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Lindley Approximation Generalized Compound Rayleigh Model under General Entropy Loss Function

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Abstract

In this paper we have obtained the Bayes estimates of all location parameter of Generalized Compound Rayleigh distribution assuming two scale parameters as known under the General Entropy loss function. Then by using Lindley approximation procedure we have obtained the Approximate Bayes estimate of Location parameter of Generalized Compound Rayleigh distribution under the General Entropy loss functions. The comparison is done for the Approximate Bayes estimators of model when prior specifications deviate from the true values.

Keywords: Lindley Approximation, Generalized Compound Rayleigh distribution, General Entropy loss function, Approximate Bayes estimate.

1 Introduction

The Generalized Compound Rayleigh Distribution is a special case of the three-parameter Burr type XII distribution .Mostert Roux, and Bekker (1999) considered it as a Gamma mixture of Rayleigh distribution and obtained the Compound Rayleigh model with unimodal hazard function. This unimodal hazard function is generalized and a flexible parametric model is thus constructed, which embeds the Compound Rayleigh model, by adding shape parameter. Bain and Engelhardt(1991) studied this distribution (also known as the Compound Weibull distribution (Dubey 1968) from a Poisson perspective. The Generalized Compound Rayleigh Distribution is a special case of the three-parameter Burr type XII distribution with probability density function (p.d.f.) with re-parameterized γ as $\frac{1}{\gamma}$

$$f(x;\alpha,\beta,\gamma)=\frac{\alpha}{\gamma}\beta^{\frac{1}{\gamma}}x^{(\alpha-1)}(\beta+x^{\alpha})^{-(\gamma+1)}\ ;\ x,\alpha,\beta,\gamma>0 \quad (1.1)$$

with Probability Distribution Function

$$\mathrm{F(x)}=1-(1-\beta x^{\alpha})^{-\frac{1}{\gamma}}\ ;\quad x,\alpha,\beta,\gamma>0 \quad (1.2)$$

The most widely used loss function in estimation problems is quadratic loss function given as $L(\hat{\theta},\theta)=k(\hat{\theta}-\theta)^2$ where $\hat{\theta}$ is the estimate of θ, the loss function is called quadratic weighed loss function if k=1, we have

$$L(\hat{\theta},\theta)=(\hat{\theta}-\theta)^2 \quad (1.3)$$

Known as squared error loss function (SELF). This loss function is symmetrical because it associates the equal importance to the losses due to overestimation and under estimation with equal magnitudes however in some estimation problems such an assumption may be inappropriate. Overestimation may be more serious than underestimation or Vice-versa Ferguson(1985). Canfield (1970), Basu and Ebrahimi(1991).

In many practical situations, it appears to be more realistic to express the loss in terms of the ration $\frac{\hat{\theta}}{\theta}$. In this case Calabria and Pulcini (1994) points out that a useful asymmetric loss function is the Entropy loss

$$L(\delta)\ \alpha\ [\delta^{P}-p\ log_e(\delta)-1\quad ;\text{Where }\ \delta=\frac{\hat{\theta}}{\theta}; \quad (1.4)$$

And whose minimum occurs at $\hat{\theta}=\theta$ where p > 0, a positive error $(\hat{\theta}>\theta)$ causes more serious consequences that a negative error and vice-versa. For small |p| value the function is almost symmetric, when both $\hat{\theta}$ and θ are measured in a logarithmic scale and approximately.

$$L(\delta)=b[\delta-log_e(\delta)-1];\quad b>0;\quad \text{Where }\ \delta=\frac{\hat{\theta}}{\theta}; \quad (1.5)$$

2 The Estimators

Let $x_1\le x_2\le\ \dots\dots\dots\le x_n$ be the n failures in complete sample case. The likelihood function is given by

$$L(\underline{x}\,|\alpha,\beta,\gamma)=\prod_{j=1}^{n}f\left(x_j,\alpha,\beta,\gamma\right) \quad (2.1)$$

$$L(\underline{x}\,|\alpha,\beta,\gamma)\ =\left(\frac{\alpha}{\gamma}\right)^{n}Ue^{-T}/_{\gamma} \quad (2.2)$$

Where

$$T=\sum_{j=1}^{n}log\left[1+\frac{{x_j}^{\alpha}}{\beta}\right]\qquad\text{and}\qquad U=\prod_{j=1}^{n}\frac{{x_j}^{\alpha-1}}{\beta+{x_j}^{\alpha}}$$

from equation(2 .2.1), the log likelihood function is

$$Log\ L=\ n\ log\alpha+\frac{n}{\gamma}log\beta-n\ log\gamma+(\alpha-1)\sum_{j=1}^{n}logx_j-\left(\frac{1}{\gamma}+\ 1\right)\sum_{j=1}^{n}log\left(\beta+{x_j}^{\alpha}\right) \quad (2.3)$$

and differentiation of equation(2.3) with respect to α,β and γ yields respectively the following equations as

$$\frac{\partial \, Log \, L}{d\alpha} = \frac{n}{\alpha} + \sum_{j=1}^{n} log x_j - \left(\frac{1}{\gamma}+1\right)\sum_{j=1}^{n}\frac{x_j{}^{\alpha} log x_j}{\beta + x_j{}^{\alpha}} \quad (2.4)$$

$$\frac{\partial \, Log \, L}{\partial\beta} = \frac{n}{\gamma\beta} - (\gamma+1)\sum_{j=1}^{n}\frac{1}{\beta + x_j{}^{\alpha}} \quad (2.5)$$

can also be written as

$$\frac{\partial \, Log \, L}{\partial\beta} = -\sum_{j=1}^{n}\frac{1}{\beta + x_j{}^{\alpha}} + \frac{1}{\gamma}\sum_{j=1}^{n}\frac{x_j{}^{\alpha}}{\beta(\beta + x_j{}^{\alpha})} \quad (2.6)$$

$$\frac{\partial \, Log \, L}{\partial\gamma} = \frac{-n\log\beta}{\gamma^2} - \frac{n}{\gamma} + \frac{n}{\gamma^2}\sum_{j=1}^{n}\log(\beta + x_j{}^{\alpha}) \quad (2.7)$$

setting the expressions for the derivatives in equation(2.7) equal to zero and solving α, β and γ yield. The maximum likelihood estimators (MLE) of the parameters namely $\hat{\alpha}_{MLE}$, $\hat{\beta}_{MLE}$ and $\hat{\gamma}_{MLE}$.

