

An Energy-Efficient Wireless Sensor Networks Utilizing LMS Filter and Matrix Completion

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Abstract: The *energy consumption* is one of the challenges that faces wireless sensor networks (WSNs) applications which require long lifetimes. Fortunately, most sensing data are spatially and temporally correlated. Low-rank *matrix completion*, which is an extension to compressive sensing, capable of recovering a sensed signal from a small number of random measurements, far below the traditional Nyquist-Shannon limit. The presence of sparsity suggests an adaptive and efficient algorithm to extract only the relevant information for recovery process. In this paper, a new approach hybridized LMS adaptive filter with matrix completion to minimize the necessary information that sensors transmit at WSN is proposed. It is verified that the proposed method reduce WSN energy consumption as compared to traditional method with high reconstruction accuracy.

AMS subject classification:

Keywords: Wireless sensor networks, matrix completion, sparsity, LMS filter, adaptive sampling, sliding window.

1. Introduction

Wireless Sensor Networks (WSNs) face a challenge of limited energy. To overcome this problem, data reduction or compression technique is one key ingredient in prolonging network lifetime. In these data-driven approaches, the goal is typically to reduce the amount of information sent to the sink while achieving the smallest distortion possible in reconstructing the signal. Principal Component Analysis (PCA) has

been widely used to reduce the amount of transmitted data among sensor nodes by finding a small set of uncorrelated linear combinations of original readings. The PCA reduce the traffic by aggregate data form sensors into fewer packets. This distributed technique is executed in intermediate nodes to combine all the incoming packets instead of forwarding them to destinations [1]. Data prediction techniques help for data reduction by building a model describing the sensed phenomenon and is used to generate the predicted data instead of the actually sensed data [2]. This technique exploits the obtained model to reduce the number of data samples thereby reducing the amount of data to be transmitted to the sink. Whenever the requested accuracy is not satisfied, the model needs to be updated, or re-estimated, so that adapted to the changing pattern of the sensed data and to predict it. The adaptive filters used with *dual prediction scheme* (DPS) where sensor and sink both has same filtering model [3], [4], [5], [6]. The LMS adaptive filters, are generally used where there is no knowledge of the sensed signal available [7], [8]. Moreover, it reduces the need for significant training compared to filters such as the Kalman filter. Borgne *et al.*, [5], present *Adaptive Model Selection*, which attempts to reduce the training time for autoregressive models. Several models are run simultaneously in training phase and a racing algorithm is implemented to select the best performing model for data prediction. Unfortunately, these algorithms faces drawbacks presented at lacking in the ability to handle several aspects of real world deployments such as *transmission loss* and *node failure detection*.

The WSN data are very often correlated both over space and time. Sparse modelling algorithms can be used to

detect these redundancies or sparseness, and to recover a high-dimensional data from relatively low number of noisy observations. *Compressive sensing* [9], [10], [11], [12] and *matrix completion* [13]–[14] provide efficient ways to accurately recover the whole data from incomplete information can reduce both sensors activity and communication bandwidth. Both of compressive sensing and matrix completion can recover the signal from fewer samples by solving an optimization problem to significantly reduce the number of transmissions over communication channel [15]. The compressive sensing studies the process of acquiring and reconstructing a signal utilizing the prior knowledge that it is sparse. This sparse signal can be reconstructed from limited number of observations [16], [17]. Similarly matrix completion allows the recovery of a randomly under-sampled low-rank matrix under some conditions [13]. This low-rank matrix completion is considered as an extension from sparse vector to low rank matrix, which compute the temporal and spatial subspaces simultaneously [18]. However, unlike compressive sensing-based methods, matrix completion-based methods do not require the prior dictionary to sparsify the original signal. Cheng *et al.* [19] proposed adaptive sampling approach that is focused on the efficiency of energy consumption. Their work proposed an approach using a new matrix called EDCA (Efficient Data Collection Approach) to lower the sampling rate and make sure fewer packets are transmitted. Recently, a Spatio-Temporal Compressive Data Collection (STCDG) extends the EDCA data gathering system which integrates the low-rank and short-term stability features to reduce the amount of traffic in WSNs and achieves an acceptable recovery accuracy [20].

