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Run Length Distribution of Synthetic Double Sampling Chart

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Abstract

The average run length (ARL) has been used as a common characteristic to examine the performance of a control chart. The computation of ARL requires quality practitioners to determine the shift size in advance. This requirement is restricted as the quality practitioners may not know the shift size in advance. In light of this, expected average run length (EARL) is proposed to evaluate the performance of a synthetic type chart when the shift size is random. In this paper, synthetic double sampling (SDS) chart was investigated based on ARL and EARL. Results demonstrate that EARL can be employed as an alternative performance measure when the process shift size is unknown in advance.

Keywords: Quality; control chart; expected average run length.

INTRODUCTION

Quality is a crucial element to be considered in the world of production and manufacturing. This is because quality products or services will increase customers' satisfaction towards these products or services. In view of this, improving quality is a key factor for a successful business. Statistical Process Control (SPC) is a collection of statistical tools, with the objective of reducing variability in the process. Hence, this will indirectly produce high quality products or services. Control chart is one of the SPC tools that is widely adopted in the manufacturing and service industries [1].

In 1924, Dr. Walter A. Shewhart developed the first control chart and named it as the Shewhart chart. The Shewhart chart is commonly used in detecting large process mean shifts. However, one major limitation of the Shewhart chart is that it is insensitive in detecting small and moderate mean shifts. This has motivated many researchers to search for ways to improve the sensitivity of the control chart. The synthetic type chart is an alternative method proposed to enhance the performance of the Shewhart chart. It is well known that the synthetic type chart is a powerful statistical process control tool in monitoring the process mean.

There are two types of synthetic chart, namely, the synthetic chart and synthetic double sampling (SDS) chart. In this paper, the SDS chart is considered due to its advantages in speeding up detection ability in comparison to the synthetic chart and double sampling (DS) chart [2].

The performance of a control chart is important in order to select the appropriate control chart to be used in a process. One common performance measure is average run length (ARL). The ARL refers to the average number of samples plotted on a control chart before it signals an out-of-control [3]. By using ARL as the performance measure, quality practitioners need to specify the magnitude of shift.

Nevertheless, in practice, practitioners may not know the shift size in advance. In view of this, it is crucial to examine the expected average run length (EARL) as an alternative performance measure, where the computation of EARL does not require the practitioners to determine the process shift size. Instead, the EARL computes the expected value of the ARL over the distribution function of the shift size [4]. Other studies evaluating the performance of the control chart when the process shift size is unknown can be found in [5] and [6], to name a few.

In this regard, the performance of the SDS chart was investigated using ARL and EARL. The paper is structured as follows: Section 2 briefly introduces the SDS chart, and this is followed by reviewing the run length properties of the SDS chart. Performance analysis using ARL and EARL is presented in Section 4. Finally, a conclusion is drawn.

OVERVIEW OF THE SDS CHART

The SDS chart was introduced by [2]. It is an integration of the double sampling (DS) sub-chart and a conforming run length (CRL) sub-chart. In 1974, Croasdale [7] introduced the first DS control chart. Daudin et al. [8] and Daudin [9] developed a modified DS chart. Irianto and Shinozaki [10] demonstrated that the performance of the newly developed DS chart ([8]; [9]) is superior to the original DS chart ([7]). Since then, the DS control chart has been extensively studied by several researchers including among other, [11] and [12].

A graphical view of the DS sub-chart is presented in Figure 1. Here, the regions in Figure 1 are defined as: $I_1 = [-L, L]$, $I_2 = [-L_1, -L) \cup (L, L_1]$, $I_3 = (-\infty, -L_1) \cup (L_1, +\infty)$ and $I_4 = [-L_2, L_2]$. Meanwhile, the CRL sub-chart has only one

