International Journal of Applied Engineering Research ISSN 0973-4562 Volume 12, Number 24 (2017) pp. 14208-14217

Evaluation of Laney p' Chart Performance

Muhammad Ahsan[1#], **Muhammad Mashuri**[2#] **and Hidayatul Khusna**[3#]

[#] *Department of Statistics, Institut Teknologi Sepuluh Nopember, Surabaya, Indonesia.*
Orcid: 0000-0003-3444-2766

Orcid: 0000-0001-9889-2884

Abstract

The problem occurs when p control chart for large sample size, the control limits are narrow and many observation will fall outside the control limits. To solve that problem, laney p' control chart was proposed. The chart can maintain the problem of large sample size. However, there is no strong evidence said that the p' control chart is better than p chart, for all condition of data. This paper will discuss the performance of p' control chart based on graphic visualization and Average Run Length (ARL), then compare its performance with the conventional p control chart. The simulation process would be used to determine the performance of p and p' control chart for all condition of sample sizes.

Keyword: p chart, Laney p' chart, Average Run Length

INTRODUCTION

Control Chart methods based on attribute data were first proposed by Shewhart in 1926. The *p* and *np* chart are widely used, primarily to monitor the fraction of non-conforming products. The p and np control chart limit and performance measure are typically based on the binomial distribution [1]. The problem occurs when a control chart for attributes data is based on large sample size, one often has to cope with the fact that the control limits are narrow and many observation will fall outside the control limits [2,3]. The narrow limits, which are caused by large sample size, make an evidence of the natural variation in the probability of producing nonconforming unit from the underlying process [4]. With very large sample sizes (in the thousands, for example) the statistical uncertainty associated with within-sample variation is 'averaged out' by the large subgroup size, resulting in limits that are right on top of each other [2].

To solve the problem, reference [2] proposed p' control chart that can maintain the problem of large sample size. The Laney p' chart is similar to a traditional p chart. Both charts could be used to monitor the proportion of defective items that are produced by the process. However, Laney p' chart could be useful in the situations when process data have large subgroups and exhibit overdispersion. To calculate the control limit p'-chart does not have to choose between intra-subgroup variation (as in the p chart) and inter-subgroup variation (as in the X-chart). It uses all the variation in the data. If there is any batch-to-batch variation, its control limits are appropriately farther away from the center line than in a p-chart. In addition, if there is a variation in subgroup sizes, its control limits will vary.

Laney p' control chart is widely used health care quality monitoring that has a very large sample size. Reference [5] using p' control chart to monitor over-dispersion in health care performance data. Reference [6] found that p' control chart produced fewest false alarms but could not detect the outlier in very small samples among small proportions, and regardless of sample size among large proportions. The concept of p' control chart also used to develop xmr control chart [7] and attribute control charts for the Weibull Distribution [8].

According to [2], the p' control chart proposed using some examples from reference [4], there is no performance testing of the p' control chart. Consequently, there is no strong evidence said that the p' control chart is better than p chart because it just presented by some examples with different conditions. This paper will discuss the performance of p' control chart based on Average Run Length and compares it with the conventional p control chart for same and different sample size. Thus, could be recommended to users when they need to use p 'or p chart. The paper is organized as follows. In Section 2, we briefly review conventional p chart. Section 3 presents Laney's p' chart. Section 4 presents Average Run Length. Section 5 contains simulation studies to evaluate the performance of Laney p' and traditional p control chart. Finally, Section 6 contains conclusion and future research.

CONVENTIONAL P CHART

The concept of p-charts is derived from the Binomial Distribution and are used to track the proportion of the samples (p) that are defective within a variable sample size. If D is the number of defective units in a random sample of size n, then our sample proportion defective will be:

$$\hat{p} = \frac{D}{n} \tag{1}$$

Since these samples come from a binomial distribution, and assuming that we knew the true proportion defective in all the product was p, then the probability that the number of

defectives (D) in a sample of size n is exactly x units is given by:

$$P\{D = x\} = \binom{n}{x} p^x (1-p)^{n-x}$$

where $x = 0, 1, 2, ...$

If we took a large enough number of samples, we would find that the mean proportion defectives in the distribution (μ) would be very close to p, and that the population variance would be given by:

$$\sigma_{\hat{p}}^2 = \frac{p(1-p)}{n} \quad (2)$$

Using plus and minus three standard deviations from the centerline (Shewhart style), the trial control limits for the proportion defective in any particular sample are:

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$
$$CL = \bar{p} \quad (3)$$
$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Note that if the subgroup sizes vary, the control limits are different [9].

