International Journal of Applied Engineering Research ISSN 0973-4562 Volume 12, Number 24 (2017) pp. 15319-15325

Design and Development of Reliability Sampling Plans for Intermittent Testing Based on Type-I Censoring

D. Ramya [1] and S. Devaarul [2]

[1]Assistant Professor, Department of Statistics, P.S.G. College of Arts and Science, Coimbatore, India & a part time research scholar at R & D Centre, Bharathiar University, India.

Orcid Id: 0000-0002-6663-4220

[2] Assistant Professor, Department of Statistics, Government Arts College, Coimbatore, India.

Orcid Id: 0000-0001-5650-8020

Abstract

In this article reliability sampling plans are developed for intermittent test batches based on type-I censoring data. In case of intermittent testing the main problem faced by the quality control practitioner is the process average, which may deviate due to sporadic productions. Hence, one cannot assure exact reliability of the products while disposing the lots. To overcome this difficulty, minimum reliability as a guarantee is obtained before acceptance of the lot or test batches through the probability of rejection function. In this paper, reliability sampling plans are determined with an assurance of high reliability of the products for a two parameter Erlang distribution. These reliability sampling plans are formulated and designed in such a way that they are more reliable with respect to the acceptance of the products or batches. Designing and simulation procedures are given for the construction of necessary tables to facilitate easy selection and testing of products.

Keywords: Reliability, Erlang distribution, Type-I-Censoring, Intermittent production, Operating Ratio.

INTRODUCTION

Many authors have developed Reliability Sampling Plans which depend on the sample size and the acceptance constant without a guarantee on the reliability of the products for intermittent testing lots or batches. Customers may expect the reliability of the product while accepting the batches or lots after testing. If the reliability of the product is acquired during testing, then there will be a smooth sailing of lots to be accepted. Moreover, if the production is not continuous, then there may be variations in the process average and hence number of failures may be more in the lots and the customer may not be satisfied with such products. Thus, for such an intermittent process, minimum reliability should be acquired before the acceptance of the batch or process. Therefore Reliability Sampling Plans indexed through minimum reliability has been developed. Fertig and Mann [1] have discussed life test sampling plans for two parameter Weibull populations. Kantam et.al [2] has developed sampling plans based on log logistic model. Fang [3] presented the study of the hyper-Erlang distribution model and its applications in wireless networks and mobile computing systems. The two parameter Erlang distribution provides flexibility in the choice of the shape and scale parameter, a wide variety of lifetime data fit quite adequately to it. When the gamma distribution has an integral parameter α, it is known to be Erlang distribution. To acquire the maximum reliability (r_m), the probability of rejection function in terms of reliability is subjected to differentiation twice and equated to zero. The equation of normal to the tangent is obtained and is used as a measure of sharpness (r_n). Finally the operating reliability rate is determined which is the vital parameter to determine the parameters of the reliability sampling plans for the intermittent production process.

FORMULATION OF THE RELIABILITY SAMPLING PLANS

The minimum sample size necessary to ensure the required reliability of the products are obtained under the assumption that the life time variable follows two parameter Erlang distribution. The cumulative distribution function F(x) and probability density function f(x) of a two parameter Erlang distribution respectively are given by

$$F(t,k,\lambda) = 1 - \sum_{X=0}^{k-1} \frac{1}{X!} e^{-t\lambda} (t\lambda)^X \qquad (1)$$

$$f(t) = \frac{\lambda^k t^{k-1} e^{-\lambda t}}{(K-1)!} \qquad (2)$$
$$(k,\lambda) > 0, 0 < t < \infty$$

Where k is the shape parameter and λ is the scale parameter.

Suresh.K.K and Latha. M [4] have developed Bayesian single sampling plans for a gamma prior distribution. Devaarul .S [5, 6] designed and developed maximum acquired reliability sampling plans. Abdelkader [7] computed moments of order statistic from nonidentically distributed Erlang variables. Devaarul.S and Jemmy Joyce.V [8] have developed reliability sampling plans based on minimum angle technique. Willmot and Lin [9] presented a review of analytical and computational properties of the mixed Erlang distribution in the context of risk analysis. Muhammed Aslam and Chi-Hyuck Jun [10] designed time truncated acceptance sampling plans by using two-point approach. Muhammed Aslam et.al [11] developed a group sampling plan based on truncated life tests for gamma distribution. Gupta and Groll [12] studied gamma distribution in acceptance sampling based on life tests. Soundarrajan [13] has developed single sampling plans by attributes inspection through maximum allowable percent defectives. Lio et.al [14] have developed acceptance sampling plans for the Birnbaum-Saunders distribution for percentiles when the life test is truncated at a pre-specified time.