In this research, a novel approach that utilizing both of adaptive filters and matrix completion for WSNs is proposed. The importance of the proposed approach are as follows: (i) it deals with limited resources of the sensors, (ii) it allows sensing nodes to adaptively samples the sensed data based on changing pattern and randomly, (iii) it reconstructs the missing data with excellent precision at the sink which collect the data from sensors. The proposed approach is evaluated using real environment data and the results indicate that it accomplishes reduction in energy consumption with higher recovery accuracy in comparison with traditional approach. This paper presents the following key contributions:

- It proposes a new approach that utilizing matrix completion and LMS adaptive filter to recover missed or lossy data.
- It prolongs the lifetime of WSN by reducing the number of transmitted samples.

- It evaluates the proposed approach accuracy in data recovery through experimental results and compared with traditional approach.

The remainder of this paper is organized as follows. Section 2, describes a brief mathematical theory of matrix completion. Sections 3 and 4 presents the model description, problem formulation and proposed approach. In Section 5, the data set used in this paper is described. The verification and evaluation of the approach by simulation is presented in Section 6. In the end, Section 7, concludes our work.

2. Matrix Completion

Various algorithms have been proposed to solve matrix completion problem and a recent review of those different algorithms can be found at [21]. Those algorithms can be classified as two categories: exact and approximation algorithms. Exact algorithms solve exactly the nuclear-norm minimization matrix completion problem while approximation algorithms usually make reasonable modification to the formulation of nuclear-norm minimization matrix completion problem. A more general trace norm minimization problem can be formulated as the following optimization problem

$$\min_{\mathbf{X}} \mu \|\mathbf{X}\|_* + f(\mathbf{X}) \quad (2.1)$$

where $f(\mathbf{X})$ can be any differentiable evaluation function on the matrix \mathbf{X} , which defined as the difference between \mathbf{X}_{ij} and \mathbf{A}_{ij} for $(i, j) \in \Omega$. The Ω is the observed entries, \mathbf{X} is the matrix to be recovered and \mathbf{A} is the partially observed matrix. It is possible to recover the low-rank matrix from a few number of its entries using a *convex optimization* program which can be solved directly by some *semi-definite programming* (SDP) packages such as SDPT3 [22], CVX [23] and TFOCS [24]. There are several efficient algorithms have been proposed in literature they use either alternative solvers instead of semi-definite programming (SDP) in nuclear norm minimization or approximating the rank function using other forms rather than the nuclear norm, including Singular Value Thresholding (SVT) [25], Atomic Decomposition for Minimum Rank Approximation (ADMIRA) [26], Fixed Point Continuation with Approximate (FPCA) [27], Accelerated Proximal Gradient (APG) [28], Subspace Evolution and Transfer (SET) [29], Singular Value Projection (SVP) [30], OptSpace [14], and LMafit [31]. It is also possible to generalize the greedy methods of compressive sensing to the rank minimization problem; for instance, ADMIRA [26] generalizes the CoSaMP [32].

3. Model Description and Problem Formulation

Let us assume a static network whose task is to collect at adaptive time instants sensor measurements, and to send them to a destination node. This scheme is standard for real-time WSN applications like environment monitoring. The destination node is commonly referred to as the sink node. We consider a set \mathbf{S} with $S = |\mathbf{S}|$ static sensor nodes that are randomly placed in the area A , at positions $\mathbf{P}_i = (u_i, v_i), i = 1, \dots, S$. Data about the environment monitoring phenomena, such as temperature, represented as scalar value. Using LMS adaptive filter, each sensor node sense and transmit a signal in *non-uniform* basis that obey the dynamic of sensed signal. The transmissions are only made when the predicted state of the signal differs from the current sensor reading by an error condition. Formally, the filter coefficient vector is then updated by

$$\mathbf{w}(t + 1) = \mathbf{w}(t) + \eta \frac{\partial L(t)}{\partial \mathbf{w}(t)} = \mathbf{w}(t) + \eta e(t) \mathbf{x}(t) \quad (3.2)$$