LANEY P' CHART

Improved p chart, Laney p' chart is chart that was developed by combining the principle of Z chart and Donald Wheeler's concepts [10,11]. Before p' chart explained, Z chart and improved Z chart will be introduced first [2].

A standard method for handling attribute data, usually when the display of variable control limits is undesirable, is to convert each p-value to a z-score (the number of sample standard deviations between that point and the overall mean), and then plot these numbers on a "z-chart". Since the theoretical mean of the z-scores is zero, this is what is used for the center line of the chart. Moreover, since the standard deviation of z is assumed to be unity, the control limits are set at +3 and -3. The "z-transformation "automatically adjusts each point for its unique intra-subgroup variation, thus producing flat control limits:

$$Z_i = \frac{p_i - \bar{p}}{\sigma_{p_i}} \quad (4)$$

$$CL = 0$$

$$UCL / LCL = \pm 3$$

If X chart and Z chart concept put together, convert the p-values to z-scores (thus correcting in advance for variable sample sizes) and then plot the z's in an individual's chart.

$$R_i = |z_i - z_{i-1}|, i = 2, 3, \ldots, k \quad (5)$$

Calculate $\bar{R}_i$ using equation :

$$\bar{R}_i = \frac{1}{k-1}\sum_{i=2}^{k} R_i \quad (6)$$

Control limit can be obtained by:

$$CL = 0 \quad (7)$$

$$UCL / LCL = \pm 3\sigma_z \quad (8)$$

where, $\sigma_z = \frac{\bar{R}_i}{1.128}$

With this concept assumption, the standard deviation equal to zero is no longer. All that remains now is to unravel the z-transformation and put our results back into the meaningful units of the p-plane.

Equation (4) can be written as:

$$p_i = \bar{p} + \sigma_{p_i} Z_i, \quad (9)$$

standart deviation of p_i can be calculated using:

$$sd(p_i) = \sigma_{p_i}\sigma_z. \quad (10)$$

Finally the control limit for p' chart would be:

$$UCL / LCL = \bar{p} \pm 3\sigma_{p_i}\sigma_z \quad (11)$$

AVERAGE RUN LENGTH

The average run length (ARL) is widely used by researchers for measuring the performance of memory type control charts. The performance is assessed by two types of ARLs that are, ARL_0 and ARL_1. ARL_0 is the expected number of samples before an out of control point is detected when the process is actually in control. ARL1 is the expected number of samples before an out of control signal is received when the process is actually shifted to an out of control state. For a fixed value of ARL_0, a chart is considered to be more effective than other charts if it has a smaller ARL_1 [12].

If the process is in control:

$$ARL_0 = \frac{1}{\alpha} \quad (12)$$

If the process is out of control:

$$ARL_1 = \frac{1}{1-\beta} \quad (13)$$

Where α is the probability of a Type I error and β the probability of a Type II error.

SIMULATION STUDIES

The simulation process will be used in this paper to determine the performance of p and p' control chart. Evaluation process of p' chart performance of will be used in same sample size and different sample size. The number of sample size will divide into three categories, the first category is small sample size around 10-100, next is a category with medium sample size 100-1000 subgroup, and another category is large sample size between 1000 and 10000. Then, graphs, based on simulation data for each control chart, will be presented. Finally, the performance from each control chart will be evaluated used Average Run Length for all category sample size with the constant shift of proportion is 0.05 with α=0.0027 refers to three-sigma. Number of run for this simulation is 30 with 1000 times generate binomial distribution data for each run.

SIMULATION RESULT

Performance of p and p' Charts for Same Sample Size

The evaluation of p and p' performance for same sample size will be divided into small, medium and large sample size. With this way, can be determined when p and p' chart effective to use.

Small Sample Size

From data generation of binomial distributed data with p=0.05, graphic illustrated condition of p and p' chart with same sample size for small number of subgroup using three-sigma control limit.

