

Experimental Identification of Dynamic Parameters of Carbon-Fiber-Reinforced Plastic (CFRP) and Their Use in a National Experimental Complex for Finite Element Analysis

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Abstract

This research is aimed at making an experimental identification of such dynamic parameters of carbon-fiber-reinforced plastic (CFRP) as natural frequencies, and dissipation factors characterizing an orthotropic material of the ply. For this purpose, samples of various symmetrical packings of 4-ply CFRP was used as the beam-strip. Samples fixation was carried out in a horizontal plane at the nodes of the first natural shape of the beam with free ends with a special type holder. Excitation of longitudinal forced oscillations of samples was performed by vibrator plates through the air environment. Precision of elastic characteristics of orthotropic plies controlled by the coincidence of the detected first natural frequency with theoretical value and value received in the finite element formulation of the problem in the experimental sample of a Russian software package. This paper deals with the Classical Theory of Laminates, without taking into account the transverse shear strain. The energy dissipation theory of anisotropic bodies was the basis for calculating the damping factors. According to this theory, the dissipation factor is the ratio of energy loss to the total energy value stored per cycle of bending vibrations by Ni-Adams. According to the calculated logarithmic decrements of each natural sample oscillations by the method of least squares damping capacities in ply directions were determined. To do this, first the samples were analyzed with the longitudinal, transverse and cross plies. Then, the calculated values of the damping capacities in the longitudinal and transverse direction of the ply were used to define damping capacity in the shear direction for angle-ply samples. Damping capacities are necessary to simulate laminated composites of different types in the experimental sample of the Russian software package for dynamic processes such, as a near-resonant forced vibration analysis of CFRP structures.

Keywords: CFRP, natural frequency, elasto-dissipative performance, finite element method.

INTRODUCTION

Quite a lot of research has been done to investigate energy dissipation in laminate composites [1,2,3]. Two basic approaches – visco-elastic and energetic – are used to describe energy dissipation in composites.

Development of computational and experimental techniques relies on the energetic approach to describing internal friction in composites, and it was used as the basis for the technical theory of damping. These approaches have been developed in particular by the scientific school of the Bauman Moscow State Technical University for over two decades, and their accomplishments have been recognized globally now [4,5,6].

The energetic theory of dissipative parameters of anisotropic bodies and fiber composites provides overall evaluation of the dissipated energy; however, it does not reveal actual mechanisms of energy dissipation. The fact damping parameters of modern carbon fiber and carbon class composites do not depend on the frequency at low vibration amplitudes in the range of unities to thousands of Hertz has been determined experimentally [3,7,8].

Energetic approach is about maintaining a linear dependence of stresses and deformations for describing natural damped oscillations, while introducing independent dissipative constants of the material (logarithmic oscillation decrements, relative dissipation factors etc.), along with elastic constants. Application of the energetic approach is usually reduced to determining oscillation frequencies and modes of a conservative system and further using energy balance equation for approximate determination of the oscillation amplitude.

Damping factors in composite structural members depend, as a rule of thumb, on oscillation mode [9], and thus are determined not only by the properties of the material, but dimensions of the structure as well.

A structural phenomenological approach is used for calculating structures of reinforced materials [10,11], which renders a unidirectional or cross-reinforced orthotropic material ply the smallest structural unit. Properties of a multi-layer structural member are functions of ply parameters, structural parameters of a multi-layer packing, and member dimensions and shape. Two coordinates are used for this

purpose: structural member coordinates xyz and natural ply coordinates LTT' .

Structural member coordinates are determined by the member geometry. For beams, x is the center line, and flexion occurs in the xz plane, while xy plane usually coincides with the medial plane.

Natural ply coordinates are determined by the direction the fiber is laid: the L axis coincides with the direction of reinforcement, the T axis is perpendicular to L axis in the plate plane, and T' determines normal to the plate. These are coordinates, where elastic and damping parameters are defined.

The technical theory of damping is developed on the energetic approach by analyzing the energy balance with account for losses for internal frictions for each natural oscillation mode. At the same time, members of the energy equation are determined approximately by modes and frequencies of absolutely elastic oscillations. The energy equation is provided for two amplitude positions of a mechanical system, divided by the oscillation period. As long as velocities of system points equal to zero in these points, the energy equation represents a change in potential energy of the system due to losses per cycle of oscillations. It is also assumed that relative energy losses do not depend on the oscillation amplitude for low amplitudes.