MEASURES OF RELIABILITY SAMPLING PLANS

Let r_m be the reliability of each component in the lot. Let n denote the number of units to be tested in time period t. Let S, $0 \leq S \leq n$, denote the number of successful units during the time period t. Thus r_m is the probability that any item tested during period t will result in a success, or it is reliable towards success such that $r_m = 1-F(x)$. In sampling plan literature, the probability of acceptance function of the process is given as

$$P_a(p) = \sum_{x=0}^{c} \binom{n}{x} p^x (1-p)^{n-x} \quad (3)$$

where p is the process average or fraction defective of the incoming lot or batch and x is the number of failure units in testing.

where $p = 1 - \sum_{x=0}^{k-1} \frac{1}{x!} e^{-t\lambda} (t\lambda)^x, (k,\lambda) > 0, 0 < t < \infty$ (4)

When S = n-c , is the minimum number of survival units required in the sample of testing before acceptance of the batch then the probability of rejection function if r_m is the reliability is given by

$$P(r) = \sum_{x=c+1}^{n} \binom{n}{x} p^x (1-p)^{n-x} \quad (5)$$

Where $1 - p = \sum_{x=0}^{k-1} \frac{1}{x!} e^{-t\lambda} (t\lambda)^x = r_m, (k,\lambda) > 0, 0 < t < \infty$ (6)

And S = n - c

Equation (5) can be rewritten as,

$$P(r) = 1 - \sum_{x=0}^{c} \binom{n}{x} p^x (1-p)^{n-x} \quad (7)$$

Formulation:

Let

n= sample size.

t = Life time random variable.

S = Successful units during testing.

S* = Minimum Number of Successful units required during testing.

ALGORITHM FOR SENTENCING A BATCH AFTER TYPE – I CENSORING

Step 1: Draw a random sample of size n and put them into life test for specified time t (type I censoring)

Step 2: Count the number of specimens successful in the test. Let it be S.

Step 3: If $S \geq S^*$ then accept the batch or lot.

Step 4: If $S < S^*$, then reject the batch.

DESIGNING THE RELIABILITY SAMPLING PLANS FOR INTERMITTENT TESTING

The probability of rejection function if r_m is reliability with type II probabilities is defined as

$$P(r) = 1 - \sum_{x=0}^{c} \frac{e^{-n(1-r_m)} \left(n(1-r_m)\right)^x}{x!} \quad (8)$$

If C is the acceptance number and n is the sample size then $S^* = n - c$ is the minimum number of survival required during the testing for acceptance of the batch. The tangent to the probability of rejection curve at the deflection point touches the x-axis at

$$r_t = \left(1 - \frac{c}{n}\right) - \frac{c!}{nc^c e^{-c}} \left(1 - \sum_{x=0}^{c} \frac{e^{-c} c^x}{x!}\right)$$

Equation of tangent passing through $\left(\left(1-\frac{c}{n}\right), 1 - \sum_{x=0}^{c} \frac{e^{-n(1-r_m)} \left(n(1-r_m)\right)^x}{x!}\right)$ with slope $\frac{n\left(n(1-r_m)\right)^c e^{-n(1-r_m)}}{c!}$ is given by the equation

$$f(r_t)-\left(1-\sum_{x=0}^{c}\frac{e^{-n(1-r_m)}(n(1-r_m))^x}{x!}\right)=\frac{n(n(1-r_m))^c e^{-n(1-r_m)}}{c!}\left(r_t-\left(1-\frac{c}{n}\right)\right) \quad (9)$$

The root of the above equation is obtained by solving $f(r_t)=0$

$$r_t-\left(1-\frac{c}{n}\right)=-\frac{c!}{n(n(1-r_m))^c e^{-n(1-r_m)}}\left(1-\sum_{x=0}^{c}\frac{e^{-n(1-r_m)}(n(1-r_m))^x}{x!}\right) \quad (10)$$

At the inflection point,

$$n(1-r_m)= c \quad (11)$$

therefore, the point at which the tangent touches the x-axis is given by

$$r_t=\left(1-\frac{c}{n}\right)-\frac{c!}{nc^c e^{-c}}\left(1-\sum_{x=0}^{c}\frac{e^{-c}(c)^x}{x!}\right) \quad (12)$$