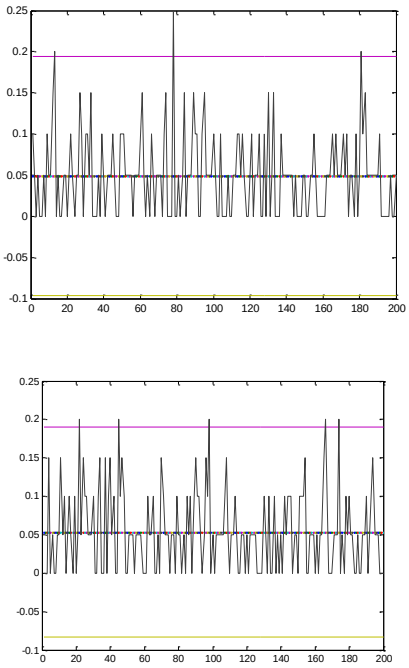

Figure 1 Graphic Visualization of p (left) and p' (right) for n=20

From figure 1 reveals that p and p' chart have same control limit, although p' have slightly narrower, but it can be concluded from the plot that for small sample size p and p' have the same performance. From the graphical visualization, it can be seen that there was no significant difference between p and p' chart for small and same sample size. To strengthen this finding, Average Run Length (ARL) will be used to measure the performance of two control charts. According to table 1, from three different sample size, we can see that ARL_0 size resulted almost same value. The value of ARL_0 increasing while number of sample size become bigger. In figure 2, the three plots of ARL give same performance of two control charts to detect shift. The ARL of p chart is represented by blue line and p' chart is represented by red line.

Table 1: ARL Result for same and small sample size

		n=100		n=70		n=20	
		p	**p'**	**P**	**p'**	**p**	**p'**
ARL_0	p=0.050	246.11	218.696	124.016	123.799	62.903	62.083
ARL_1	p=0.051	192.294	205.804	111.977	111.743	61.449	58.398
	0.052	158.274	171.506	94.279	96.688	55.707	51.956
	0.053	153.156	152.63	83.083	86.231	52.392	50.839
	0.054	144.078	136.112	79.85	75.216	47.337	47.389
	0.055	107.996	118.064	73.891	71.792	47.659	45.251
	0.056	93.592	95.158	63.874	63.306	42.909	44.762
	0.057	85.488	82.656	53.821	59.194	41.134	44.056
	0.058	78.732	73.786	51.181	52.073	37.575	38.209
	0.059	63.922	66.472	46.084	45.572	37.627	35.348

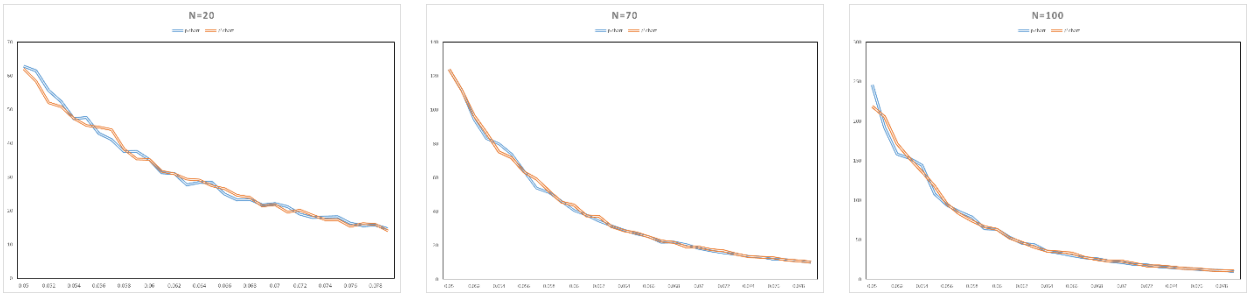

Figure 2: ARL for small sample size

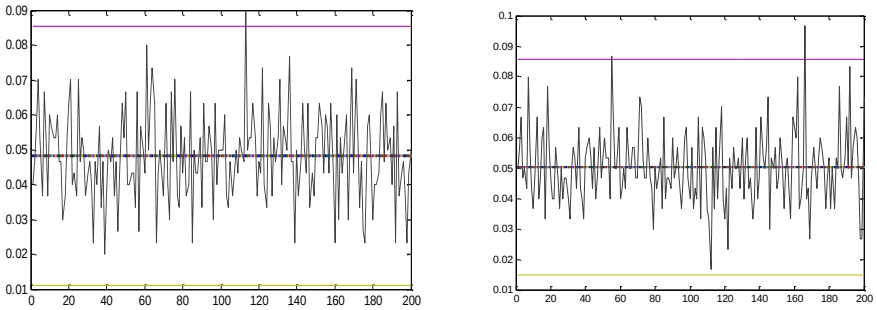

Figure 3 Graphic Visualization of p (left) and p' (right) for n=300

Medium Sample Size

Same as before, first the graphic visualization will be presented, according to figure 3, can be temporary concluded that p and p' chart have almost same control limits, although p' have thinner control limits. However, it cannot be determined yet if chart has same performance or not.