Suppose that a subset $K \leq S$ of sensors are sent their data to the sink which lower energy and bandwidth cost than transmitting from all S sensors. Formally, a sensed phenomenon by the wireless sensor nodes can be expressed as a discrete time signal $x_i(t) \in \mathcal{R}$, where $t \in T$ and each $x_i \in \mathbf{X}, i = 1, \dots, K$ is data value by K sensing nodes readings. The data prediction of the other sensors can be cast as a matrix completion problem. In this setting, there is an unknown matrix, \mathbf{X} , whose columns represent sensors and whose rows represent time steps. An entry of \mathbf{X} is a sensor reading at a time instance t .

$$\mathbf{X}_t = \begin{bmatrix} x_1(1) & x_1(2) & x_1(3) & \dots & x_1(T) \\ x_2(1) & x_2(2) & x_2(3) & \dots & x_2(T) \\ x_3(1) & x_3(2) & x_3(3) & \dots & x_3(T) \\ \dots & \dots & \dots & \dots & \dots \\ x_S(1) & x_S(2) & x_S(3) & \dots & x_S(T) \end{bmatrix} \quad (3.3)$$

4. Utilizing Adaptive Sampling and Matrix Completion

At each time instant, the sensor node adaptively sensed its environment non-uniformly based on the dynamic of the signal using LMS and forwards the data to the sink through a single-hop way. Where at the sink construct $S \times T$ matrix, where T means the maximum window length, i.e., $\mathbf{X} \in \mathbb{R}^{S \times T}$. The matrix will be windowed using a rectangular sliding window of length $0 < w \leq T$, where the sink can

instantaneously reconstruct current data samples from previously completed and collected samples. The window slides one sample at a time, such that, the data inside the sliding window of size w contains the current WSN samples and previous $w - 1$ time instance samples. However, due to the adaptive sampling scheduling, lossy transmission and the failure of sensor nodes, some data are missing and the matrix is incomplete and only K samples are received by sink as listed in algorithm 1. Thus adopting a linear operator $P_\Omega(\cdot)$ to represent such lossy or selection effect(s). The proposed approach is listed in algorithm 2. As demonstrated in Fig. 1, the data will be sampled or collected and transmitted to the sink based on hybridization the LMS filter and Bernoulli random generator. The LMS filter alone is not able to collect the sufficient samples to construct the original signal. It is needed $|\Omega| \geq cnr \log n$ samples [33], [34]. Where \mathbf{X} with size $n \times n$ matrix of rank r . Also the Bernoulli sampling can not capture the dynamic of the signal specially in abrupt changes.

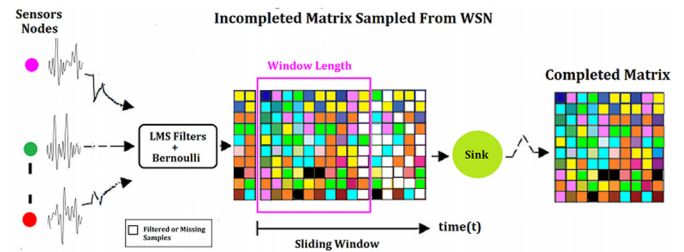


FIG. 1. Representation of the operations of proposed system in a WSN. The sensor nodes use LMS filter and Bernoulli probability to transmit sampled data to the sink. The sink complete these uncompleted or lossy data using matrix completion algorithm.

These collected samples used to recover the missing or lost ones based on the linear operator $P_\Omega(\cdot)$ by exploiting the *spatio-temporal correlation* via matrix completion algorithm. From these known sensor readings we wish to recover, predict or complete, the full matrix \mathbf{X} . Since \mathbf{X} has low-rank structure, the key principle in matrix completion is to fit a low-rank model which captures the important aspects of the data matrix from available data. This low-rank assumption has been used successfully in other matrix completion contexts [25], [35]. The best possible algorithm that will be used in this paper is fixed point continuation algorithm (FPCA) to solve the matrix completion problem [27]. FPCA algorithm is based on Proximal Gradient (PG) with a continuation technique to accelerate convergence and uses a fast Monte Carlo algorithm for SVD calculations implemented to solve the regularized linear least problem:

$$\min_{\mathbf{X}} \mu \|\mathbf{X}\|_* + \frac{\lambda}{2} \|P_\Omega(\mathbf{E})\|_F^2 \quad \text{s.t. } P_\Omega(\mathbf{A}) = P_\Omega(\mathbf{X}) + P_\Omega(\mathbf{E}) \quad (4.4)$$

where μ and λ are regularisation parameters. Solving the formulated problem state in Equ. 4.4 will find the low-rank component corresponding the sensors measurements. The complete proposed algorithms is listed in Algorithm. 2.

Algorithm 1: The adaptive sampling of sensor node

Input: A finite set $S = \{s_1, s_2, \dots, s_S\}$ of sensors

Output: A transmitted data of sensed sensor

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for  $i \leftarrow 1$  to  $S$  do
     $x_i = \text{sense}(s_i)$ ;
     $\hat{x}_i = \text{LMS}(s_i)$ ;
    if  $|\hat{x}_i - x_i| > \theta_{max}$  then
        |  $\text{Transmit}(x_i)$ ;
    else
        if  $\text{Bernoulli}(p) > \delta$  then
            |  $\text{Transmit}(x_i)$ ;
        else
            |  $\text{Discard}(x_i)$ ;
    
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Algorithm 2: WSN data prediction using matrix completion

Input: A finite set $S = \{s_1, s_2, \dots, s_S\}$ of sensors

Input: sliding window of size w

Output: A completed matrix **A**

Data: A matrix **X** with partially observed data

Sliding_Window_Processing(**X**);

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for  $i \leftarrow 1$  to  $S$  do
    if  $\hat{x}_i = \text{Received}(x_i)$  then
        |  $X[i, w] = \hat{x}_i$ ;
    else
        |  $X[i, w] = \text{missed}$ ;
    
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$A = \text{MatrixCompletion}(X)$;

return A;

5. Experimental Dataset

The experimental dataset used to test the proposed approach for WSNs is from Intel Berkeley Research Lab [36]. The collected data consists of real-time data for temperature, humidity, light and voltage sensed by 54 Mica2Dot sensor nodes in a WSN. There are 3000 samples were collected every 31 sec between 6-8 March. The sensor nodes were settled in the lab according to Figure 2. The locations are also given for each node relative to the upper right corner of the lab, which is shown in Figure 2. Temperature data will be used in this paper.

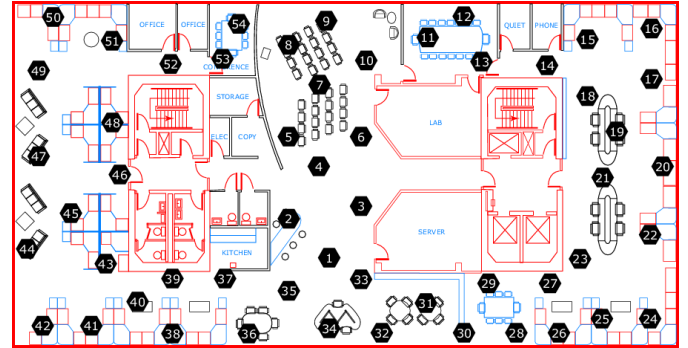


FIG. 2. WSN of Intel Berkeley Research Lab

6. Simulation and Experimental Results

In this section, we seek to evaluate its effectiveness by using convex optimization and K-nearest neighbour (KNN) approach as a traditional one in which each sensor in the network [37]. The following measures are used to evaluate the proposed approach:

- Transmission reduction (or compression ratio) — The percentage of suppressed transmissions.
- Reconstructed signal accuracy — The accuracy of the reconstructed signal compared to the sensed signal measured using Normalized Mean Squared Error (NMSE).

