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Vibtration Transimmition of Human Femuer Bone Due to Massage Device

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Abstract

Studies on the capacity to approximate the accepted natural frequency and response of the bone are currently in the limelight. Of late, investigators utilized resonant phenomena with mechanical loads in the vicinity of the natural frequency to augment bone apposition. Investigations targeted the parameters of natural frequency, deflection, displacement and acceleration. Experimental and numerical models developed in the past were saddled with issues related to CPU time and precise bone geometry (2D or 3D) acquirement. This study recommends an uncomplicated beam model that is a close replication of the bone. Three bone situations with varying load case studies were taken into account. These were 'concentrated, concentrate harmonic and distributed harmonic' loads. Through a series of tests conducted on bone boundary conditions, the simply supported bone model was revealed to be the most appropriate for bone fixation. The disparity of the response equation was probed in relation to bone mass, damping and stiffness. The outcomes for natural frequency were in agreement with those generated by other researchers.

Keywords: Vibration, bone, continuous beam, natural frequency near

INTRODUCTION

The therapeutic implementation of manual vibration on the human body has been in practice for hundreds of years. Its utilization for evaluating bone osteoporosis and determining Bone Mineral Density (BMD) eliminates the requirement for dual x-rays which subject the human body to unwarranted levels of radiation.

While the massaging process utilizing mechanical appliances can be deemed contemporary, vibration itself as a massaging procedure has been in existence for thousands of years. Many people are of the opinion that vibration therapy enhances their health by promoting blood flow, lessening muscle tension and reducing neurological excitability. However, as studies on this topic have been rather limited, more in-depth investigations ought to be in the pipeline.

The application of vibration analysis for bone materials emerged in 1949. In 1956 it was employed for investigations on the mechanical characteristics of bones [1]. Mather 1967 [2] linked the geometrical strength of the human femur bone to Young's modulus (E). This facilitated an approximation of Young's modulus of bone and paved the way for the determination of bone quality. Jurist 1970 [3] exploited a formula founded on transverse resonant frequency and length of the bone with the speed of sound and expanded it for computed tomography. He then proceeded with investigations on vibrational analysis by comparing the reaction of an osteoporotic ulna to that of a normal ulna. In 1975, (Spiegel and jurist) [4] assessed the capability of their model for forecasting the ulna resonant frequency of a subject whose ulna strength was gauged and mineral content ascertained. The efforts of Pugh, Rose et al. 1973 [5] focused on the cancellous bone which apparently has two resonances. Their investigation employed the Fitzgerald theory in order to study the data for best fit observed frequency.

(Steele, Zhou et al. 1988)[6] generated a 7-parameter lumped model of the ulna founded on the beam theory to act as a computational algorithm for the purpose of curve fitting the actual and conjured stiffness of the ulna. This is exhibited in Figure 1.

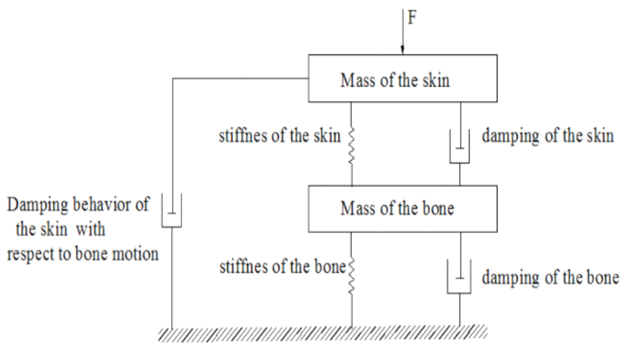

Figure 1: 7-Parameters model by (Steele, 1988)

Roberts and associates [1996][7] fashioned a 6-parameter mechanical analogue for the flexible monkey tibia to integrate stated conditions (refer to figure 2). The authentication of the 6-parameter model for long bone vibration in clamped end circumstances was realized through tests on aluminium rods with padding to duplicate skin.