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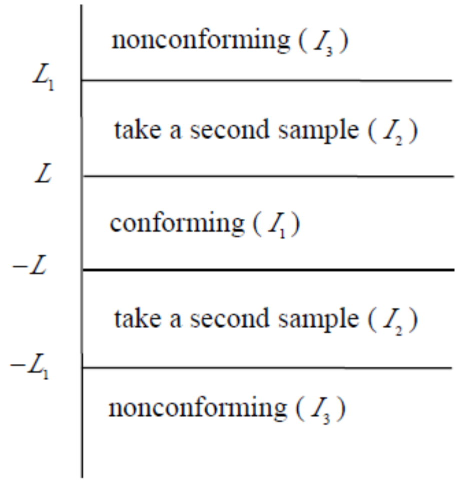
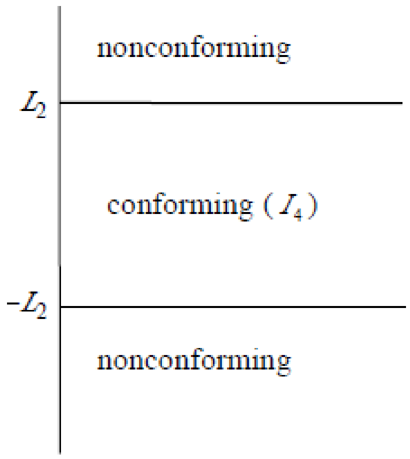
lower limit, i.e. L_3 . A CRL value is the number of conforming units between two nonconforming units (inclusive the ending nonconforming unit).

$$Z_{1,i} = \frac{\bar{Y}_{1,i} - \mu_0}{\sigma_0 / \sqrt{n_1}}$$



(a) First sample

$$Z_i = \frac{\bar{Y}_i - \mu_0}{\sigma_0 / \sqrt{n_1 + n_2}}$$



(b) Combined samples

Figure 1: DS sub-chart

The operation of the SDS chart is elaborated as follows. Take the first sample of size n_1 from the monitoring process. Then, compute the sample mean and standardised statistic, $\bar{Y}_{1,i} = \sum_{j=1}^{n_1} Y_{1j} \big/ n_1$ and $Z_{1,i} = \left[\left(\bar{Y}_{1,i} - \mu_0\right)\sqrt{n_1}\right] \big/ \sigma_0$, respectively. Using Figure 1,

(a) if $Z_{1,i}$ is in I_1 , the i^{th} sampling time is conforming, or

(b) If $Z_{1,i}$ is in I_3 , the i^{th} sampling time is nonconforming, or

(c) If $Z_{1,i}$ is in I_2 , take the second sample with n_2 , and calculate the

(i) sample mean, $\bar{Y}_{2,i} = \sum_{j=1}^{n_2} Y_{2j} \big/ n_2$, and

(ii) combined samples, $Z_i = \left[\left(\bar{Y}_i - \mu_0\right)\sqrt{n_1 + n_2}\right] \big/ \sigma_0$.

Note that $\bar{Y}_i = \left(n_1\bar{Y}_{1,i} + n_2\bar{Y}_{2,i}\right) \big/ \left(n_1 + n_2\right)$. If Z_i is in I_4 , the i^{th} sampling time is conforming. Otherwise, the i^{th} sampling time is nonconforming.

Note that if $Z_{1,i}$ is in I_3 or ($Z_{1,i}$ is in I_2 and Z_i not in I_4), it merely indicates the existence of a nonconforming sample. The CRL sub-chart is required to determine whether or not the sample is out-of-control. If CRL > L_3 , the process is in-control. Otherwise, the process is declared as out-of-control. Thus, appropriate action(s) are required to eliminate the assignable cause(s).