These two measures are used because they enable us to determine the compression ratio that the proposed approach let the sensors to collect more valuable information that has lower error compared to that of the non-adaptive sampling and KNN algorithm using MATLAB *knnimpute* function. The dataset used to analyse the proposed framework consists of data samples from each of the 54 sensors except the nodes 5 and 20 due to irregular readings. The LMS filter initial tap weights are assigned to be zeros and length is chosen to be 5 (i.e. $N = 5$). The value of η for LMS is selected as 0.00001. The temperature threshold level θ_{max} is set to ± 0.5 .

The parameter setting for FPCA was $tol = 10^{-3}$, $\mu = 0.01$, $\lambda = 1$ and $iterations_{MAX} = 500$. All other parameters were set to their respective default values. Figure 4 shows selected number of samples that is transmitted within the range of 500 – 1000 using LMS filter with different temperature error θ_{max} . The 10% of samples are transmitted at $\theta_{max} = 0.5$ degree.

As shown in Figure 3, this percentage is not sufficient for recovering the signals. So, integrating Bernoulli random number generator will increase the percentage to approximately

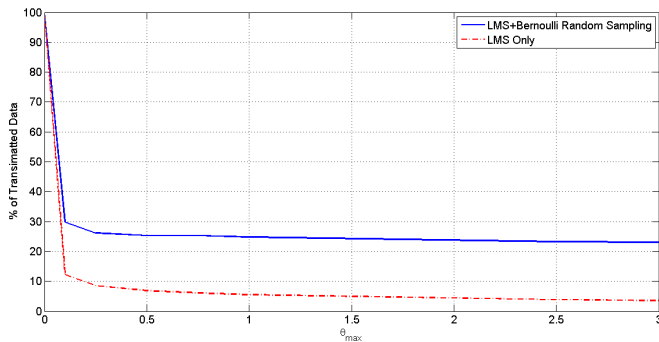


FIG. 3. The transmission percentage of samples using LMS for different temperature error θ_{max}

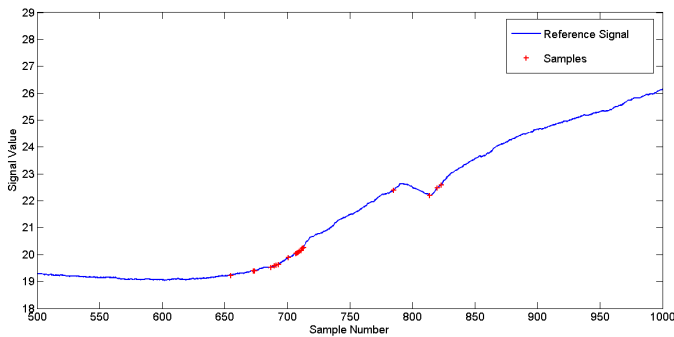


FIG. 4. The adaptive samples using LMS $\eta = 0.00001$ and $\theta_{max} = 0.5$ degree

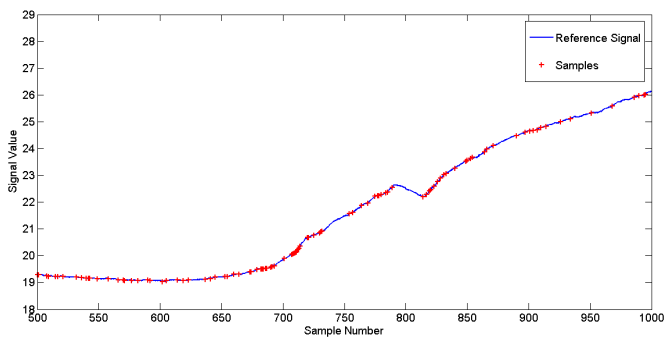


FIG. 5. Hybridization LMS with $\eta = 0.00001$ and $\theta_{max} = 0.5$ degree with Bernoulli random generator with ($p = 0.2$)

28% which enable the recovery algorithms capable to interpolate the complete signals. Thus 72% compression ratio in the transmitted signal which adaptively changed based on the nature of the signal. The transmitted samples for the number of sample of range of 500 – 1000 are shown in Figure 5. The results of applying both of KNN and FPCA recovery algorithms at the sink to predict the missing entries are shown in Figure 6 and Figure 7. In Figure 8, a comparison between the FPCA and the KNN algorithms in terms of the NMSE