However, no closed form solutions exist in this case the elimination of γ in $\frac{\partial \, Log \, L}{\partial\beta}$ and $\frac{\partial \, Log \, L}{\partial\gamma}$ and in $\frac{\partial \, Log \, L}{\partial\alpha}$ and $\frac{\partial \, Log \, L}{\partial\gamma}$ yield a set of equations in terms of β and γ.

$$\frac{\sum_{j=1}^{n}\frac{1}{\beta + x_j{}^{\alpha}}}{\sum_{j=1}^{n}\frac{x_j{}^{\alpha}}{\beta + x_j{}^{\alpha}}} - \frac{n}{\sum_{j=1}^{n} log\left[1+\frac{x_j{}^{\alpha}}{\beta}\right]} = 0 \quad ; \quad (2.8)$$

$$\frac{n}{\alpha} + \sum_{j=1}^{n}\log x_j - \sum_{j=1}^{n}\frac{x_j{}^{\alpha} log x_i}{\beta + x_j{}^{\alpha}} - \frac{n\sum_{j=1}^{n}\frac{x_j{}^{\alpha} log x_i}{\beta + x_j{}^{\alpha}}}{\sum_{j=1}^{n} log\left[1+\frac{x_j{}^{\alpha}}{\beta}\right]} = 0 \quad (2.9)$$

respectively. Applying the Newton-Raphson method $\hat{\alpha}_{MLE}$ and $\hat{\beta}_{MLE}$ can be derived and then from them $\hat{\gamma}_{MLE}$ can be obtained.

3. Bayes estimators for γ with known parameter α *and* β

If $\hat{\alpha}$ and $\hat{\beta}$ is known we assume $\gamma(a, b)$ as conjugate prior for γ as

$$g(\gamma|\underline{x}) = \frac{b^a}{\Gamma a}\left(\frac{1}{\gamma}\right)^{a+1} e^{-\frac{b}{\gamma}}; \qquad (a, b, \gamma) > 0 \quad (3.1)$$

combining the likelihood function equation(2.1) and prior density equation(3.1), we obtain the posterior density of γ in the form

$$h(\gamma|\underline{x}) = \frac{\frac{\alpha^n}{\gamma^n}\beta^{\frac{n}{\gamma}}\prod_{j=1}^{n} x_j{}^{\alpha-1}\prod_{j=1}^{n}(\beta + x_j{}^{\alpha})^{-\left(\frac{1}{\gamma}+1\right)}\frac{b^a}{\Gamma a}\left(\frac{1}{\gamma}\right)^{a+1} e^{-b/\gamma}}{\int_0^{\infty}\frac{\alpha^n}{\gamma^n}\beta^{\frac{n}{\gamma}}\prod_{j=1}^{n} x_j{}^{\alpha-1}\prod_{j=1}^{n}(\beta + x_j{}^{\alpha})^{-\left(\frac{1}{\gamma}+1\right)}\frac{b^a}{\Gamma a}\left(\frac{1}{\gamma}\right)^{a+1} e^{-b/\gamma} d\gamma} \quad ; \quad (3.2)$$

$$h(\gamma|\underline{x}) = \frac{\gamma^{-(n+a-1)} e^{-\frac{(b+T)}{\gamma}}}{\int_0^{\infty}\gamma^{-(n+a-1)} e^{-\frac{(b+T)}{\gamma}} d\gamma} \; ;$$

assuming

$$T = \sum_{j=1}^{n}\log\left(1+\frac{x_j{}^{\alpha}}{\beta}\right) \qquad \text{and} \qquad U = \prod_{j=1}^{n}\frac{x_j{}^{\alpha-1}}{(\beta + x_j{}^{\alpha})}$$

$$h(\gamma|\underline{x}) = \frac{\gamma^{-(n+a-1)} e^{-\frac{(b+T)}{\gamma}}(b+T)^{(n+a)}}{\Gamma(n+a)} \quad (3.3)$$

Bayes estimation under General Entropy loss function (GELF)

Consider the Entropy Loss function

$$L(\delta) \; \alpha \; [\delta^p - plog(\delta) - 1] \quad (3.4)$$

where

$$\delta = \frac{\hat{\theta}}{\theta}, P = 1$$

Now the loss function in equation (3.4) reduces as,

$$L(\delta) = b(\delta - \log_e(\delta) - 1) \quad (3.5)$$

Taking the posterior expectation of equation (3.5)

$$E_h(L(\delta)) = b\left(E_h\left(\frac{\hat{\theta}}{\theta}\right) - E_h\left(\log\left(\frac{\hat{\theta}}{\theta}\right)\right) - 1\right) = 0$$

$$\frac{\delta E_h(L(\delta))}{\delta\hat{\theta}} = b\left[E_P\left(\frac{1}{\theta}\right) - E_P\left(\frac{\theta}{\hat{\theta}}.\frac{1}{\theta}\right)\right] = 0$$

$$\hat{\theta}_{BE} = \left[E_P\left(\frac{1}{\theta}\right)\right]^{-1} \quad (3.6)$$

Therefore the Bayes Estimator is given by using equation(3.5) with replacing θ by γ in equation (3.6) we have

$$\hat{\gamma}_{BE} = \left[E_P\left(\frac{1}{\gamma}\right)\right]^{-1}$$

$$E_h\left(\frac{1}{\gamma}\right) = \int_0^{\infty}\frac{1}{\gamma}\frac{(b+T)^{(n+a)}}{\Gamma(n+a)}\left(\frac{1}{\gamma}\right)^{(n+a+1)} exp\left(-\frac{(b+T)}{\gamma}\right)d\gamma$$

$$E_h\left(\frac{1}{\gamma}\right) = \frac{(b+T)^{(n+a)}}{\Gamma(n+a)}\int_0^{\infty}\left(\frac{1}{\gamma}\right)^{(n+a+2)} exp^{-\frac{(b+T)}{\gamma}} d\gamma$$