THE RUN LENGTH PROPERTIES OF THE SDS CHART

Assume that the process is independently and identically distributed (i.i.d) having a normal distribution with known in-control mean, μ_0 , and in-control standard deviation, σ_0 . Let P be the probability of a nonconforming sampling time and denoted as follows [2]:

$$P = 1 - P_a - P_b \tag{1}$$

with

$$P_a = \Pr\left(Z_{1,i} \in I_1\right)$$
$$= \Phi\left(L + \delta\sqrt{n_1}\right) - \Phi\left(-L + \delta\sqrt{n_1}\right) \tag{2}$$

and

$$P_b = \Pr\left(Z_i \in I_4 \text{ and } Z_{1,i} \in I_2\right)$$
$$= \int_{z \in I_2^*} \left[\Phi\left(cL_2 + rc\delta - z\sqrt{\frac{n_1}{n_2}}\right) - \Phi\left(-cL_2 + rc\delta - z\sqrt{\frac{n_1}{n_2}}\right)\right]\phi(z)dz. \tag{3}$$

Here, $\Phi(\ .\ )$ and $\phi(\ .\ )$ are the cumulative distribution function (cdf) and probability density function (pdf) of the standard normal distribution, respectively. In addition, $I_2^* = \left[-L_1 + \delta\sqrt{n_1}, -L + \delta\sqrt{n_1}\right) \cup \left(L + \delta\sqrt{n_1}, L_1 + \delta\sqrt{n_1}\right]$, and $r = \sqrt{n_1 + n_2}$ and $c = r \big/ \sqrt{n_2}$.

Finally, the ARL is computed as:

$$\text{ARL} = \frac{1}{P} \times \frac{1}{1-\left(1-P\right)^{L_3}}, \quad (4)$$

where L_3 is the lower control limit for the CRL sub-chart. Computation of the ARL requires the practitioners to determine the process shift size in advance. In practical application, practitioners may not know the process shift size in advance as they do not have historical knowledge of the process. In view of this, the EARL criterion is used as the performance measure of the SDS chart and is defined as:

$$\text{EARL} = \int_{\delta_{\min}}^{\delta_{\max}} f_\delta\left(\delta\right) \text{ARL}\ \text{d}\delta. \quad (5)$$

Note that $f_\delta\left(\delta\right)$ is the probability density function of the magnitude of the shift in a process, i.e. δ.

PERFORMANCE ANALYSIS OF THE SDS CHART

In general, performance of the control chart is measured using average run length (ARL). There are two common ARLs used, namely, the in-control ARL $\left(\text{ARL}_0\right)$ and out-of control ARL $\left(\text{ARL}_1\right)$. Table 1 displays the ARL_1 for different combinations of sample size, n = {3, 5} and shift size, δ = {0.2, 0.5, 0.9, 1.2, 1.5, 1.9}.

In Table 1, the optimal charting parameters $\left(n_1, n_2, L, L_1, L_2, L_3\right)$ are displayed in columns 3 – 8. Results in Table 1 were obtained based on these parameters. Note that the optimal charting parameters $\left(n_1, n_2, L, L_1, L_2, L_3\right)$ will give an intended $\text{ARL}_0 = 370.4$ when δ = 0. For example, when n = 3 and δ = 0.5, the optimal charting parameters are $\left(n_1, n_2, L, L_1, L_2, L_3\right)$ = (2, 6, 1.3830, 5.2804, 2.1867, 18) and the corresponding ARL_1 is 10.41, and yet achieved $\text{ARL}_0 = 370.4$.

In reality, the shift size, δ, may not be known in advance because quality practitioners do not have any historical knowledge on the process or without any experience in monitoring the process to determine the shift size. Furthermore, if a practitioner determines a particular shift size and employs the corresponding optimal charting parameter, the performance of the SDS chart will be significantly deteriorated if a different shift size is occurred in the process. Therefore, it is crucial to measure the process using the performance measure that takes into account a random shifts, i.e. EARL.

Here, two EARLs are usually of interest, namely the in-control EARL, EARL_0 and out-of-control EARL, EARL_1. Note that the EARL_0 is set at 370.4. For comparison purposes, Table 2 summarises the EARL_1 for the same n combinations. The shift interval, i.e. $\left(\delta_{\min}, \delta_{\max}\right)$, is shown in columns 2 – 3.