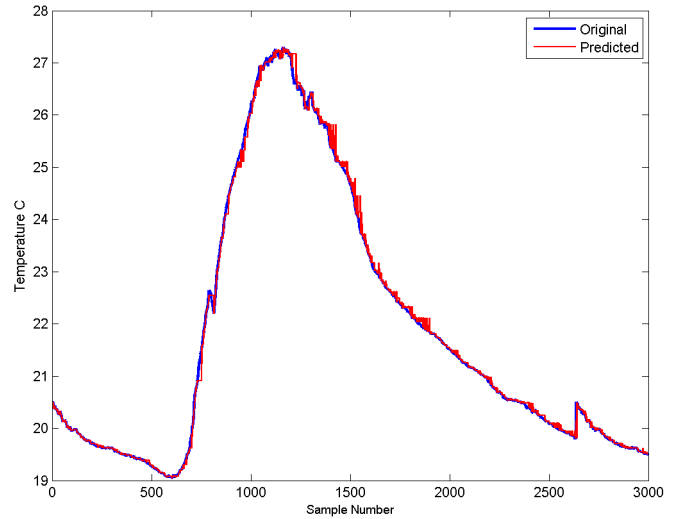


FIG. 6. The predicted signal for sensor one using KNN algorithm at sink

performance. It can be seen that the FPCA has minimum error values for all recovered samples.

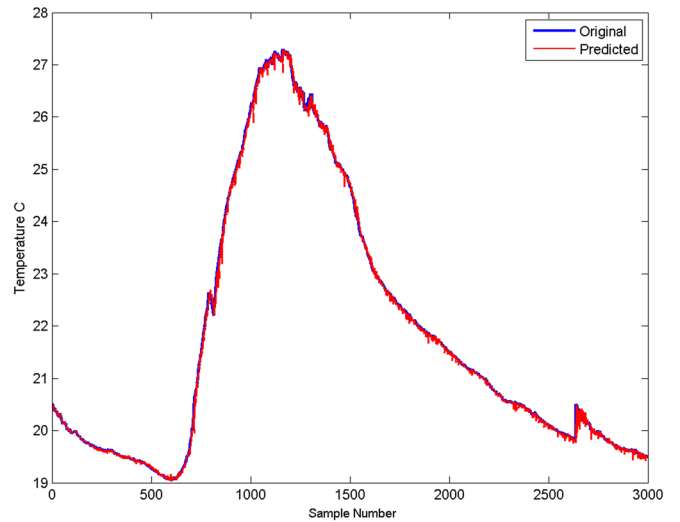


FIG. 7. The predicted signal for sensor one using FPCA algorithm at sink

7. Conclusion

The adaptive sampling concept can also be applied along the time dimension by changing the sampling rate as a function of the signal history. This work proposes a new approach to the problem of energy efficiency in WSNs. The sensors are adaptively sensing the dynamic of the signal using LMS predictor. The sink predict the missing values based on the low-rank

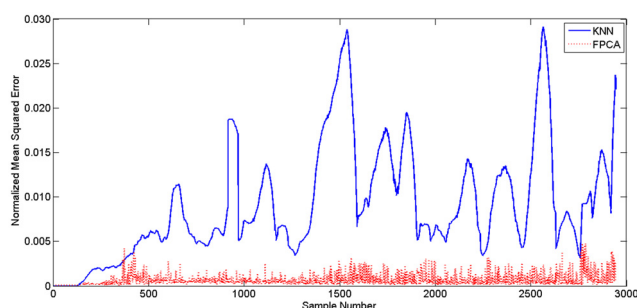


FIG. 8. The Normalized Mean Squared Error using FPCA and KNN algorithms

matrix completion algorithms. However, due to insufficient number of sampling to recover the signal accurately, LMS filter is hybridized with a Bernoulli sampling with low rate. This approach give approximately 72% compression ratio for the transmitted data. Our studies in the future will be planned to study the influence of parameters for suggested algorithm with formulation to its performance with *noise* and *outlier*.

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