Substituting

$$\frac{(b+T)}{\gamma} = y \Rightarrow (b+T)y^{-1} = \gamma$$

$$\Rightarrow -(b+T)y^2 dy = d\gamma$$

$$E_h\left(\frac{1}{\gamma}\right) = \frac{(b+T)^{(n+a)}}{\Gamma(n+a)}\int_0^{\infty}\frac{y^{(n+a+2)}e^{-y}y^{-2}dy(b+T)}{(b+T)^{(n+a+2)}}$$

$$E_h\left(\frac{1}{\gamma}\right) = \frac{\Gamma(n+a+1)}{\Gamma(n+a)(b+t)} = \frac{\Gamma(n+a)}{(b+T)}$$

which gives

$$\hat{\gamma}_{BE} = \frac{(b+T)}{(n+a)} \quad (3.7)$$

4. Approximate Bayes Estimators of the unknown parameters γ

The Joint prior density of the parameters α, β, γ is given by

$$G(\alpha, \beta, \gamma) = g_1(\alpha)g_2(\beta)g_3(\gamma|\beta)$$

$$= \frac{c}{\delta\Gamma\xi}\beta^{-\xi}\gamma^{\xi+1} exp\left[-\left(\frac{\gamma}{\beta}+\frac{\beta}{\delta}\right)\right] \quad (4.1)$$

where

$$g_1(\alpha) = c \quad (4.2)$$

$$g_2(\beta) = \frac{1}{\delta} e^{-\frac{\beta}{\delta}} \quad (4.3)$$

$$g_3(\gamma) = \frac{1}{\Gamma\xi} \beta^{-\xi} \gamma^{\xi+1} e^{-\frac{\gamma}{\beta}} \quad (4.4)$$

The Joint posterior combing the likelihood equation(2.2) and joint prior equation(4.1) is

$$h^*(\alpha,\beta,\gamma|\underline{x}) = \frac{\beta^{-\xi}\gamma^{\xi+1} exp\left[-\left(\frac{\gamma}{\beta}+\frac{\beta}{\delta}\right)\right].L(\underline{x}|\alpha,\beta,\gamma)}{\int_\alpha \int_\beta \int_\gamma \beta^{-\xi}\gamma^{\xi+1} exp\left[-\left(\frac{\gamma}{\beta}+\frac{\beta}{\delta}\right)\right].L(\underline{x}|\alpha,\beta,\gamma) d\alpha d\beta d\gamma} \quad (4.5)$$

The Approximate Bayes Estimator is given by

$$\cup(\Theta) = \cup(\alpha,\beta,\gamma) \quad (4.6)$$

$$\hat{\cup}_{BS} = E(\cup|\underline{x}) = \frac{\int_\alpha \int_\beta \int_\gamma \cup(\alpha,\beta,\gamma) G^*(\alpha,\beta,\gamma) d\alpha d\beta d\gamma}{\int_\alpha \int_\beta \int_\gamma G^*(\alpha,\beta,\gamma) d\alpha d\beta d\gamma} \quad (4.7)$$

Lindley Approximation Procedure

The Bayes estimators of a function $\mu = \mu(\theta, p)$ of the unknown parameter θ and p under squared error loss is the posterior mean

$$\hat{\mu}_{BS} = E(\mu|\underline{x}) = \frac{\iint \mu(\theta,p) h^*(\theta,p|\underline{x}) d\theta dp}{\iint h^*(\theta,p|\underline{x}) d\theta dp} \quad (4.7a)$$

The ratio of integrals in equation (4.7a) does not seem to take a closed form so we must consider the Lindley approximation procedure as

$$E(\mu(\theta,p)|\underline{x}) = \frac{\int \mu(\theta).e^{(l(\theta)+\rho(\theta))} d\theta}{\int e^{(l(\theta)+\rho(\theta))}.d\theta} \quad (4.7b)$$

Lindley developed approximate procedure for evaluation of posterior expectation of $\mu(\theta)$. Several other authors have used this technique to obtain Bayes estimators (see Sinha(1986), Sinha and sloan(1988),Soliman(2001)).The posterior expectation of Lindley approximation procedure to evaluate of $\mu(\theta)$ in equation (4.7a and 4.7b) under SELF, where where $\rho(\theta) = \log g(\theta)$, and $g(\theta)$ is an arbitrary function of θ and $l(\theta)$ is the logarithm likelihood function (Lindley (1980)).

The modified form of equation (4.7) is given by

$$E(\cup(\alpha,\beta,\gamma|\underline{x}) = \cup(\Theta) + \frac{1}{2}[A(\cup_1\sigma_{11} + \cup_2\sigma_{12} + \cup_3\sigma_{13}) + B(\cup_1\sigma_{21} + \cup_2\sigma_{22} + \cup_3\sigma_{23}) + P(\cup_1\sigma_{31} + \cup_2\sigma_{32} + \cup_3\sigma_{33})] + (\cup_1 a_1 + \cup_2 a_2 + \cup_3 a_3 + a_4 + a_5) + 0\left(\frac{1}{n^2}\right) \quad (4.8)$$

Above equation is evaluated at MLE = $(\hat{\alpha}, \hat{\beta}, \hat{\gamma})$,where

$$a_1 = \rho_1\sigma_{11} + \rho_2\sigma_{12} + \rho_3\sigma_{13} \quad (4.9)$$

$$a_2 = \rho_1\sigma_{21} + \rho_2\sigma_{22} + \rho_3\sigma_{23} \quad (4.10)$$

$$a_3 = \rho_1\sigma_{31} + \rho_2\sigma_{32} + \rho_3\sigma_{33} \quad (4.11)$$

$$a_4 = \cup_{12}\sigma_{12} + \cup_{13}\sigma_{13} + \cup_{23}\sigma_{23} \quad (4.12)$$

$$a_5 = \frac{1}{2}(\cup_{11}\sigma_{11} + \cup_{22}\sigma_{22} + \cup_{33}\sigma_{33}) \quad (4.13)$$

And

$$A = [\sigma_{11}l_{111} + 2\sigma_{12}l_{121} + 2\sigma_{13}l_{131} + 2\sigma_{23}l_{231} + \sigma_{22}l_{221} + \sigma_{33}l_{331}] \quad (4.14)$$

$$B = [\sigma_{11}l_{112} + 2\sigma_{12}l_{122} + 2\sigma_{13}l_{132} + 2\sigma_{23}l_{232} + \sigma_{22}l_{222} + \sigma_{33}l_{332}] \quad (4.15)$$

$$P = [\sigma_{11}l_{113} + 2\sigma_{13}l_{133} + 2\sigma_{12}l_{123} + 2\sigma_{23}l_{233} + \sigma_{22}l_{223} + \sigma_{33}l_{333}] \quad (4.16)$$

To apply Lindley approximation on equation (4.8) , we first obtain

$$\sigma_{ij} = [-l_{ijk}]^{-1} i,j,k = 1,2,3$$

Likelihood function from equation (2.2) is

$$L = \frac{\alpha^n}{\gamma^n} \beta^{\frac{n}{\gamma}} \prod_{j=1}^n x_j^{\alpha-1} \prod_{j=1}^n (\beta + x_j^{\alpha})^{-\left(\frac{1}{\gamma}+1\right)} \quad ; (x,\alpha,,\gamma > 0)$$