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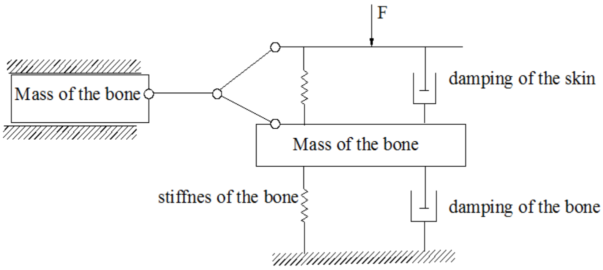

Figure 2: 6-Parameters model (Steele, 1996)

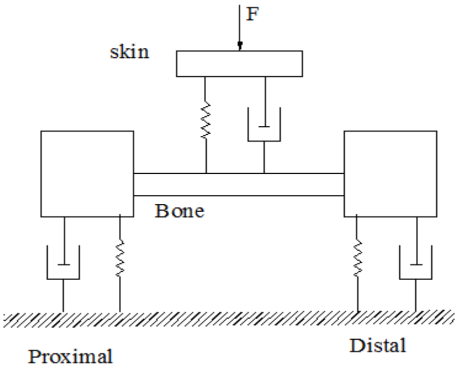

Figure 3: 12-Parameters model for the human tibia

The 12-parameter model, which is portrayed in Figure 3, permits the unhindered vibration of the proximal as well as the distal ends (knee and ankle joints) of the tibia. This model is designed to eradicate supporting hardware for the lower limb and limit the flaws related to the training extent of the technician (Callaghan, 2003)[8].

Earlier investigations utilized a restricted number of bone models for probing the influence of vibration on human bone as well as the utilization of vibration analysis for the forecasting and assessment of human bone behaviour. The objective of this study is to develop an innovative and uncomplicated continuous beam bone model to replace costly standard mass-damper spring models. This model will facilitate studies on a range of lateral vibrations in relation to the human body and provide passable outcomes from the induction of external vibration (for instance, vibration therapy) on the human bone.

Theoretical consideration

The recommended model is based on a straightforward continuous beam theory utilizing fundamental vibration rules and equations (refer to Figure 4). This model simplifies investigations on bone response to external vibrations and embraces an array of boundary circumstances and mode profiles.

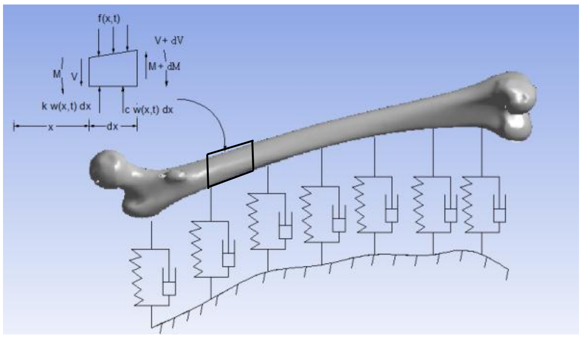

Figure 4: Proposed Model

the natural frequency[9] $\mathbf{f_n = h\alpha_n^{\,2}\sqrt{1+\frac{k}{EI\alpha_n^{\,2}}}}$ ... (1)

Assuming: $\boldsymbol{\alpha_n}^{\mathbf{4}} = \left(-\frac{\mathbf{k}}{\mathbf{EI}}+\frac{\mathbf{w_n}^{\mathbf{2}}}{\mathbf{h^2}}\right)$, $\quad \mathbf{h} = \sqrt{\frac{\mathbf{EI}}{\boldsymbol{\rho}\mathbf{A}}}$

where E, I, ρ and A are: Young's modulus (N/m^2), moment of inertia ($1/m^4$), density (kg/cm^3) and mean cross-sectional area (m^2) of the bone respectively.

ii- when forced vibrations are studied the equation of motion

$$\mathbf{EI\frac{\partial^4 w(x,t)}{\partial x^4}+\rho A\frac{\partial^2 w(x,t)}{\partial t^2}+C\frac{\partial w(x,t)}{\partial t}+kw(x,t) = F(x,t)} \quad ...(2)$$