The operating reliability rate is defined as

$$z_t=\frac{r_t}{r_m} \quad (13)$$

$$z_t=\frac{\left(1-\frac{c}{n}\right)-\frac{c!}{nc^c e^{-c}}\left(1-\sum_{x=0}^{c}\frac{e^{-c}(c)^x}{x!}\right)}{\left(1-\frac{c}{n}\right)} \quad (14)$$

$$z_t=\frac{r_m-\frac{(1-r_m)(c-1)!}{c^c e^{-c}}\left(1-\sum_{x=0}^{c}\frac{e^{-c}(c)^x}{x!}\right)}{r_m} \quad (15)$$

This z_t is purely a function of c having $r_m=1-\frac{c}{n}$ = S^* / n, as the reliability standard. The next parameter to be related is the measure of sharpness arguing similar to designing procedure of Soundararajan [12]. Hence r_t the point which touches x-axis of the tangent at the deflection point of probability of rejection curve can be used as measure of sharpness of inspection. A lot or process of quality with reliability r_m will be accepted and less than r_m will be rejected. By using equation (15), table (1) is constructed so that the minimum number of successful items required in type I censoring can be determined. Once the operating reliability rate and the parameter λ are known, one can determine minimum value of S^*, the successful units in the test and the sample size n.

Similarly, if c is the maximum allowable failure during type I censoring testing and n is the sample size then the number of successful items in the test is S^* = n – c, hence the normal to the probability of rejection curve at the deflection point touches the x-axis is given by

$$r_n=\left(1-\frac{c}{n}\right)+\frac{nc^c e^{-c}}{c!}\left(1-\sum_{x=0}^{c}\frac{e^{-c}c^x}{x!}\right)$$

Equation of the normal passing through $\left(\left(1-\frac{c}{n}\right),1-\sum_{x=0}^{c}\frac{e^{-n(1-r_m)}(n(1-r_m))^x}{x!}\right)$ with slope $-\frac{c!}{n(n(1-r_m))^c e^{-n(1-r_m)}}$ is given by the equation

$$f(r_n)-\left(1-\sum_{x=0}^{c}\frac{e^{-n(1-r_m)}(n(1-r_m))^x}{x!}\right)=\frac{-c!}{n(n(1-r_m))^c e^{-n(1-r_m)}}\left(r_n-\left(1-\frac{c}{n}\right)\right) \quad (16)$$

The root of the above equation is obtained by equating $f(r_n)=0$

$$r_n-\left(1-\frac{c}{n}\right)=\frac{n(n(1-r_m))^c e^{-n(1-r_m)}}{c!}\left(1-\sum_{x=0}^{c}\frac{e^{-n(1-r_m)}(n(1-r_m))^x}{x!}\right) \quad (17)$$

$$r_n=\left(1-\frac{c}{n}\right)+\frac{n(n(1-r_m))^c e^{-n(1-r_m)}}{c!}\left(1-\sum_{x=0}^{c}\frac{e^{-n(1-r_m)}(n(1-r_m))^x}{x!}\right) \quad (18)$$

At the inflection point, $n(1-r_m) = c$

therefore, the point at which the normal touches the x-axis is given by

$$r_n=\left(1-\frac{c}{n}\right)+\frac{nc^c e^{-c}}{c!}\left(1-\sum_{x=0}^{c}\frac{e^{-c}c^x}{x!}\right) \quad (19)$$

Let the operating reliability rate be

$$Z^*=\frac{r_m}{r_n} \quad (20)$$

$$Z^*=\frac{\left(1-\frac{c}{n}\right)}{\left(1-\frac{c}{n}\right)+\frac{nc^c e^{-c}}{c!}\left(1-\sum_{x=0}^{c}\frac{e^{-c}c^x}{x!}\right)} \quad (21)$$

$$Z^* = \frac{r_m}{r_m + \frac{c^c e^{-c}}{(1-r_m)(c-1)!}\left(1-\sum_{x=0}^{c}\frac{e^{-c}c^x}{x!}\right)} \quad (22)$$

By using equation (22), table (2) is constructed so that the minimum number of successful items required in type I censoring is determined. Once the operating reliability rate of normal point and the parameter λ is known one can determine the value of S^* and sample size n.