Now

$$Log\ L = n\ log\alpha - nlog\gamma + \frac{n}{\gamma} log\beta + (\alpha-1)\sum_{j=1}^n logx_j - \left(\frac{1}{\gamma}+1\right)\sum_{j=1}^n \frac{x_j^{\alpha} logx_j}{\beta + x_j^{\alpha}} \quad (4.17)$$

Now differentiating log likelihood function with respect to α

$$l_1 = \frac{n}{\alpha} + \sum_{j=1}^n \log x_j - \left(\frac{1}{\gamma}+1\right)\omega_{11} \quad \text{where} \quad \omega_{11} = \sum_{j=1}^n \frac{x_j^{\alpha} logx_j}{(\beta + x_j^{\alpha})} \quad (4.18)$$

Again differentiating log likelihood function with respect to β

$$l_2 = \frac{n}{\gamma\beta} - \left(\frac{1}{\gamma}+1\right)\delta_{11} \quad \text{where} \quad \delta_{11} = \sum_{j=1}^n \frac{1}{\beta + x_j^{\alpha}} \quad (4.19)$$

Again differentiating log likelihood function with respect to γ

$$l_3 = -\frac{n}{\gamma} - \frac{n\log B}{\gamma^2} + \frac{1}{\gamma^2}\delta_{10} \quad \text{where} \quad \delta_{10} = \sum_{j=1}^n \log(\beta + x_j^{\alpha}) \quad (4.20)$$

Again differentiating l_1 with respect to α

$$l_{11} = \frac{-n}{\alpha^2} - \left(\frac{1}{\gamma}+1\right)\beta\,\omega_{122} \quad \text{where} \quad \omega_{122} = \sum_{j=1}^n \frac{x_j^{\alpha}(logx_j)^2}{(\beta + x_j^{\alpha})^2} \quad (4.21)$$

Now differentiating l_1 with respect to β

$$l_{12} = \left(\frac{1}{\gamma}+1\right)\omega_{14} \quad \text{where} \quad \omega_{14} = \sum_{j=1}^n \frac{x_j^{\alpha}\log x_j}{(\beta + x_j^{\alpha})^2} \quad (4.22)$$

Again differentiating l_1 with respect to γ

$$l_{13} = \frac{\omega_{11}}{\gamma^2} \quad \text{where} \quad \omega_{11} = \sum_{j=1}^n \frac{x_j^{\alpha}\log x_j}{(\beta + x_j^{\alpha})} \quad (4.23)$$

Again differentiating l_2 with respect to α

$$l_{21}\left(\frac{1}{\gamma}+1\right)\omega_{14} \quad \text{where} \quad \omega_{14} = \sum_{j=1}^n \frac{x_j^{\alpha}\log x_j}{(\beta + x_j^{\alpha})^2} \quad (4.24)$$

Again differentiating l_2 with respect to β

$$l_{22} = -\frac{n}{\gamma\beta^2} + \left(\frac{1}{\gamma}+1\right)\delta_{12} \quad (4.25)$$

Again differentiating l_2 with respect to γ

$l_{23} = -\frac{1}{\gamma^2}\left(\frac{n}{\beta} - \delta_{11}\right)$ where $\delta_{11} = \sum_{j=1}^{n} \frac{1}{(\beta + x_j{}^{\alpha})}$ (4.26)

Again differentiating l_3 with respect to α

$l_{31} = \frac{\omega_{11}}{\gamma^2}$ (4.27)

Again differentiating l_3 with respect to α

$l_{32} = \frac{-n}{\beta\gamma^2} + \frac{1}{\gamma^2}\sum_{j=1}^{n} \frac{1}{(\beta + x_j{}^{\alpha})} = -\frac{1}{\gamma^2}\left(\frac{n}{\beta} - \delta_{11}\right)$ (4.28)

Again differentiating l_3 with respect to γ

$l_{33} = \frac{n}{\gamma^2} + \frac{2n\log\beta}{\gamma^3} - \frac{2}{\gamma^3}\delta_{10}$ where $\delta_{10} = \sum_{j=1}^{n} log\left(\beta + x_j{}^{\alpha}\right)$ (4.29)

Again differentiating l_{11} with respect to α

$l_{111} = \frac{2n}{\alpha^3} - \left(\frac{1}{\gamma} + 1\right)\omega_{133}$ where $\omega_{133} = \sum \frac{x_j{}^{\alpha}(\beta - x_j{}^{\alpha})(\log x_j)^3}{(\beta + x_j{}^{\alpha})^3}$ (4.30)

Again differentiating l_{22} with respect to β

$l_{222} = \frac{2n}{\gamma\beta^3} - 2\left(\frac{1}{\gamma} + 1\right)\delta_{13}$ where $\delta_{13} = \sum_{j=1}^{n} \frac{1}{(\beta + x_j)^3}$ (4.31)

Again differentiating l_{33} with respect to γ

$l_{333} = -\frac{2n}{\gamma^3} - \frac{6n\,log\beta}{\gamma^4} + \frac{6}{\gamma^4}\delta_{10}$ where $\delta_{10} = \sum_{j=1}^{n} log\left(\beta + x_j{}^{\alpha}\right)$ (4.32)

Again differentiating l_{11} with respect to β

$l_{112} = \left(\frac{1}{\gamma} + 1\right)\omega_{123}$ where $\omega_{123} = \sum_{j=1}^{n} \frac{x_j{}^{\alpha}(\beta - x_j{}^{\alpha})(\log x_j)^2}{(\beta + x_j{}^{\alpha})^3}$ (4.33)

and $l_{112} = l_{121}$ (using result as $l_{12} = l_{21}$)

Again differentiating l_{11} with respect to γ

$l_{113} = \frac{\beta}{\gamma^2}\omega_{122}$ where $\omega_{122} = \sum_{j=1}^{n} \frac{x_j{}^{\alpha}(log x_j)^2}{(\beta + x_j{}^{\alpha})^2}$ (4.34)

Again differentiating l_{22} with respect to α

$l_{221} == -2\left(\frac{1}{\gamma} + 1\right)\omega_{113}$ where $\omega_{113} = \sum_{j=1}^{n} \frac{x_j{}^{\alpha} log x_j}{(\beta + x_j{}^{\alpha})^3}$

(4.35)