Therefore, each natural oscillation mode of a structural member is assigned a dissipation factor Ψ (absorption constant or relative hysteresis), which is defined as a ratio of mass energy loss per oscillation cycle ΔW and amplitude energy, which equals to maximum potential energy W per oscillation cycle [12]:

$$\Psi = \frac{\Delta W}{W}$$

Product of the dissipation factor and its natural frequency represents dissipation power q , which determines relative energy loss per unit of time for monoharmonic damped oscillations of the structural member [4].

Determination of the dissipation factor spectrum Ψ_i and corresponding natural frequencies f_i and dissipation powers q_i is the task of damping parameter analysis of structural composite members. At the same time, each dissipation power determines damping rate of natural oscillations for this particular mode, and each dissipation factor determines maximum amplitude of forced oscillations in the resonant mode at this particular natural frequency.

An experimental model of a software package to calculate dynamic parameters (DPSP (EM)) was designed locally for time computer-aided simulation of dynamic processes of composite laminate structures, applying finite element analysis. Based on damping factors of a ply, which were determined in experiments, this DPSP (EM) makes possible adequate simulation of such dynamic processes as force oscillations in near-resonant modes, for carbon fiber structures.

THE THEORY.

According to the energetic approach [7], potential deformation energy in the element of volume of the plate δV can be divided into three components that are associated with stresses in the direction of orthotropy parameters, such as L and T :

$$\delta W = \delta W_L + \delta W_T + \delta W_{LT}.$$

Energy δW_L is associated with the deformation energy that was stored in extension- compression in the longitudinal direction, δW_T is associated with the deformation energy that was stored in extension- compression in the transverse direction, and δW_{LT} is associated with the deformation energy that was stored in in-plane shear:

$$\begin{cases} \delta W_L = \frac{1}{2} \cdot \sigma_L \cdot \varepsilon_L \delta V, \\ \delta W_T = \frac{1}{2} \cdot \sigma_T \cdot \varepsilon_T \delta V, \\ \delta W_{LT} = \frac{1}{2} \cdot \sigma_{LT} \cdot \gamma_{LT} \delta V. \end{cases}$$

Deformation energy dissipation in the direction of orthotropy is proportional to the corresponding dissipation factors in this direction:

$$\begin{cases} \delta(\Delta W_L) = \psi_L \cdot \delta W_L, \\ \delta(\Delta W_T) = \psi_T \cdot \delta W_T, \\ \delta(\Delta W_{LT}) = \psi_{LT} \cdot \delta W_{LT}. \end{cases}$$

Thus and so, total dissipation energy of the laminate plate for the oscillation period is calculated as follows:

$$\Delta W = \int_V \delta(\Delta W) = \int_V \delta(\Delta W_L) + \delta(\Delta W_T) + \delta(\Delta W_{LT}) = \Delta W_L + \Delta W_T + \Delta W_{LT}.$$

Finally, the dissipation factor of the laminate plate is calculated as follows:

$$\Psi = \frac{\Delta W}{W} = \Psi_L + \Psi_T + \Psi_{TL}, \quad (1)$$

Where W is stored potential deformation energy that is written in plate coordinates:

$$W = \int_V \delta(W) = \frac{1}{2} \int_V (\sigma_{xx} \cdot \varepsilon_{xx} + \sigma_{yy} \cdot \varepsilon_{yy} + \sigma_{xy} \cdot \gamma_{xy}).$$

Let us consider a symmetrical laminate plate that which is composed of N layers of an orthotropic material (ply), see Figure 1. Total plate thickness is H . k -layer thickness is h_k - h_{k-1} , like laying angles φ_k relative to the longitudinal direction of the plate plane Ox - coincide.

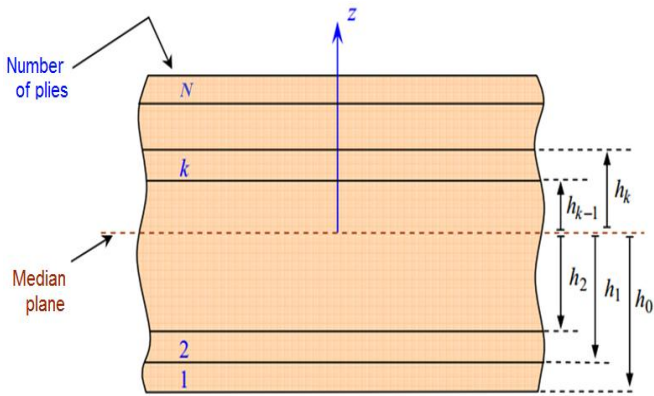


Figure 1. Overall view of a laminate plate that is comprised of N layers.