Using the mode superposition principle and assuming

$$\mathbf{w(x,t)=\sum_{i=1}^{\infty} W_n(x)q_n\,(t)} \quad ...(3)$$

$$\mathbf{\frac{d^2q_n(t)}{dt^2}+\frac{c}{\rho A}\frac{dq_n(t)}{dt}+\left[\frac{k}{\rho A}+\omega_n^{\,2}\right]q_n(t)=\frac{Q_n(t)}{\rho A}} \quad \text{r} ...(4)$$

$$\mathbf{Q_n(t)=\int_0^l F(x,t)W_n(x)dx} \quad ...(5)$$

Through the multiplication of both sides of equation by $W_n(x)$ and integration from zero to l, the steady state solution is realized as:

$$\mathbf{q_n(t)=\frac{1}{\rho Awn\sqrt{1-\xi^2}}\int_0^t Q_n(\tau)e^{-\zeta wn(t-\tau)}sinwn\sqrt{1-\zeta^2}(t-\tau)d\tau} \quad ...(6)$$

where ζ is the damping ratio of the bone.

RESULTS AND DISCUSSIONS

Bone properties are a function of several parameters and possibly also a function between them. The bone properties utilized are displayed in Table 1.

Table 1: Mechanical properties data of human bone[10]

Property	**Magnitude**
Tensile Young modulus; E human femur (Et)	10.9-20.6 Gpa
Bending Young modulus (E)	9.82-15.7 Gpa
Density (Cancellous bone) (ρ)	0.43 or g/cm 3430 kg/m3
Density (Cortical bone) (ρ)	1.9 g/cm or 1900 kg/m3
Mass of bone (m)	7 kg
Femur means cross section area (A)	3.6 x10-4 m2
Bone mean length (l)	300 mm

It is likely that this continuous beam model will provide an excellent estimation as it utilizes distributed properties instead of lumped properties. The dependability of the recommended model was evaluated through the utilization of bone data (Table 1) in concurrence with the forms for the natural frequency equation (1). It is observed that the fundamental natural frequency is 278.65, where ref [11]is 252 with disparities of 26.6Hz and 9% for boundary conditions of simply supported. This is exhibited in Table 2. As shown in Table 3, there is a disparity in the approximation of the natural frequency for fixed-fixed, fixed-simply supported and fixed-free in relation to other boundary conditions. This disparity suggests that among the boundary conditions, replication of the human bone is best realized through simply supported beams. Notably, during tests, a comparatively different natural frequency is procured with the utilization of second and third modes of vibration.

Table 2: Natural frequency simply supported boundary conditions.

Type of fixation	**Equation of fn**	**Ref [11]**	**Fundamental Natural frequency f_1 (Hz)**
Simply supported-simply sup.	* $(n\pi)^2 . (\frac{EI}{\rho A L^4})^{0.5}$	252	278.6541833
Fixed-fixed	$\left(\frac{(2n+1)\pi}{2}\right)^2 . (\frac{EI}{\rho A L^4})^{0.5}$		626.9
Fixed-simply sup.	$\left(\frac{(4n+1)\pi}{4}\right)^2 . (\frac{EI}{\rho A L^4})^{0.5}$		435.3
Fixed-free	$\left(\frac{(2n-1)\pi}{2}\right)^2 . (\frac{EI}{\rho A L^4})^{0.5}$		69.6

*where n=1 for fundamental natural frequency

Investigations on bone response (continuous beam) involved three (simply supported) bone case studies which were acquired from a variety of vibration loads. These were ‘concentrate, harmonically concentrate and harmonically varied’. Bone response was also tested in relation to alterations in mass, damping and stiffness of bone.

i- Signifies the response of the bone with regard to a concentrated step load

The appliance of the response equation (6) for the three cases chosen resulted in the following closed form response equations:

$$\mathbf{w(x,t) = \alpha \pm \beta e^{-\zeta wnt} sin(\omega_d t) \pm \gamma e^{-\zeta wnt} cos(\omega_d t)} \ ...\ (7)$$

With the load equations $\mathbf{f(x,t) = Fo\delta(x-a)}$, n signifies the number of polynomial coefficients and w_d denotes the damped natural frequency.