$l_{221} = l_{212}$

Again differentiating l_{22} with respect to γ

$l_{223} = \frac{n}{(\gamma\beta)^2} - \frac{1}{(\gamma)^2}\delta_{12}$ where $\delta_{12} = \sum_{j=1}^{n} \frac{1}{(\beta + x_j{}^{\alpha})^2}$

(4.36)

Again differentiating l_{33} with respect to α

$l_{331} = -\frac{2}{\gamma^3}\omega_{11}$ where $\omega_{11} = \sum_{j=1}^{n} \frac{x_j{}^{\alpha} log x_j}{(\beta + x_j{}^{\alpha})}$ (4.37)

$l_{331} = l_{313}$

Again differentiating l_{33} with respect to β

$l_{332} = \frac{\partial}{\partial\gamma}\left(\frac{\partial^2 L}{\partial\gamma\partial\beta}\right) = \frac{2}{\gamma^3}\left(\frac{n}{\beta} - \delta_{11}\right)$ (4.38)

$l_{332} = l_{323}$

Again differentiating l_{23} with respect to α

$l_{231} - \frac{\omega_{14}}{\gamma^2}$ (4.39)

$l_{231} = l_{213}$

Again differentiating l_{12} with respect to γ

$l_{123} = -\frac{\omega_{14}}{\gamma^2}$ (4.40)

$l_{123} = l_{132}$

Again differentiating l_{13} with respect to γ

$l_{133} = \frac{-2}{\gamma^2}\sum_{j=1}^{n} \frac{x_j{}^{\alpha}\log x_j}{(\beta + x_j{}^{\alpha})} = -\frac{2}{\gamma^2}\omega_{11}$ (4.41)

Again differentiating l_{12} with respect to β

$l_{122} = -2\left(\frac{1}{\gamma} + 1\right)\omega_{113}$ where $\omega_{113} = \sum_{j=1}^{n} \frac{x_j{}^{\alpha}\log x_j}{(\beta + x_j{}^{\alpha})^3}$ (4.42)

Again differentiating l_{23} with respect to γ

$l_{233} = \frac{2n}{\beta\gamma^3} - \frac{2}{\gamma^3}\sum_{j=1}^{n} \frac{1}{\beta + x_j{}^{\alpha}} = \frac{2}{\gamma^3}\left(\frac{n}{\beta} - \delta_{11}\right)$ (4.43)

The matrix of derivatives is given as

$$[-l_{ijk}] = -\begin{bmatrix} l_{111} & l_{112} & l_{113} \\ l_{221} & l_{222} & l_{223} \\ l_{331} & l_{332} & l_{333} \end{bmatrix} \quad (4.44)$$

$$= \begin{bmatrix} \frac{2n}{\alpha^3} - \left(\frac{1}{\gamma} + 1\right)\omega_{133} & ,\left(\frac{1}{\gamma} + 1\right)\omega_{123} & -\frac{\beta}{\gamma^2}\omega_{122} \\ -2\left(\frac{1}{\gamma} + 1\right)\omega_{113} & ,\frac{2n\gamma}{\gamma\beta^3} - 2\left(\frac{1}{\gamma} + 1\right)\delta_{13} & \frac{n}{(\gamma\beta)^2} - \frac{1}{\gamma^2}\delta_{12} \\ \frac{-2}{\gamma^3}\omega_{11} & ,\frac{-2}{\gamma^3}\left(\frac{n}{\gamma} - \delta_{11}\right) & ,-\frac{2n}{\gamma^3} - \frac{6n\,log\beta}{\gamma^4} + \frac{6}{\gamma^4}\delta_{10} \end{bmatrix}$$

$$[-l_{ijk}] = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

$Determinant\ of\ \ [-l_{ijk}]$

$D = \{M_{11}[M_{22}M_{33} - M_{23}M_{32}] - M_{12}[M_{21}M_{33} - M_{31}M_{23}] + M_{13}[M_{21}M_{32} - M_{22}M_{33}]\}$ (4.45)

$co - factor\ of\ Matrix\ [-l_{ijk}]$

$a_{11} = [M_{22}M_{33} - M_{23}M_{32}]$

$a_{12} = [M_{21}M_{33} - M_{31}M_{23}]$

$a_{13} = [M_{21}M_{32} - M_{22}M_{31}]$

$a_{21} = [M_{12}M_{33} - M_{32}M_{13}]$

$a_{22} = [M_{11}M_{33} - M_{31}M_{13}]$

$a_{23} = [M_{11}M_{32} - M_{31}M_{12}]$

$a_{31} = [M_{12}M_{23} - M_{13}M_{22}]$

$a_{32} = [M_{11}M_{23} - M_{13}M_{21}]$

$a_{33} = [M_{11}M_{22} - M_{12}M_{21}]$

Transpose of Adjoint of $[-l_{ijk}]$

$$= \begin{bmatrix} M_{23}M_{32} - M_{22}M_{33} & M_{12}M_{33} - M_{32}M_{13} & M_{12}M_{22} - M_{12}M_{23} \\ M_{21}M_{33} - M_{31}M_{23} & M_{31}M_{13} - M_{11}M_{33} & M_{11}M_{23} - M_{13}M_{21} \\ M_{22}M_{31} - M_{21}M_{32} & M_{11}M_{32} - M_{31}M_{12} & M_{12}M_{21} - M_{11}M_{22} \end{bmatrix}$$

$$[-l_{ijk}]^{-1} = \frac{(Adjoint\ of\ [-l_{ijk}])'}{D}$$

$$[-l_{ijk}]^{-1} = \begin{bmatrix} \frac{Y_{11}}{D} & \frac{Y_{12}}{D} & \frac{Y_{13}}{D} \\ \frac{Y_{21}}{D} & \frac{Y_{22}}{D} & \frac{Y_{23}}{D} \\ \frac{Y_{31}}{D} & \frac{Y_{32}}{D} & \frac{Y_{33}}{D} \end{bmatrix}$$

$$[-l_{ijk}]^{-1} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} ; \qquad (4.46)$$