Based on table 2, the ARL_0 of p and p' chart gives an almost same result for 300 and 700 sample size with ARL_0 of p' a little bit lower than p. For 700 sample size, ARL_0 of p' is a little bit smaller than its ARL_0 for 300 and 1000 sample size, this happens because of data is generated randomly, however, its performance is not significantly different from the other sample size. For 1000 sample size, ARL_0 of p is lower than p' chart. In addition, ARL_0 for p is decreasing when the number of subgroups increases, indicating that the performance of p becomes poorer when the number of sample size increases. In general, for a medium number of sample size ARL_0 of each control chart is stable around 300, that indicates that the p and p' charts are better to use in this sample size because the result for their ARL_0 converges to 370. In figure 4, the three plots give the performance of two control chart to detect shift for medium sample size. The ARL of p chart is represented by blue line and p' chart is represented by red line

Table 2 ARL Result for same and medium sample size

		n=1000		n=700		n=300	
		p	**p'**	**p**	**p'**	**p**	**p'**
ARL_0	p= 0.050	290.67	332.204	326.34	216.66	347.474	331.448
ARL_1	p=0.051	240.762	223.548	281.184	187.092	264.804	279.556
	0.052	167.226	163.132	206.654	135.556	218.544	216.242
	0.053	110.224	105.752	147.58	97.192	178.004	181.588
	0.054	79.124	74.27	100.58	71	139.438	137.296
	0.055	50.71	56.426	76.592	51.952	116.062	105.636
	0.056	39.24	38.446	61.34	36.85	91.574	84.636
	0.057	26.964	26.824	42.762	31.06	71.058	63.412
	0.058	20.502	23.91	30.672	21.656	63.404	53.85
	0.059	15.15	15.166	23.706	16.572	50.058	51.07
	0.060	11.876	12.472	19.244	13.81	40.224	39.838

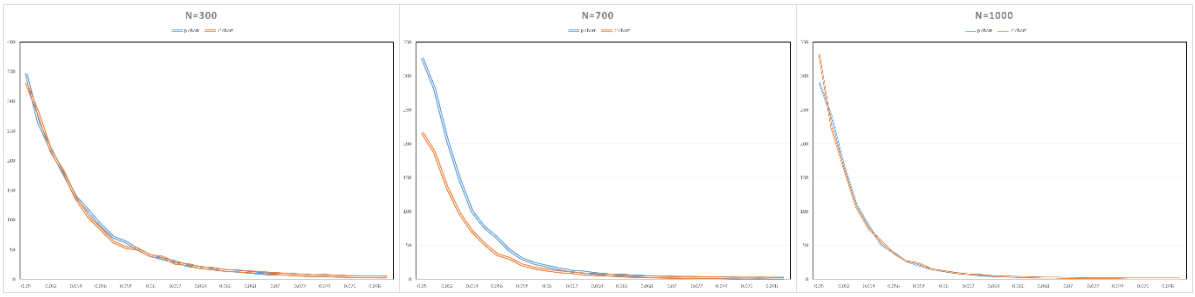

Figure 4: ARL for medium sample size

Large Sample Size

The plot of control charts for large sample size in figure 5 shows that the control limit for p chart not significantly different from p' chart. However, it appears that the control limits of the control chart p become slightly narrower than the p 'chart. From the graphic visualization, it cannot be determined which chart has a good performance for large sample size. Therefore, the ARL will be presented to cater this problem.