Where α, β, and γ are the equation constants, the response at t=0 for various numbers of the series coefficient are provided in Table 5.

Table 3: Response parameters for case one

n	**α**	**β**	**γ**
3	0.010702	$\mathbf{-1.7822 \times 10^{-5}}$	-0.0107028
5	0.016054	$\mathbf{-2.6734 \times 10^{-5}}$	-0.01605426
7	0.021405	$\mathbf{-3.5645 \times 10^{-5}}$	-0.02140568
10	0.026757	$\mathbf{-4.4557 \times 10^{-5}}$	-0.02675710

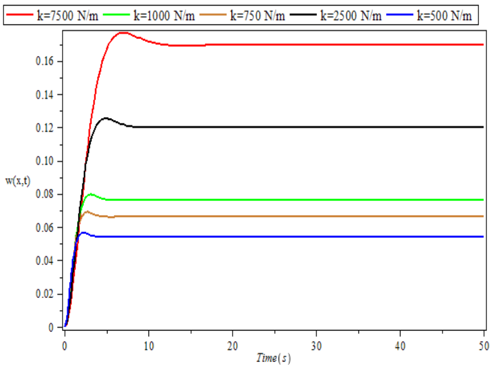

Figure 5: Bone response in relation to stiffness variations for a concentrated load case

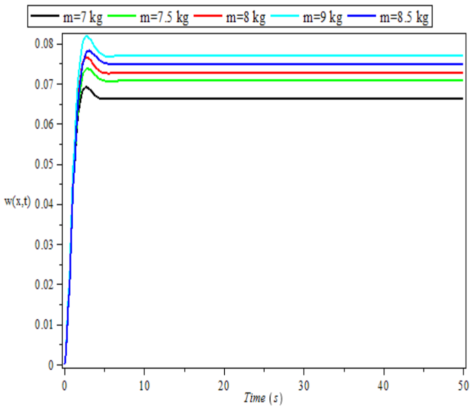

Figure 6: Bone response in relation to mass variations for a concentrated load case

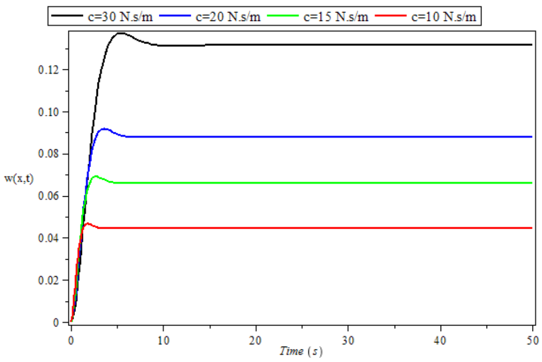

Figure 7: Bone response in relation to damping variations for a concentrated load case

ii- Case two

For this case, the closed form response equation (7) is:

$$\mathbf{w(x,t)} \\ = \mathbf{c_1 sin(\Omega t) \pm c_2 cos(\Omega t) \pm \beta e^{-\zeta wnt} sin(\omega_d t)} \\ \pm \mathbf{\gamma e^{-\zeta wnt} cos(\omega_d t)} \quad \text{... (8)}$$

With the load equations

$$\mathbf{f(x,t) = Fosin\Omega t\delta(x-\xi)Q_n = F_o sin\Omega t}$$

where C1, C2, β and γ are the equation constants as exhibited in table 4 and Ω represents the excitation frequency.

