-1-u)\times 1} = \mathbf{Q}^{\overline{P},}_{(q-1-u)\times 1} - \mathbf{W}^{\overline{P},P}_{(q-1-u)\times u}\mathbf{U}^{P,}_{0u\times 1}.$$

Voltage source controlled by voltage. A branch b_d, containing a voltage-controlled voltage source (VCVS) (Fig.1a), is a $z_Ø$-branch. In the matrix line $\mathbf{W}_{(q-1)\text{x}(q-1)}$ (8), corresponding to $z_Ø$- node v_g, the information that $K^{-1}(U_{0g} - U_{0f}) - (U_{0t} - U_{0k}) = 0$ is entered. This is equivalent to replacing in the matrix $\mathbf{W}_{(q-1)\text{x}(q-1)}$ a line with the number g by a line containing 4 nonzero elements:

$$\boldsymbol{W}_{g,\,j=1\div(q-1)}:\ W_{g,g} = +K^{-1},\ W_{g,f} = -K^{-1},\ W_{g,t} = +1,\ W_{g,k} = -1. \quad (9)$$

If $\beta = \infty$ ($\beta^{-1} = 0$) VCVS transforms into an ideal operational amplifier.

Voltage source controlled by current. A branch b_d, containing a current-controlled voltage source (CCVS) (Fig. 1b), is a $z_Ø$-branch. In the matrix line $\mathbf{W}_{(q-1)\text{x}(q-1)}$, corresponding to $z_Ø$-node v_g, the information that

$(U_{0g} - U_{0f})\cdot\alpha^{-1} - I_a = 0$ is entered.

Potential difference at the ends of the branch b_d: $U_d = (\mathbf{A}_{(q-1)\text{ x }p})^T\,\mathbf{U_0}^{;(q-1)\text{x}1}$. Current in the branch b_a can be found on the basis of (5) and (7): $I_\alpha = \mathbf{D}^{;\alpha}_{1\div p}\ \mathbf{Y}^{;p\text{ x }p}\ (\mathbf{A}^{;(q-1)\text{x}p})^{T;}\ \mathbf{U_0}^{;(q-1)\text{x}1} + \mathbf{D}^{;\alpha}_{1\div p}\mathbf{I}^{I;p\text{x}1}$

Hence,

$$\left[(\alpha^{-1}\mathrm{A}_{1\div(q-1),d})^T - D_{\alpha,1\div p}Y_{pxp}(\mathrm{A}_{(q-1),xp})^T\right]U_{0_{(q-1)x1}} = D_{\alpha,1\div p}I^{I}_{px1}$$

In order to replace in the system of equations (8) the equations corresponding to $z_Ø$-node v_g by the resulting expression we must replace in the matrix $\mathbf{Q}_{(q-1)\text{x}1}$ an element with index g by $Q_g = \mathbf{D}_{a,1\div p}\mathbf{I}^{I}_{p\times 1}$,and in the matrix $\mathbf{W}_{(q-1)\text{x}(q-1)}$ the line with index g must be replaced by

Error!

Current controlled current source. For CCCS represented in Figure 2a, the branch current b_d is ρ mal greater than the branch currant b_a. To introduce information into the system of equations (8) that $I_d - \rho I_a = 0$ in the matrix $\mathbf{K}_{p\times p}$ an element with indices (*d*, *a*) is assigned the value *ρ* ($K_{a}^{II;d} = \rho$).

Voltage controlled current source. VCCS in the branch b_d (Fig. 2*b*) is controlled by the branch b_a (possibly having zero conductivity, as in Fig. 2*b*). To introduce information into the system of equations (8) that $(U_{0t} - U_{0k})\eta - I_d = 0$, in the matrix $\mathbf{Y}_{p\times p}$ an element with indices (*d, a*) is assigned the value *η* ($Y_{d,a} = \eta$).

Inductively coupled branches. In this article, under the set of inductively coupled branches $\{t_1, \dots, t_k\}$ will be understood an ideal *k*-winding transformer with voltage transformation coefficients relative to one of the windings (for example, *k*-number) $\beta_1, \beta_2, \dots, \beta_{k-1}$. In such a transformer, the sum of the powers for all the windings is zero, that is $\sum_{i=1}^{k} I_{t_i} U_{t_i} = 0$, wherein $U_{t_i} = \beta_i U_{t_k}$, $\forall i = 1 \div (k-1)$.