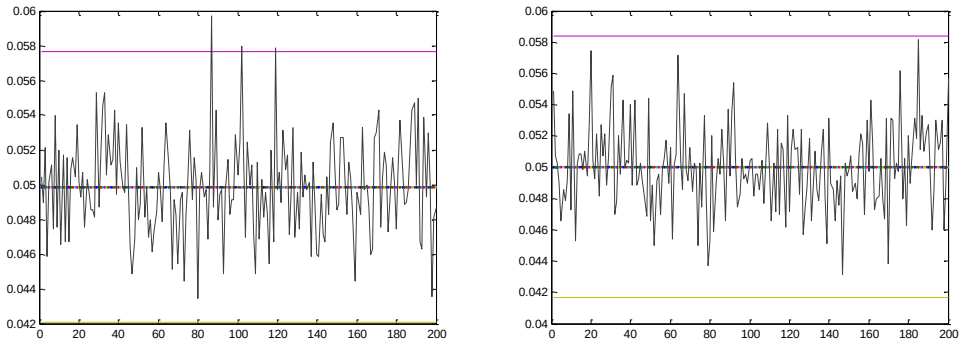

Figure 5 Graphic Visualization of p (left) and p' (right) for n=7000

Table 3 ARL Result for same and large sample size

		n=10000		**n=7000**		**n=3000**	
		P	**p'**	**p**	**p'**	**p**	**p'**
ARL_0	p= 0.050	339.404	410.556	346.232	387.68	293.092	403.886
ARL_1	p= 0.051	153.352	185.362	195.462	198.688	213.328	250.616
	0.052	53.2	56.124	73.016	69.926	108.116	153.618
	0.053	18.788	19.196	26.126	30.852	56.084	75.026
	0.054	8.168	8.826	12.468	13.938	33.274	38.752
	0.055	3.914	4.31	7.222	7.052	18.296	23.084
	0.056	2.39	2.774	4.222	4.162	13.178	13.816
	0.057	1.834	1.74	2.718	2.898	8.034	8.752
	0.058	1.374	1.444	1.92	1.95	5.372	6.172
	0.059	1.184	1.196	1.59	1.506	4.134	4.5
	0.060	1.07	1.074	1.288	1.298	2.85	3.35

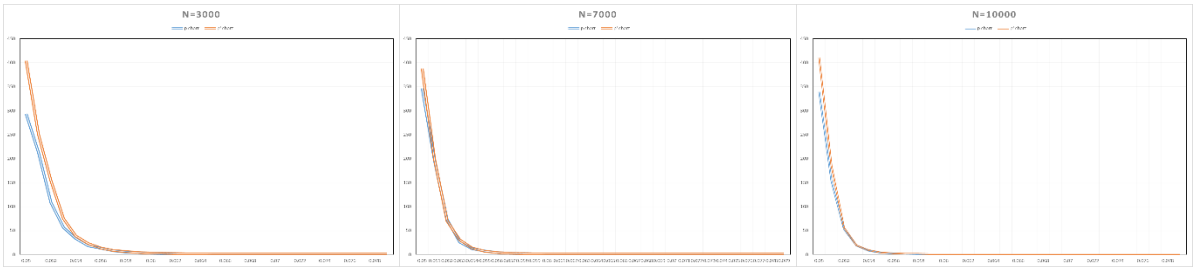

Figure 6 ARL for large sample size

According to table 3, the ARL_0 of the p' chart is always bigger than that of the p chart. However, for same sample size the main problems of narrower control limit from p chart were not clearly visible yet. The ARL_0 of p' chart is stable around 350 until 400 for large sample size. Based on Figure 12, it clearly can be seen that p chart is slightly more sensitive than p' chart based on ARL_1. In general, for same sample size or with same number of subgroups it can be concluded that for small and medium sample size p and p' have same performance, so using either p or p 'chart will produce not many different results. However, for large sample size the p chart becomes a little bit more sensitive than the p chart. Figure 6 shows the performance of p and p' control charts for large sample size.

5.2 Performance of p and p' Charts for Different Sample Size

The main problem of p chart that has narrow control limit for large sample size is not clearly visible for same sample size so this paper will continue the exploration of two control charts when the number of sample size is different. Same as before, the category of sample size will be divided into three categories.

Small Sample Size

The graphic of control charts in figure 7 shows that p and p' charts have the same control limits for small sample size.

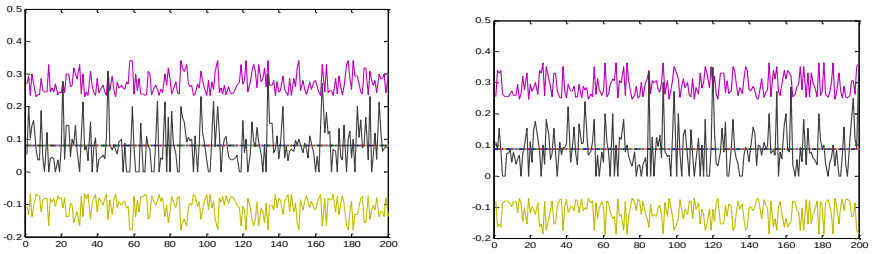

Figure 7: Graphic Visualization of p (left) and p' (right) for n between 10 and 30