Differentiating equation (8) with respect to r_m twice and equating to zero, we get

$$\frac{d(P(r_m))}{dr_m} = \frac{n(n(1-r_m))^c e^{-n(1-r_m)}}{c!} \quad (23)$$

$$\frac{d^2(P(r_m))}{dr_m^{\;2}} = \frac{n(n(1-r_m))^c e^{-n(1-r_m)}}{c!}\frac{(c-n(1-r_m))}{r_m} = 0 \quad (24)$$

The reliability (r_m) acquired in the lot is obtained from equation (24),

$$r_m = 1 - \frac{c}{n} = \frac{n-c}{n} = \frac{s}{n} \quad (25)$$

$$n = \frac{c}{(1-r_m)} \quad (26)$$

The sample size n can be determined using equation (26) and the same is tabulated in table (3).

Interpretation: During life testing with type-I censoring if in the sample of n units the reliability $r \geq r_m$, then the batch is accepted otherwise it is rejected.

SIMULATION PROCEDURE

The parameters of the failure distribution are k and λ in case of Erlang distribution. Hence the cumulative failure rate values are simulated using MATLAB program for the known Scale and Shape parameters of the Erlang distribution. Twenty random values were generated and hence the corresponding F(x) values are obtained using equation (1). Thereafter r_m is obtained by using equation (4). For the shape parameter k=2, the different values of the sample size are obtained. Hence table (3) is constructed for easy selection of reliability sampling plans. By using equation (22), table (2) is constructed to determine the operating reliability rate at the normal point.

Step 1:

In the MATLAB editor window, go to the APPS menu and select the MuPAD notebook option. (A MuPAD notebook performs and documents symbolic calculations using the Symbolic Math Toolbox).

Step 2:

In the MuPAD notebook type the following script:

```
stats:: erlangRandom (2, 1.5)();
```

Step3:

When executed the call in the previous step, random numbers with shape parameter = 2 and scale parameter = 1.5 are generated

Step 4:

Along with the procedure in step 3, put $ k=1..20. So that the script will be

```
stats::erlangRandom (2, 1.5) ( ) $ k=1..20;
```

This returns a sequence of 20 random numbers each of which is called only once.

Step 5:

The simulated data are tabulated as tλ values in the tables and further calculations are made to determine the reliability sampling plans.

Table 1: Values of the Operating Rate (Z_t) for the maximum allowable failure (C) at tangent point

tλ	F(x)	r_m	C	Z_t
0.36	0.0512	0.9488	1	0.9613
0.56	0.1089	0.8911	2	0.9270
0.66	0.1420	0.8580	3	0.9131
0.89	0.2239	0.7761	4	0.8630
1.19	0.3338	0.6662	5	0.7807
1.38	0.4012	0.5988	6	0.7262
1.43	0.4185	0.5815	7	0.7231
1.54	0.4555	0.5445	8	0.6948
1.98	0.5886	0.4114	9	0.5023
2.00	0.5940	0.4060	10	0.5124

Table 2: Operating reliability rate for the maximum allowable failure (C) at normal point

tλ	F(x)	r_m	C	Z*
0.36	0.0512	0.9488	1	0.0337
0.56	0.1089	0.8911	2	0.0548
0.66	0.1420	0.8580	3	0.0601
0.89	0.2239	0.7761	4	0.0762
1.19	0.3338	0.6662	5	0.0887
1.38	0.4012	0.5988	6	0.0894
1.43	0.4185	0.5815	7	0.0856
1.54	0.4555	0.5445	8	0.0830
1.98	0.5886	0.4114	9	0.0777
2.00	0.5940	0.4060	10	0.0744

In table (2) for each value of C, Z^* is calculated using equation (22). Z^* is the operating reliability ratio at normal which is obtained by taking the ratio of r_m and r_n.