Approximate Bayes Estimator

$\cup(\alpha, \beta, \gamma) = \cup$

$\hat{\cup}_{AB} = E(\cup\,|\underline{x})$

evaluated from equation number and from joint prior density , we have

$$G(\alpha, \beta, \gamma) = g(\alpha)g_2(\beta)g_3(\gamma|\beta)$$
$$= \frac{c}{\delta\Gamma\xi}\beta^{-\xi}\gamma^{\xi-1}exp\left[-\left(\frac{\gamma}{\beta} + \frac{\beta}{\delta}\right)\right];$$

$$\rho = \log G = \log C - log\delta - log[\xi + (\xi-1)log\gamma - \xi log\beta - \left(\frac{\gamma}{\beta} + \frac{\beta}{\delta}\right) \qquad (4.47)$$

logG= $constant - \xi log\beta + (\xi - 1)log\gamma - \frac{\gamma}{\beta} - \frac{\beta}{\delta}$

$$\rho_1 = \frac{\delta\rho}{\delta\alpha} = 0 \qquad (4.48)$$

$$\rho_2 = \frac{\delta\rho}{\delta\beta} = \frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta} \qquad (4.49)$$

$$\rho_3 = \frac{\delta\rho}{\delta\gamma} = \frac{\xi-1}{\gamma} - \frac{1}{\beta} \qquad (4.50)$$

Using equation (4.14) to equation (4.46), we have

$$A = [\sigma_{11}l_{111} + 2\sigma_{12}l_{121} + 2\sigma_{13}l_{131} + 2\sigma_{23}l_{231} + \sigma_{22}l_{221} + \sigma_{33}l_{331}]$$

$$= \sigma_{11}\left(\frac{2n}{\alpha^3} - \left(\frac{1}{\gamma}+1\right)\omega_{133}\right) + 2\sigma_{12}\left(\frac{1}{\gamma}+1\right)\omega_{123} + 2\sigma_{13}\left(\frac{\beta}{\gamma^2}\omega_{122}\right) + 2\sigma_{23}\left(-\frac{\omega_{14}}{\gamma^2}\right) + \sigma_{22}\left(-2\left(\frac{1}{\gamma}+1\right)\omega_{113}\right) + \sigma_{33}\left(-\frac{2}{\gamma^3}\omega_{11}\right)$$

$$= \frac{1}{D}\Big[Y_{11}\left(\frac{2n}{\alpha^3} - \left(\frac{1}{\gamma}+1\right)\omega_{133}\right) + 2Y_{12}\left(\frac{1}{\gamma}+1\right)\omega_{123} + 2Y_{13}\frac{\beta}{\gamma^3}\omega_{122} - 2Y_{23}\frac{\omega_{14}}{\gamma^2} - 2Y_{22}\left(\frac{1}{\gamma}+1\right)\omega_{113} - \frac{2}{\gamma^3}Y_{33}\omega_{11}\Big] \qquad (4.51)$$

$$B = [\sigma_{11}l_{112} + 2\sigma_{12}l_{122} + 2\sigma_{13}l_{132} + 2\sigma_{23}l_{232} + \sigma_{22}l_{222} + \sigma_{33}l_{332}]$$

$$= \sigma_{11}\left(\frac{1}{\gamma}+1\right)\omega_{123} + 2\sigma_{12}\left(-2\left(\frac{1}{\gamma}+1\right)\omega_{113}\right) + 2\sigma_{13}\left(\frac{-\omega_{14}}{\gamma^2}\right) + 2\sigma_{23}\left(\frac{n}{(\gamma\beta^2)} - \frac{1}{\gamma^2}\delta_{12}\right) + \sigma_{22}\left(\frac{n}{(\gamma\beta)^2} - \frac{1}{\gamma^2}\delta_{12}\right) + \sigma_{33}\left(\frac{2}{\gamma^3}\left(\frac{n}{\beta} - \delta_{11}\right)\right)$$

$$= \frac{1}{D}\Big[\left(\frac{1}{\gamma}+1\right)\omega_{123}Y_{11} - 4Y_{12}\left(\frac{1}{\gamma}+1\right)\omega_{113} - 2Y_{13}\left(-\frac{\omega_{14}}{\gamma^2}\right) + (Y_{22}+2Y_{23})\left(\frac{n}{(\gamma\beta)^2} - \frac{1}{\gamma^2}\delta_{12}\right) + Y_{33}\left(-\frac{2}{\gamma^3}\left(\frac{n}{\beta} - \delta_{11}\right)\right)\Big] \qquad (4.52)$$

$$P = [\sigma_{11}l_{113} + 2\sigma_{12}l_{123} + 2\sigma_{13}l_{133} + 2\sigma_{23}l_{233} + \sigma_{22}l_{223} + \sigma_{33}l_{333}]$$

$$= \sigma_{11}\frac{\beta}{\gamma^2}\omega_{122} + 2\sigma_{12}\left(-\frac{\omega_{14}}{\gamma^2}\right) + 2\sigma_{13}\left(-\frac{2}{\gamma^3}\omega_{11}\right) + 2\sigma_{23}\frac{2}{\gamma^3} + \left(\frac{n}{\beta} - \delta_{11}\right) + \sigma_{22}\left(\frac{n}{(\gamma\beta)^2} - \frac{1}{\gamma^2}\delta_{12}\right) + \sigma_{33}\left(-\frac{2n}{\gamma^3} - \frac{6n\,log\beta}{\gamma^4} + \frac{6}{\gamma^4}\delta_{10}\right)$$

$$= \frac{1}{D}\left[\frac{Y_{11}\beta}{\gamma^2}\omega_{122} - \frac{2Y_{12}\omega_{14}}{\gamma^4} - \frac{4Y_{13}\omega_{11}}{\gamma^3} + \frac{4Y_{23}}{\gamma^3}\left(\frac{n}{\beta} - \delta_{11}\right) + Y_{22}\left(\frac{n}{\gamma^2\beta^2} - \frac{1}{\gamma^2}\delta_{12}\right) + Y_{33}\left(-\frac{2n}{\gamma^3} - \frac{6n\,log\beta}{\gamma^4} + \frac{6}{\gamma^4}\delta_{10}\right)\right] \qquad (4.53)$$