Table 4: ARL Result for different and small sample size

		n=10-30		n=20-40		n=30-55		n=50-75		n=70-100	
		P	p'	P	p'	p	p'	p	p'	P	p'
ARL0	p= 0.050	67.7	91.8	80.8	105.8	122.3	161.5	172.9	200.7	180.3	225.7
ARL1	p= 0.051	62.1	83.4	76.7	100.7	111.5	146.3	153.6	179.4	154.2	191.7
	0.052	59.1	78.9	71.0	93.0	100.5	132.5	137.7	159.6	134.8	166.0
	0.053	53.5	71.7	65.4	86.2	91.5	120.6	119.8	138.2	116.5	141.2
	0.054	52.6	68.8	61.2	78.9	83.0	108.9	105.3	122.1	105.6	128.4
	0.055	49.0	65.4	55.0	72.0	71.9	95.6	95.5	110.0	92.9	112.2
	0.056	47.4	61.7	51.8	68.3	66.6	87.3	84.7	99.1	80.2	99.8
	0.057	43.0	57.9	49.0	64.6	61.0	78.6	75.6	87.7	74.1	89.4
	0.058	41.4	54.4	44.9	59.6	54.8	72.8	68.3	79.4	64.1	77.1
	0.059	39.1	51.0	41.0	56.7	50.3	65.6	61.9	71.3	57.3	69.0
	0.060	36.3	47.2	39.7	52.1	47.5	60.4	55.4	63.7	50.9	62.8

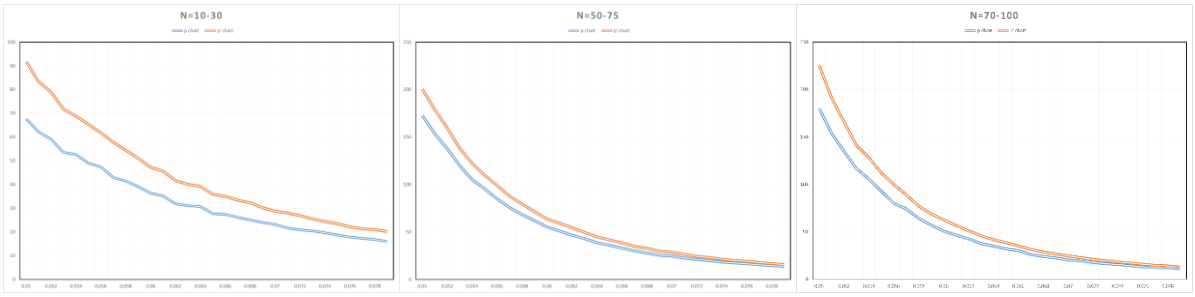

Figure 8 ARL for different and small sample size

Based on table 4, the ARL_0 of p the chart is smaller than that of the p chart in all number of sample size in this case. Furthermore, when the sample size increases, the ARL_0 value of each chart increases, too. Figure 8 shows the ARL of p chart (represented by a blue line), that always is below the red line that represents that of the p' chart. So it can be concluded that p chart is more sensitive in this case. However, the performances of the p and p' charts are not significantly different.

Medium Sample Size

Figure 9 clearly shows the difference between the p and p' charts. The control limits of the p chart become narrower than that of the p' chart. According to table 5, we can see the significant difference between p and p' charts. The ARL0 of the p chart became very small that indicated that the p chart method had poorer performance than the p' chart, in medium sample. In this case, we can see that ARL_0 of p' chart became smaller when number of sample size increasing. The ARL_0 of p' chart is around 300 until 400, that is a realistic result for three-sigma control limit. Figure 10 shows the difference performance between the p and the p' charts. The blue line of the p chart is always below the red line. From the plot, it can be concluded that the p chart became oversensitive and inappropriate for monitoring the process.

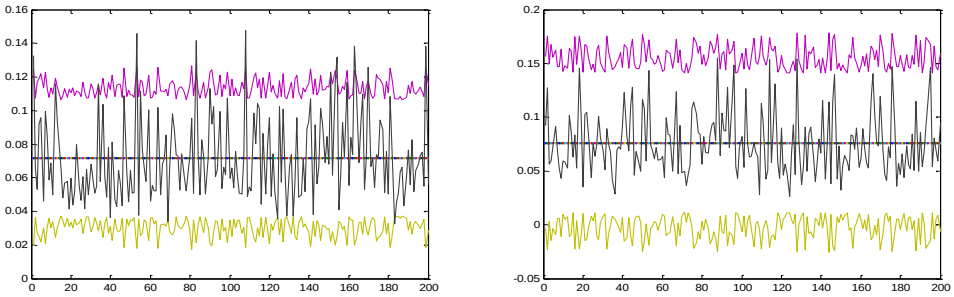

Figure 9 Graphic Visualization of p (left) and p' (right) for n between 200 and 500