Table 4: Response parameters for case two load

Ω rad/s	C1	C2	β	γ
10	0.0389	- 0.000986	- 0.010972	0.000986
20	0.0522	-0.0036	-0.0299	0.00369
30	0.1279	-0.00357	-0.109098	0.0357
40	-0.0733	-0.41	0.0907	0.4104

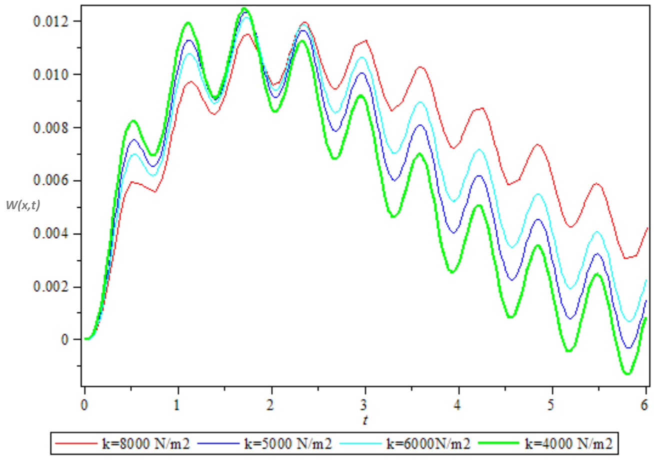

Figure 8: Bone response in relation to stiffness variations for a concentrated harmonic load case

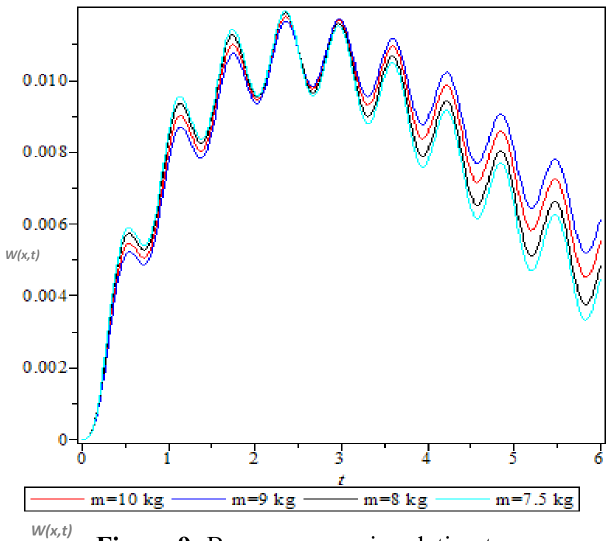

Figure 9: Bone response in relation to mass variation for concentrated

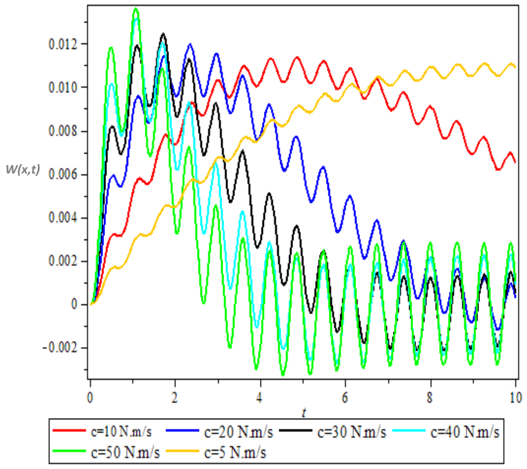

Figure 10: Bone response in relation to damping vibrations for a concentrated harmonic load case

iii-Case three

For this load case the response equations are:

$$\boldsymbol{w(x,t) = c_O \pm c_1 e^{-\zeta wnt} sin((\Omega + \omega_d)t) \pm c_2 e^{-\zeta wnt} cos((\Omega + \omega_d)(t) \pm c_3 e^{-\zeta wnt} sin((\Omega - \omega_d)t) \pm c_4 e^{-\zeta wnt} cos((\Omega - \omega_d)(t)} \ldots (9)$$

With the load $\boldsymbol{fosin\left(\frac{n\pi x}{l}\right) sinwt}$

where α, β and γ are the equation constants and the response is at t=0 for various numbers of series coefficient. This is displayed in Table 5.