Table 3: Values of the sample size (n) for shape parameter k=2, given the reliability r_m and maximum allowable failures C.

tλ	F(x)	r_m	C=1	C=2	C=3	C=4	C=5	C=6	C=7	C=8	C=9	C=10
0.36	0.0512	0.9488	20	39	59	78	98	117	137	156	176	195
0.56	0.1089	0.8911	9	18	28	37	46	55	64	73	83	92
0.66	0.1420	0.8580	7	14	21	28	35	42	49	56	63	70
0.89	0.2239	0.7761	4	9	13	18	22	27	31	36	40	45
1.19	0.3338	0.6662	3	6	9	12	15	18	21	24	27	30
1.38	0.4012	0.5988	2	5	7	10	12	15	17	20	22	25
1.43	0.4185	0.5815	2	5	7	10	12	14	17	19	22	24
1.54	0.4555	0.5445	2	4	7	9	11	13	15	18	20	22
1.98	0.5886	0.4114	2	3	5	7	8	10	12	14	15	17
2.00	0.5940	0.4060	2	3	5	7	8	10	12	13	15	17
2.13	0.6280	0.3720	2	3	5	6	8	10	11	13	14	16
2.41	0.6937	0.3063	1	3	4	6	7	9	10	12	13	14
2.43	0.6980	0.3020	1	3	4	6	7	9	10	11	13	14
3.17	0.8248	0.1752	1	2	4	5	6	7	8	10	11	12
3.39	0.8520	0.1480	1	2	4	5	6	7	8	9	11	12
3.72	0.8856	0.1144	1	2	3	5	6	7	8	9	10	11
4.24	0.9245	0.0755	1	2	3	4	5	6	8	9	10	11
4.49	0.9384	0.0616	1	2	3	4	5	6	7	9	10	11
5.83	0.9799	0.0201	1	2	3	4	5	6	7	8	9	10
6.87	0.9918	0.0082	1	2	3	4	5	6	7	8	9	10

Table 4: Reliability values for shape parameter k=2, C=3, n=10 and the corresponding probability of rejection P(r)

tλ	F(x)	r_m	P(r)
0.36	0.0512	0.9488	0.0153
0.56	0.1089	0.8911	0.0974
0.66	0.1420	0.8580	0.1714
0.89	0.2239	0.7761	0.3876
1.19	0.3338	0.6662	0.6481
1.38	0.4012	0.5988	0.7637
1.43	0.4185	0.5815	0.7877
1.54	0.4555	0.5445	0.8325
1.88	0.5605	0.4395	0.9179
2.00	0.5940	0.4060	0.9353
2.13	0.6280	0.3720	0.9494
2.41	0.6937	0.3063	0.9689
2.43	0.6980	0.3020	0.9699
3.17	0.8248	0.1752	0.9887
3.39	0.8520	0.1480	0.9909
3.72	0.8856	0.1144	0.9930
4.24	0.9245	0.0755	0.9949
4.49	0.9384	0.0616	0.9954
5.83	0.9799	0.0201	0.9967

From table (4) it is found that whenever reliability of the lot decreases the probability of rejection is more. In other words, when the reliability of the product decreases then the probability of rejection curve shows an increase in the probability of rejection of a particular batch or lot. A Curve is drawn by taking Reliability of the lot in the x-axis and the probability of rejection in the y-axis by using the corresponding values in table (4). The point at which the tangent touches the x-axis is marked as (r_t) and the point at which the normal touches the x-axis is marked as (r_n) in this curve. The point r_m is the Reliability obtained which is known to be maximum value. Hence for the intermittent production process maximum reliability is obtained after type-I censoring test and is a satisfactory one for a customer. This guarantees the customer the required reliability before acceptance of the lots or batches.

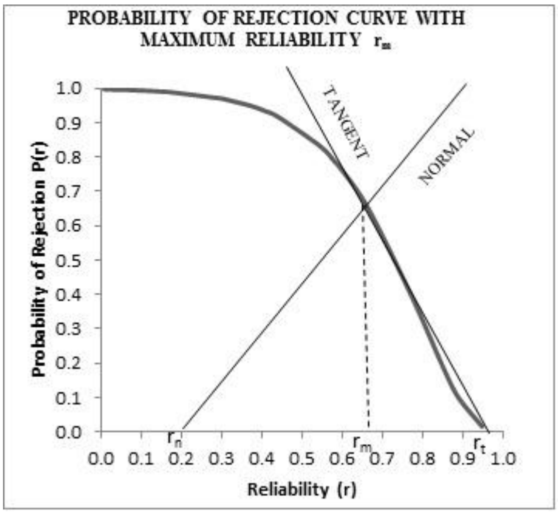

Figure 1: Probability of rejection Curve

ILLUSTRATION

The operating reliability rate expected by the customer is Z^*=0.0762 with type I censoring and the test time is 44.5 hours. Determine Reliability Sampling Plans if the process shows intermittent production with Erlangian scale 2.