Table 5: ARL Result for different and medium sample size

		n=200-500		n=300-600		n=400-700		n=700-1000	
		p	p'	p	p'	P	p'	p	p'
ARL_0	p= 0.050	13.3	401.9	18.6	353.9	28.7	323.5	67.3	318.5
ARL_1	p= 0.051	13.8	328.8	14.8	276.9	24.3	242.7	53.8	234.8
	0.052	14.2	259.9	12.5	201.6	19.6	184.3	40.3	162.8
	0.053	14.0	207.1	10.3	152.3	15.6	133.4	30.3	113.1
	0.054	13.9	167.4	8.7	116.1	12.8	103.5	22.5	86.7
	0.055	13.6	129.1	7.1	88.6	10.5	72.0	16.7	59.3
	0.056	13.2	105.3	5.8	69.8	9.0	55.4	12.6	43.1
	0.057	12.5	85.3	5.0	53.7	7.6	42.1	9.6	31.9
	0.058	11.9	69.5	4.3	40.9	6.6	31.7	7.4	23.9
	0.059	11.5	60.5	3.9	35.0	6.0	24.8	5.9	17.7

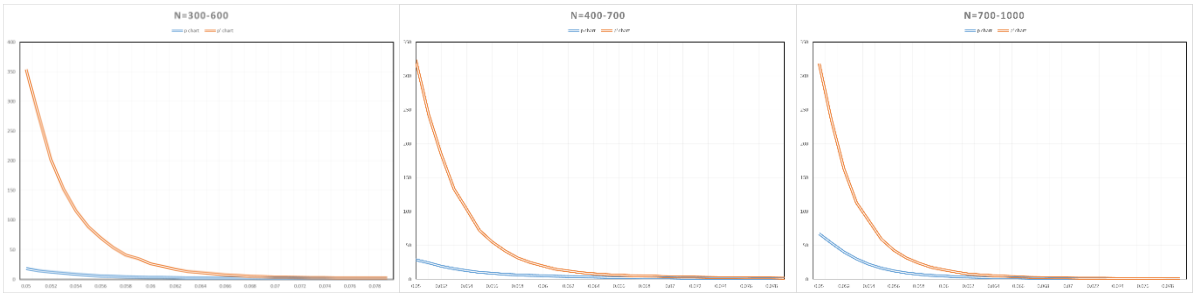

Figure 10 ARL for different and medium sample size

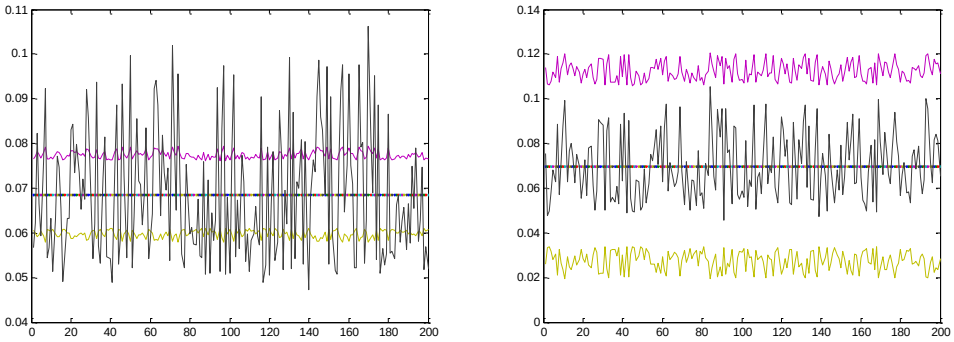

Figure 11 Graphic Visualization of p (left) and p' (right) for n between 5000 and 10000

Large Sample Size

Figure 11 shows the graphic visualization of the p and p' charts for large sample size. In the graphic visualization, it is clearly visible that the p chart has narrower control limits so there are many observations that fall outside of control limits. Vice versa, the p' chart has wide control limits that make all of the observations fall inside the control limits.