Table 5: Response parameters for case three

Ω rad/s	C0	C1	C2	C3	C4
10	0.00045	-0.0062	0.00019	0.0112	-0.00065
20	0.0016	-0.0052	0.00013	0.018	-0.0018
30	0.016	-0.0042	0.00009	0.054	-0.016
40	0.013	-0.0037	0.00007	0.049	-0.013
50	0.001	-0.0032	0.00005	0.018	-0.0016

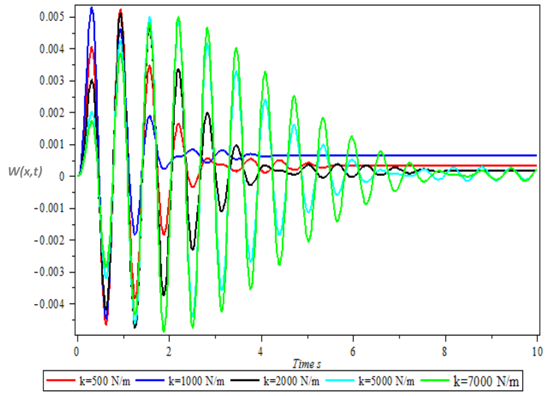

Figure 11: Bone response in relation to stiffness variations for a harmonically varied load case

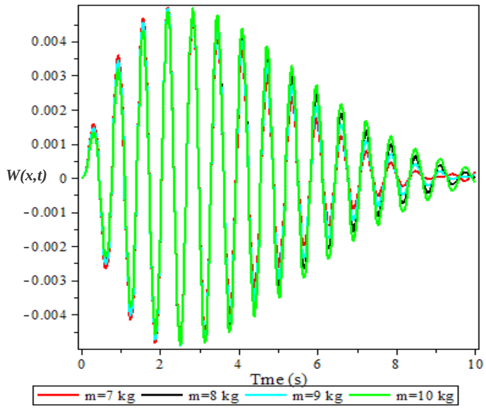

Figure 12: Bone response in relation to mass variations for a harmonically varied load case

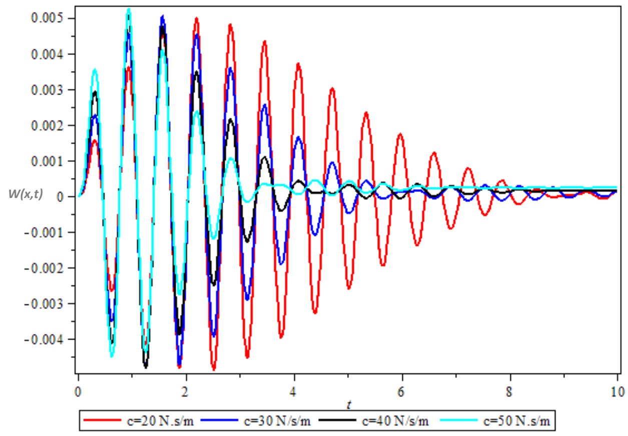

Figure 13: Bone response due to damping variation for harmonically varied load case

The responses of the bone are examined with alterations in mass, damping and stiffness. This is portrayed in figures 5 to 13.

CONCLUSIONS

The simply supported boundary condition was deemed the superior model for bone fixation in comparison to fixed-fixed, fixed-simply supported and fixed-free boundary conditions.

The absolute accuracies of the recommended continuous model in relation to ref [11] are 9%, 59 % 42% and 262% for simply supported, fixed-fixed, fixed-simply supported and fixed-free boundary conditions respectively.

An enhancement of bone responses was observed with an elevation in bone mass and stiffness, while a reduced level of response was apparent with a rise in the bone damping coefficient.

The recommended model has the capacity to precisely forecast the fundamental natural frequency and response of the bone through the application of a simple continuous bone beam theory.

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