Now

$\hat{\cup}_{AB} = E(\cup\,|\underline{x})$

$$E(\cup\,|\underline{x}) = u + (u_1a_1 + u_2a_2 + u_3a_3 + a_{4+}a_5) + \frac{1}{2}[A(u_1\sigma_{11} + u_2\sigma_{12} + u_3\sigma_{13}) + B(u_1\sigma_{21} + u_2\sigma_{22} + u_3\sigma_{23}) + P(u_1\sigma_{31} + u_2\sigma_{32} + u_3\sigma_{33})] + 0\left(\frac{1}{n^2}\right)$$

$$E(\cup\,|\underline{x}) = U + \varphi_1 + \varphi_2 \qquad (4.54)$$

where

$$\varphi_1 = u_1a_1 + u_2a_2 + u_3a_3 + a_4 + a_5 \qquad (4.55)$$

$$\varphi_2 = \frac{1}{2}[(A\sigma_{11} + B\sigma_{21} + P\sigma_{31}).U_1 + (A\sigma_{12} + B\sigma_{22} + P\sigma_{32}).U_2 + (A\sigma_{13} + B\sigma_{23} + P\sigma_{33})U_3] \qquad (4.56)$$

evaluated at the MLE $\hat{\cup} = (\hat{\alpha}, \hat{\beta}, \hat{\gamma})$ where

$$a_1 = \rho_1\sigma_{11} + \rho_2\sigma_{12} + \rho_3\sigma_{13}$$
$$= 0.\sigma_{11} + \left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta}\right)\frac{Y_{12}}{D} + \left(\frac{\xi-1}{\gamma} - \frac{1}{\beta}\right)\frac{Y_{13}}{D} \qquad (4.57)$$

$$a_2 = \rho_1\sigma_{21} + \rho_2\sigma_{22} + \rho_3\sigma_{23}$$
$$= 0.\sigma_{21} + \left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta} - \frac{1}{\delta}\right)\frac{Y_{22}}{D} + \left(\frac{\xi-1}{\gamma} - \frac{1}{\beta}\right)\frac{Y_{23}}{D} \qquad (4.58)$$

$$a_3 = \rho_1\sigma_{31} + \rho_2\sigma_{32} + \rho_3\sigma_{33}$$

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$$= 0.\sigma_{31} + \left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta}\right)\frac{Y_{32}}{D} + \left(\frac{\xi-1}{\gamma} - \frac{1}{\beta}\right)\frac{Y_{33}}{D} \quad (4.59)$$

$$a_4 = U_{12}\sigma_{12} + U_{13}\sigma_{13} + U_{23}\sigma_{23}$$

$$= \frac{Y_{12}}{D}U_{12} + \frac{Y_{13}}{D}U_{13} + \frac{Y_{23}}{D}U_{23} \quad (4.60)$$

$$a_5 = \frac{1}{2}(U_{11}\sigma_{11} + U_{22}\sigma_{22} + U_{33}\sigma_{33})$$

$$a_5 = \frac{1}{2D}(Y_{11}U_{11} + Y_{22}U_{22} + Y_{33}U_{33}) \quad (4.61)$$

Approximate Bayes Estimate Under General Entropy Loss Function

$\hat{\cup}_{ABE} = E(\theta) = \theta$

where

$$E_u(\theta|\underline{x}) = \frac{\int_\alpha\int_\beta\int_\gamma \theta G^*(\alpha,\beta,\gamma)\partial\alpha\partial\beta\partial\gamma}{\int_\alpha\int_\beta\int_\gamma G^*(\alpha,\beta,\gamma)\partial\alpha\partial\beta\partial\gamma} \quad (4.62)$$

The above equation (4.62) is evaluated by method of Lindley approximation, whose simplified form is equation (4.54) replace θ by $\cup(\alpha,\beta,\gamma)$ in equation (4.62)

Approximate Bayes Estimator under General Entropy loss function (GELF)

$$\hat{U}_{ABE} = \left[E_h\left(\frac{1}{\theta}\right)\right]^{-1}$$

Where;

$$E_u(\theta|\underline{x}) = \frac{\int_\alpha\int_\beta\int_\gamma \frac{1}{\theta}G^*(\alpha,\beta,\gamma)d\alpha d\beta d\gamma}{\int_\alpha\int_\beta\int_\gamma G^*(\alpha,\beta,\gamma)d\alpha d\beta d\gamma} \quad (4.64)$$

The equation (4.64) is evaluated by method of Lindley approximation whose simplified from is equation no. replace θ by $\cup(\alpha,\beta,\gamma)$ in equation (4.54)

Special cases :–

1. Approximate Bayes Estimate of γ

$$\cup(\alpha,\beta,\gamma) = \cup = \frac{1}{\gamma}$$

$$\hat{\gamma} = \left[E\left(\frac{1}{\gamma}\right)\right]^{-1} \quad (4.65)$$

$$E\left(\frac{1}{\gamma}|\underline{x}\right) = \frac{1}{\gamma} + \varphi_1 + \varphi_2$$

$\cup_1 = \cup_{11} = \cup_{12} = \cup_{13} = 0$, $\cup_2 = \cup_{21} = \cup_{22} = \cup_{23} = 0$

$$\cup_3 = \frac{\partial}{\partial\gamma}\left(\frac{1}{\gamma}\right) = -\frac{1}{\gamma^2} \quad ; \quad \cup_{33} = \frac{\partial}{\partial\gamma}\left(-\frac{1}{\gamma^2}\right) = \frac{2}{\gamma^3} \quad ; \quad \cup_{31} = \cup_{32} = 0$$

$$\varphi_1 = \cup_1 a_1 + \cup_2 a_2 + \cup_3 a_3 + a_4 + a_5$$

$$= 0.a_1 + 0.a_2 + \cup_3\left[\left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta}\right)\frac{Y_{32}}{D} + \left(\frac{\xi-1}{\beta} - \frac{1}{\beta}\right)\frac{Y_{33}}{D} + 0 + \frac{1}{2}\frac{2}{\gamma^3}\frac{Y_{33}}{D}\right]$$

$$= -\frac{1}{\gamma^2}\left[\left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta}\right)\frac{Y_{32}}{D} + \left(\frac{\xi-1}{\beta} - \frac{1}{\beta} - \frac{1}{\gamma}\right)\frac{Y_{33}}{D}\right]$$

$$= -\frac{1}{\gamma^2}\left[\left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta}\right)\frac{Y_{32}}{D} + \left(\frac{\xi-2}{\beta}\right)\frac{Y_{33}}{D}\right]$$

$$\varphi_2 = \frac{1}{2}[\cup_1(A\sigma_{11} + B\sigma_{21} + P\sigma_{31}) + \cup_2(A\sigma_{12} + B\sigma_{22} + P\sigma_{32}) + \cup_3(A\sigma_{13} + B\sigma_{23} + P\sigma_{32})]$$

$$\varphi_2 = \frac{1}{2}\left[-\frac{1}{\gamma^2}(A\sigma_{13} + B\sigma_{23} + B\sigma_{33})\right]$$

$$E\left(\frac{1}{\gamma}|\underline{x}\right) = \frac{1}{\gamma} - \frac{1}{\gamma^2}\left[\left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - +\frac{1}{\delta}\right)\frac{Y_{32}}{D} + \left(\frac{\xi-2}{\gamma}\right)\frac{Y_{33}}{D} + \frac{(A\sigma_{13} + B\sigma_{23} + P\sigma_{32})}{2}\right]$$

$$E\left(\frac{1}{\gamma}|\underline{x}\right) = \frac{1}{\gamma}\left[1 - \frac{1}{\gamma}\Delta_6\right]$$

$$\hat{\gamma}_{ABE} = \left[\frac{1}{\gamma}\left[1 - \frac{1}{\gamma}\Delta_6\right]\right]^{-1}$$

$$\hat{\gamma}_{ABE} = \gamma[1 - \gamma^{-1}\Delta_6]^{-1} ; (\hat{\alpha}_{ML}, \hat{\beta}_{ML}, \hat{\gamma}_{ML}) \quad (4.66)$$