The summary of ARL_0 and ARL_1 in table 6 confirms that phenomenon. The ARL_0 of p chart is very small around 1 to 6 while ARL_0 of p' chart is very big around 1000. This makes p' chart appropriate to use for large sample. Something different happens for 5000-10000 sample size, ARL_0 of this sample size is 1000 that means that the control chart has small probability detection of false alarm. In 5000-10000 sample size, ARL1 for p=0.051 until p= 0.056 are around 1000 that indicates that the control chart loses its sensitivity to monitor small shift of proportion. However, for sample size 7000-10000, ARL_1 for this case is constantly smaller. Figure 12 shows the difference between the p (represent by the blue line) and the p'(represent by the blue line) charts for large and different sample size.

Table 6 ARL Result for different and large sample size

		n=1000-2000		n=5000-10000		n=7000-10000	
		p	p'	p	p'	p	p'
ARL_0	p=0.050	5.6	968.6	4.1	1000.0	1.6	994.0
ARL_1	p=0.051	6.0	935.2	3.7	1000.0	1.3	951.7
	0.052	6.1	872.9	3.3	999.7	1.1	829.4
	0.053	6.2	774.8	2.9	999.3	1.1	559.6
	0.054	6.1	652.3	2.6	995.4	1.0	233.4
	0.055	6.0	522.7	2.4	980.4	1.0	94.9
	0.056	5.6	361.7	2.2	917.8	1.0	45.6
	0.057	5.4	237.2	2.1	753.3	1.0	21.3
	0.058	5.1	155.9	2.0	471.8	1.0	10.0
	0.059	4.9	100.7	1.9	237.9	1.0	5.0
	0.060	4.6	67.9	1.8	107.1	1.0	2.6

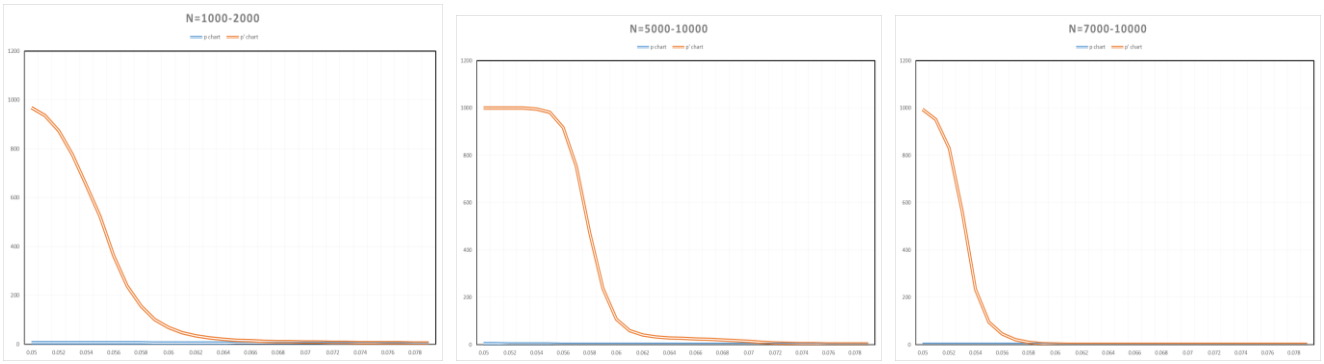

Figure 12: ARL for different and large sample size

CONCLUSION AND FUTURE RESEARCH

Based on simulation results, it was found that there are many points that are interesting findings in this research. First, for the same, particularly for small and medium sample size the p and the p' methods have the same performance, so the use of either the p or the p 'chart will produce not significantly different results. However, for large sample size p chart becomes slightly more sensitive than p' chart. However, there are similarities in the p and p' charts abilities to detect sift of proportions in this case.

For different and small sample size, although p chart is slightly more sensitive than p' chart, there are no significant differences between the two control charts. In medium and different sample size, it can be clearly seen that the p chart becomes more sensitive than the p' chart. The ARL_0 of p chart became very small that indicated that p had a poorer performance than p'. Finally, for large sample size, the p chart becomes extremely oversensitive and cannot be used to monitor the process, while the p' chart gives realistic results.

Based on the exploration of p and p' charts, it is recommended to use p' chart when the sample size is medium and large for a different number of subgroups. However, for same sample size, there are no significant differences between the p and the p' charts, so using both of them is reasonable. For future research, there is an interesting finding, that in large sample size case, for instance, for sample size 5000-10000 the p' chart have lost its ability to detect a shift of proportions. On the other hand, in 7000-10000 number of subgroups p' chart is able to detect the shift, that indicates that the big sample size is not the only factor that changes the p' chart. The future research will study this phenomenon.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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