Table 1: Optimal charting parameters $\left(n_1, n_2, L, L_1, L_2, L_3\right)$ and the corresponding ARL_1s for n = {3, 5} when ARL_0 = 370.4.

n	δ	n_1	n_2	L	L_1	L_2	L_3	ARL_1
3	0.2	2	6	1.3830	5.2804	2.4572	68	96.01
	0.5	2	6	1.3830	5.2804	2.1867	18	10.41
	0.9	2	6	1.3830	5.2804	1.9945	8	2.59
	1.2	2	4	1.1503	5.0443	2.0178	4	1.58
	1.5	2	3	0.9674	4.9920	2.0523	3	1.22
	1.9	2	3	0.9674	4.9920	2.0523	3	1.05
5	0.2	4	10	1.6449	5.1247	2.3394	55	56.60
	0.5	3	10	1.2816	5.1041	2.1216	12	5.40
	0.9	3	8	1.1503	5.0443	2.0186	5	1.67
	1.2	4	4	1.1503	5.0443	2.0615	3	1.18
	1.5	4	4	1.1503	5.0443	1.9647	2	1.04
	1.9	4	3	0.9674	4.9920	2.0363	2	1.00

In order to include the exact shifts size considered in Table 1, $(\delta_{\min}, \delta_{\max})$ = (0.2, 1.0) and $(\delta_{\min}, \delta_{\max})$ = (1.0, 2.0) were considered. For illustration, the shift interval $(\delta_{\min}, \delta_{\max})$ = (0.2, 1.0) and $(\delta_{\min}, \delta_{\max})$ = (1.0, 2.0) include the shifts, δ = {0.2, 0.5, 0.9} and δ = {1.2, 1.5, 1.9}, respectively. For example, when n = 5, $\delta_{\min}$ = 1.0 and $\delta_{\max}$ = 2.0, the optimal charting parameters $(n_1, n_2, L, L_1, L_2, L_3)$ = (4, 9, 1.5813, 3.000, 16.000, 19) yields the lowest EARL_1 = 1.08. By considering δ = 1.5 (i.e. $\delta \in (\delta_{\min}, \delta_{\max})$) for the same n value, the ARL_1 = 1.04 is obtained using the optimal charting parameters $(n_1, n_2, L, L_1, L_2, L_3)$ = (4, 4, 1.1503, 5.0443, 1.9647, 2) from Table 1. This finding shows that the ARL_1 value obtained using the optimal charting parameters, $(n_1, n_2, L, L_1, L_2, L_3)$ by minimising EARL_1 in Table 2 is almost similar with the ARL_1 value obtained using the optimal charting parameters, $(n_1, n_2, L, L_1, L_2, L_3)$ by minimising ARL_1 in Table 1, as long as $\delta \in (\delta_{\min}, \delta_{\max})$.

Table 2: Optimal charting parameters $(n_1, n_2, L, L_1, L_2, L_3)$ and the corresponding EARL_1s for n = {3, 5} when EARL_0 = 370.4.

n	$\delta_{\min}$	$\delta_{\max}$	n_1	n_2	L	L_1	L_2	L_3	EARL_1
3	0.2	1.0	2	6	1.3742	3.000	12.000	1390	30.00
	1.0	2.0	2	6	1.3742	3.000	16.000	58	1.79
5	0.2	1.0	4	9	1.5813	3.000	12.000	584	11.47
	1.0	2.0	4	9	1.5813	3.000	16.000	19	1.08

CONCLUSION

In this paper, it is clearly shown that EARL can be used in place of ARL for the SDS chart when the process shift size is unknown. In the production and manufacturing industries, it is an usual situation where quality practitioners do not have historical knowledge on which process shift size to be implemented. Hence, quality practitioners can employ the proposed optimal charting parameters based on minimising EARL_1. This study is based on the in-control process parameters, i.e. mean, μ_0 and standard deviation, σ_0 are known. In reality, the process parameters are usually unknown. Hence, this study can be extended to consider the SDS chart with estimated process parameters based on minimising EARL_1.

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