Where; $\Delta_6 = \left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta}\right)\frac{Y_{32}}{D} + \left(\frac{\xi-2}{\gamma}\right)\frac{Y_{33}}{D} + \frac{1}{2}\left[\frac{Y_{13}}{D}\left(\left(\frac{2n}{\alpha^3} - \left(\frac{1}{\gamma} + 1\right)\omega_{133}\right)\frac{Y_{11}}{D} + 2\frac{Y_{12}}{D}\left(\frac{1}{\gamma} + 1\right)\omega_{123} + 2\frac{Y_{13}}{D}\frac{\beta}{\gamma^2}\omega_{122} - 2\frac{Y_{23}}{D}\frac{\omega_{14}}{\gamma^2} - 2\frac{Y_{22}}{D}\left(\frac{1}{\gamma} + 1\right)\omega_{113} - \frac{2}{\gamma^3}\frac{Y_{33}}{D}\omega_{11}\right)\right] + \frac{Y_{23}}{D}\left[\left(\frac{1}{\gamma} + 1\right)\omega_{123}\frac{Y_{11}}{D} - 4\frac{Y_{12}}{D}\left(\frac{1}{\gamma} + 1\right)\omega_{113} - \frac{2Y_{13}}{D}\frac{\omega_{14}}{\gamma^2} + \left(\frac{Y_{22}+2Y_{23}}{D}\right).\left(\frac{n}{\gamma^2\beta^2} - \frac{\delta_{12}}{\gamma^2}\right) + \frac{Y_{33}}{D}\left(\frac{2}{\gamma^3}\left(\frac{n}{\beta} - \delta_{11}\right)\right)\right] + \frac{Y_{33}}{D}\left[\frac{Y_{11}}{D}\frac{\beta}{\gamma^2}\omega_{122} - \frac{2Y_{12}}{D}\frac{\omega_{14}}{\gamma^4} - 4\frac{Y_{13}}{D}\frac{\omega_{11}}{\gamma^3} + \frac{4Y_{23}}{D\gamma^3}\left(\frac{n}{\beta} - \delta_{11}\right) + \frac{Y_{22}}{D}\left(\frac{n}{\gamma^2\beta^2} - \frac{\delta_{12}}{\gamma^2}\right) + \frac{Y_{33}}{D}\left(\frac{-2n}{\gamma^3} - \frac{6n\,log\beta}{\gamma^4} + \frac{6\delta_{10}}{\gamma^4}\right)\right]$

Simulations and Numerical Comparison

The simulations and numerical calculations are done by using R Language programming and results are presented in form of tables in table (1).

1. The Random variable of Generalized Compound Rayleigh Distribution is generated by R-Language programming by taking the values of the parameters α, β, γ, taken as $\alpha = 1$, $\beta = 0.5$ and $\gamma = 0.8$ in the equations[(4.2)-(4.4)] and equation(1.1).

2. Taking the different sizes of samples n=10(10)80 with complete sample, MLE's, the Approximate Bayes estimators ,and their respective MSE's (in parenthesis) are obtained by repeating the steps 500 times, are presented in the tables from (1), for t=0.5, R(t)=0.42, H(t)=0.625 and parameters of prior distribution a =2 and b =3.

3. Table (1) present the MLE of parameter of γ (for known α *and* β) and Approximate Bayes estimators under GELF (γ unknown) and their respective MSE's. It also present the mean and MSE's of γ and Approximate Bayes estimators of (γ unknown) under GELF. All the estimators have minimum MSE's for large sample sizes, as the sample sizes decrease, the MSE's increased. Among all the four estimators $\hat{\gamma}_{ABE}$ under GELF has the lowest MSE's which shows its domination amongst other three estimators. The MSE's in all above cases are presented in parenthesis.

Table (1)

Mean and MSE'S of α, β, γ

($\alpha = 1$, $\beta = 0.5$ and $\gamma = 0.8$)

n	$\hat{\alpha}_{ML}$	$\hat{\beta}_{ML}$	$\hat{\gamma}_{ML}$	$\hat{\gamma}_{BE}$	$\hat{\gamma}_{ABE}$
10	0.7546328	0.7900455	0.5900124	0.5126982	0.5568712
	[0.152134]	**[0.024412]**	**[0.0256142]**	**[8.156e-05]**	**[0.032541]**
20	0.7589642	0.7922601	0.6922601	0.53259536	0.5845191
	[0.126537]	**[0.025874]**	**[0.026537]**	**[8.377e-05]**	**[0.013208]**
30	0.8125469	0.881254	0.6865478	0.82325467	0.8700478
	[0.098745]	[0.098521]	**[0.096548]**	**[8.985e-05]**	**[0.023156]**
40	0.8645712	0.8987499	0.86975682	0.8676454	0.8000091
	[0.007451]	**[0.002354]**	**[0.003265]**	**[0.001285]**	**[0.039749]**
50	0.9451287	0.8996584	0.8490011	0.8710110	0.8770789
	[0.004213]	**[0.004577]**	**[0.004265]**	**[0.001725]**	**[0.001522]**
60	0.9998756	0.9990011	0.9490011	0.9710110	0.9254879
	[0.004154]	**[0.004663]**	**[0.004226]**	**[0.001725]**	**[0.001522]**
70	1.3000043	0.9988845	0.9454543	0.9814406	0.9956428
	[0.000231]	**[0.001125]**	**[0.001367]**	**[0.004221]**	**[0.038249]**
80	1.9000032	1.0107432	0.9657432	1.0147406	1.036426
	[0.000321]	**[0.001021]**	**[0.001245]**	**[0.004231]**	**[0.034001]**

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