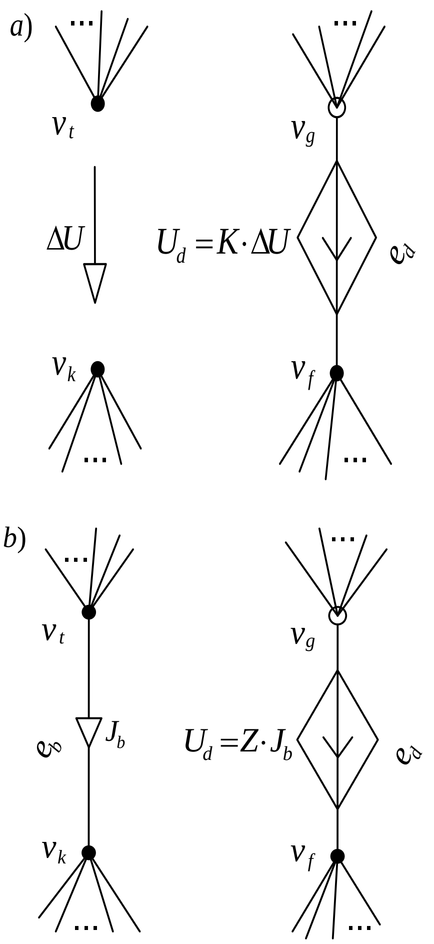

Figure 1:Controlled voltage sources: *a* –VCVS; *b* – CCVC

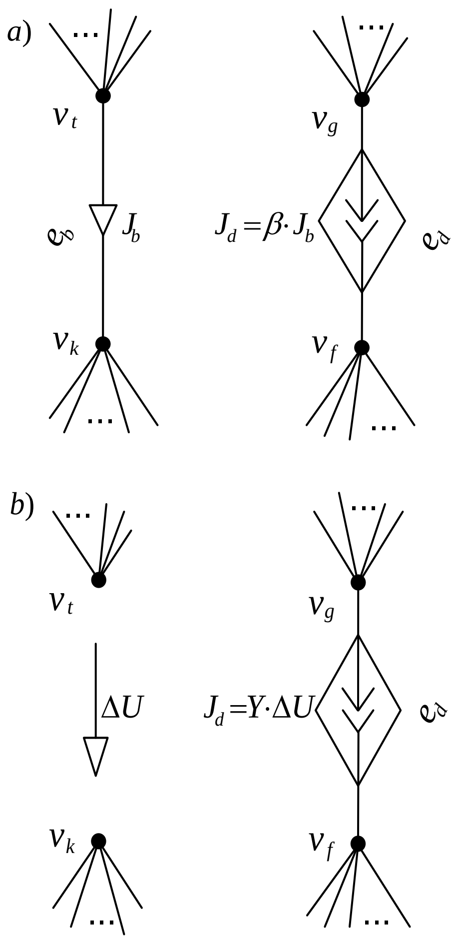

Figure 2: Controlled current sources: *a* – CCCS; *b* – VCCS

The ideal transformer can be represented by one CCCS and (*k* – 1) VCCS. Each of the branches with VCVS will be considered an $z_{Ø}$-branch. Let CCCS replaces *k*-number winding of the transformer, then: $-\sum_{i=1}^{k-1} \beta_i I_{t_i} = I_{t_k}$. To add to the system equations (8) the information on an ideal transformer, in the matrix $\mathbf{K}_{p\times p}$ elements with indices (t_k, $t_{i=1\div(k-1)}$) is assigned a value β_i: $K_{t_k;t_i} = \beta_i$, $i = 1\div(k-1)$, where t_k – is a number of the branch selected as CCCS, $t_1, \dots, t_{k-1}$ – the numbers of branches selected as VCVS, and in the matrix $\mathbf{W}_{(q-1)\times(q-1)}$ the lines, corresponding to each of $z_{Ø}$-nodes, are replaced in accordance with the formula (9).