Solution: It is given that the operating reliability rate is Z*=0.0762 and test duration is 44.5 hours. The scale parameter $\lambda = 2$, such that $t\lambda = 89$. One can find the parameters of the reliability sampling plans from table (2) and table (3).

Z^*, the operating reliability rate and the average test time $t\lambda$ are given, from table (2), it can be observed that the maximum allowable failure corresponding to these values is C=4.

Since for the same component the average test time is quoted as 89 hours, then from table (3), one can determine the required sample size. When $t\lambda$=0.89, the parameters c = 4 the sample size is n=18. Therefore, the minimum number of successful units required to accept the lot is S=14 with guaranteed reliability r_m = 77.6%

When Z^* and $t\lambda$ are given, one can determine the other parameters such as n, r_m and S^* from the tables and hence the suitable reliability sampling plans are determined. Also from table (1) the consumer is provided with the guaranteed reliability before the acceptance of the lot. So with this information the customer is satisfied with type I censoring for intermittent testing of lots.

CONCLUSION

In this article, Reliability Sampling Plans for intermittent production process has been developed by assuming two parameter Erlang distributions. It is observed that whenever the reliability value decreases the sample size increases, this pressurise the producer to maintain reliability of products and as such the consumer is also benefited. It is evident from the curve that as the reliability of the lot increases, the probability of rejection decreases. Therefore for an intermittent process, the new reliability sampling plans will be an appropriate one.

REFERENCES

[1] Abdelkader, Y.H., 2003, “Computing the moments of order statistics from non identically distributed Erlang variables.” Statistical Papers, 45, pp. 563-570.

[2] Devaarul, S., 2002, “Certain Studies relating to Mixed Sampling plans and Reliability based Sampling Plans”, Ph.D., Thesis, Department of Statistics, BharathiarUniversity, Coimbatore, Tamil Nadu, India.

[3] Devaarul, S., 2003, ”Maximum Acquired Reliability Sampling (MARS) plan”, Far East Journal of Theoretical Statistics, 10, No.1, pp 25-32.

[4] Devaarul, S and Jemmy Joyce, V., 2013, “Designing and Selection of Reliability Sampling Plans Based on Minimum Angle Technique”, International Journal of Mathematics and Computation, 20, No.3, pp. 60-65.

[5] Fang,Y., 2001, “Hyper-Erlang distribution model and its application in wireless mobile networks. Wireless Networks” , Vol.7, pp. 211–219.

[6] Fertig, K, W and Mann, N, R., 1980,”Life Test Sampling Plans for Two Parameter Weibull Populations”, Technometrics, Vol.22, pp. 165-177

[7] Gupta, S,S and Groll P, A., 1961,”Gamma Distribution in Acceptance Sampling based on Life Tests”, Journal of American Statistical Association, Vol.56, pp. 942-970.

[8] Kantam, R, R, L, Rosaiah, K, and Rao G.S., 2001, “Acceptance Sampling Based on Life Tests: Log-Logistic Models”, Journal of Applied Statistics, Vol. 28, pp. 121-128.

[9] Lio, Y.L, Tzong-Ru Tsai and Shuo-Jye Wu., 2010, “Acceptance Sampling Plans from Truncated Life Tests Based on the Birnbaum-Saunders Distribution for Percentiles”, Communications in Statistics – Simulation and Computation, 39:1, 119-136.

[10] Muhammed Aslam, Chi-Hyuck Jun., 2013, “Designing of Time Truncated Acceptance Sampling Plans by Using Two-Point Approach”, Electronic Journal of Applied Statistical Analysis Vol.6, Issue1, 18-31.

[11] Muhammed Aslam, Chi-Hyuck Jun and Ahmad. M., 2009, “A group sampling plan based on truncated life tests for gamma distribution”, Pakistan Journal of Statistics, Vol. 25:333-340

[12] Soundarrajan, V., 1975, “Maximum Allowable Per cent Defective (MAPD) Single Sampling Inspection by Attributes Plan”, Journal of Quality Technology, Vol.7, No.4, pp. 173-177.

[13] Suresh, K.K. and Latha, M., 2001, **“**Bayesian Single Sampling Plans for a Gamma Prior”, Economic Quality Control, Vol.16, No.1, pp. 93-107.

[14] Willmot, G. E. and Lin, X. S., 2011, **“**Risk modelling with the mixed Erlang distribution”. Applied Stochastic Models in Business and Industry, 27, 2-16.