EXAMPLE CALCULATION OF A LINEAR ELECTRIC CHAIN

Figure 3 shows the circuit diagram of the electrical circuit, and Table 1 shows the directions of the branches. Let $E_2 = E_{13} = 1$ V, $U_8 = 1$ V, $I_5 = I_7 = I_8 = I_{16} = 1$ A, $Z_6 = Z_{10} = Z_{12} = 1\ \Omega$, $\alpha_4 = 2\ \Omega$, $\rho_{14} = 1$, $\beta_{15} = 2$, $\eta_{17} = 1\ \Omega^{-1}$. Branches 1 and 9 are inductively coupled to branches 3 and 11 with a transmission ratio of 0.5. Numbers of $z_{Ø}$-branches: 1, 2, 4, 9, 13, 15. Numbers of $z_{\varnothing}$-nodes - 1, 3, 4, 5, 6, 7. Non-zero matrix elements: $\mathbf{K}_{17x17}$: $K_{3,1} = -2$, $K_{11,9} = -2$, $K_{14,2} = 1$; $\mathbf{Y}_{17x17}$: $Y_{6,6} = Y_{10,10} = Y_{12,12} = 1\ \Omega^{-1}$, $Y_{17,16} = -2\ \Omega^{-1}$; $\mathbf{I}^{I;17\text{x}1}$: $I^{I}_{5,1} = I^{I}_{7,1} = I^{I}_{8,1} = I^{I}_{16,1} = 1\,\text{A}$.

The obtained results fully coincided with the results of the calculation of the scheme shown in Fig. 3, in the programs *NI Multisim* and *Simulink*.

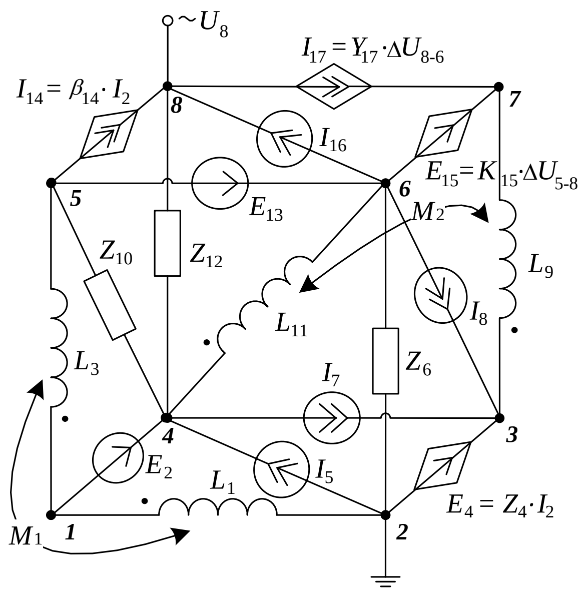

Figure 3: Example of an electrical circuit

Calculated values of potentials $\mathbf{U}_0$ (V), and currents $\mathbf{I}_{p\times 1}$ (A)

Node number	$\mathbf{U}_0$
1	-6
2	0
3	-4
4	-7
5	-3
6	-2
7	6
8	1

Branch number	$\mathbf{I}_{p\times 1}$
1	2
2	2
3	-4
4	-9
5	1
6	2
7	-1
8	-1
9	-7
10	-4
11	14
12	-8
13	-10
14	2
15	4
16	1
17	-3

Table 1.

Branch number	Node number	
1	1	2
2	1	4
3	1	5
4	2	3
5	2	4
6	2	6
7	3	4
8	3	6
9	3	7
10	4	5
11	4	6
12	4	8
13	5	6
14	5	8
15	6	7
16	6	8
17	7	8

In the following matrices $\mathbf{S}_{s\times p}$, $\mathbf{T}_{s\times p}$, and $\mathbf{D}_{p\times p}$ zero columns are omitted.

$\mathbf{S}_{s\times p}$ – matrix of summation of currents in the node:

Nod numberr	1	2	4	5	6	7	8	9	10	12	13	15	16	17
1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	-1	0	0	1	1	1	0	0	0	0	0	0
4	0	-1	0	-1	0	-1	0	-2	1	1	0	0	0	0
5	2	1	0	0	0	0	0	0	-1	0	1	0	0	0
6	0	0	0	0	-1	0	-1	2	0	0	-1	1	1	0
7	0	0	0	0	0	0	0	-1	0	0	0	-1	0	1

$\mathbf{T}_{s\times p}$ –- the matrix of the substitution of the currents of the$z_{\varnothing}$ - branches by the currents of branches from the sets *R*, *I*, *IU*:

Branch number	5	6	7	8	10	12	16	17
1	0.2	0.4	0.2	0.4	0.2	-0.2	-0.4	-0.4
2	0.2	0.4	0.2	0.4	0.2	-0.2	-0.4	-0.4
4	-0.6	-0.2	0.4	0.8	0.4	0.6	0.2	0.2
9	-0.6	-0.2	-0.6	-0.2	0.4	0.6	0.2	0.2
13	-0.6	-1.2	-0.6	-1.2	0.4	0.6	1.2	1.2
15	0.6	0.2	0.6	0.2	-0.4	-0.6	-0.2	0.8

$\mathbf{D}_{p\times p}$ – current distribution matrix:

Branch number	5	6	7	8	10	12	16	17
1	0.2	0.4	0.2	0.4	0.2	-0.2	-0.4	-0.4
2	0.2	0.4	0.2	0.4	0.2	-0.2	-0.4	-0.4
3	-0.4	-0.8	-0.4	-0.8	-0.4	0.4	0.8	0.8
4	-0.6	-0.2	0.4	0.8	0.4	0.6	0.2	0.2
5	1	0	0	0	0	0	0	0
6	0	1	0	0	0	0	0	0
7	0	0	1	0	0	0	0	0
8	0	0	0	1	0	0	0	0
9	-0.6	-0.2	-0.6	-0.2	0.4	0.6	0.2	0.2
10	0	0	0	0	1	0	0	0
11	1.2	0.4	1.2	0.4	-0.8	-1.2	-0.4	-0.4
12	0	0	0	0	0	1	0	0
13	-0.6	-1.2	-0.6	-1.2	0.4	0.6	1.2	1.2
14	0.2	0.4	0.2	0.4	0.2	-0.2	-0.4	-0.4
15	0.6	0.2	0.6	0.2	-0.4	-0.6	-0.2	0.8
16	0	0	0	0	0	0	1	0
17	0	0	0	0	0	0	0	1

Matrix $\mathrm{W}_{(q-u-1)\times(q-u-1)}$:

Node number

Node number	1	3	4	5	6	7
1	0.5	0	0	-1	0	0
3	0	-0.5	0	0.2	0.8	0
4	1	0	-1	0	0	0
5	0	0	0	1	-1	0
6	0	0	0	1	-0.5	0.5
7	0	-0.5	1	0	-1	0.5

Matrix $\mathrm{Q}_{(q-u-1)\times 1}$:

Node number	Current, A
1	0
3	-0.2
4	1
5	-1
6	1
7	0

CONCLUSIONS

The proposed method makes it possible to perform an exact calculation of nondegenerate branched linear electric circuits in steady state in the presence of branches with infinite conductivity that do not form contours without constructing a matrix of principal contours or a cross-section matrix without introducing additional current variables and transforming the circuit scheme. The electrical circuit can contain ideal voltage and current sources, inductively coupled elements, zero-resistance branches, four kinds of controlled voltage and current sources, and complex impedances.

From the point of view of computational costs, the proposed matrix method for calculating electrical distribution networks is intermediate between the node-potential method, which requires the calculation of an inverse matrix of size $(q-1)\times(q-1)$, and the method of extended nodal equations requiring the calculation of an inverse matrix of size $(q+s-1)\times(q+s-1)$. As for the operation of matrix inversion, the computational costs increase exponentially with the size of the matrix, the successive computation of two inverse matrices of sizes $(s-1)\times(s-1)$ and $(q-1)\times(q-1)$, as in the proposed method, more efficient application of the method of extended nodal equations

With the number of $z_{Ø}$-branches equal to the maximum, the proposed method becomes comparable in efficiency to the cross-section method, since in this case the $z_{Ø}$-subgraph of the electric circuit becomes a spanning tree, and all branches that are not $z_{Ø}$-branches become chords of the electric circuit graph.

Due to the fact that the method is matrix, it is easily formalized. The implementation of this method using Matlab, with the support of sparse matrices was tested in the study of intelligent medium voltage electric networks with various topological structures [8], including a hexagonal structure [9], and allowed us to calculate a simplified model of a uniformly distributed electrical network containing 10,200,000 nodes and 1,517,000 branches (including 12,800 ideal step-down transformers) in a time equal to 40 seconds.

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