Constraint equation of momenta and curvature for the symmetrical laminate plate at a net flexion in the plate coordinates xy is written as follows, according to the classical theory of laminate plates [13,7] (it does not account for transverse shear deformations):

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}. \quad (2)$$

Constraint matrix D is calculated through ply rigidity matrices that are reduced from the local coordinates to the material direction Q^k_k :

$$D_{ij} = \frac{1}{3} \sum_{k=1}^N (h_k^3 - h_{k-1}^3) (Q^k_{ij})_k, \quad i, j = 1, 2, 6.$$

For the flexion from the momentum M_x (along the longitudinal plate axis x) curvatures are written, based on (2):

$$\begin{cases} \kappa_x = D_{11}^{-1} M_x, \\ \kappa_y = D_{12}^{-1} M_x, \\ \kappa_{xy} = D_{16}^{-1} M_x. \end{cases} \quad (3)$$

Deformations in the plate coordinates for the net flexion:

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = z \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}. \quad (4)$$

Deformations in the k laminate layer in the local coordinates that are associated with the direction the fiber is laid, turned at the angle φ_k relative to the plate coordinates:

$$\begin{bmatrix} \varepsilon_L \\ \varepsilon_T \\ \gamma_{LT} \end{bmatrix}_k = \begin{bmatrix} m^2 & n^2 & mn \\ n^2 & m^2 & -mn \\ -2mn & 2mn & m^2 - n^2 \end{bmatrix}_k \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix}_k, \quad (5)$$

where the following notations are used: L is direction along the fiber, T is direction across the fiber, $m = \cos \varphi_k$, $n = \sin \varphi_k$.

In the direction the layers are laid, combined ply deformation equations (5) are written as follows by substitution of (4) and (3):

$$\begin{cases} \varepsilon_L^k = z \left[m^2 D_{11}^{-1} + n^2 D_{12}^{-1} + mn D_{16}^{-1} \right] M_x, \\ \varepsilon_T^k = z \left[n^2 D_{11}^{-1} + m^2 D_{12}^{-1} - mn D_{16}^{-1} \right] M_x, \\ \gamma_{LT}^k = -z \left[2mn D_{11}^{-1} - 2mn D_{12}^{-1} - (m^2 - n^2) D_{16}^{-1} \right] M_x. \end{cases} \quad (6)$$

Stresses in the k layer in the plate coordinates are expressed through the rigidity matrix of such layer Q^k_k :

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix}_k = \begin{bmatrix} Q^k_{11} & Q^k_{12} & Q^k_{16} \\ Q^k_{12} & Q^k_{22} & Q^k_{26} \\ Q^k_{16} & Q^k_{26} & Q^k_{66} \end{bmatrix}_k \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix}_k.$$

Substitution of equations for deformations and curvature produces:

$$\begin{cases} \sigma_{xx}^k = z \left(Q^k_{11} D_{11}^{-1} + Q^k_{12} D_{12}^{-1} + Q^k_{16} D_{16}^{-1} \right) M_x, \\ \sigma_{yy}^k = z \left(Q^k_{12} D_{11}^{-1} + Q^k_{22} D_{12}^{-1} + Q^k_{26} D_{16}^{-1} \right) M_x, \\ \sigma_{xy}^k = z \left(Q^k_{16} D_{11}^{-1} + Q^k_{26} D_{12}^{-1} + Q^k_{66} D_{16}^{-1} \right) M_x. \end{cases}$$

Stresses in the k laminate layer in the local coordinates that are associated with the direction the fiber is laid, turned at the angle φ_k relative to the plate coordinates:

$$\begin{bmatrix} \sigma_L \\ \sigma_T \\ \sigma_{LT} \end{bmatrix}_k = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix}_k \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix}_k.$$

As long as this research further considers the net flexion of a freely fixed laminate beam, it is assumed that σ_{yy} are σ_{xy} negligibly small. Then the combined equations in the direction that the layers are laid are written as:

$$\begin{cases} \sigma_L^k = z \left[Q^k_{11} D_{11}^{-1} + Q^k_{12} D_{12}^{-1} + Q^k_{16} D_{16}^{-1} \right] M_x \cdot m^2, \\ \sigma_T^k = z \left[Q^k_{11} D_{11}^{-1} + Q^k_{12} D_{12}^{-1} + Q^k_{16} D_{16}^{-1} \right] M_x \cdot n^2, \\ \sigma_{LT}^k = -z \left[Q^k_{11} D_{11}^{-1} + Q^k_{12} D_{12}^{-1} + Q^k_{16} D_{16}^{-1} \right] M_x \cdot mn. \end{cases} \quad (7)$$

After stresses and deformations in the direction the k -laminate ply is laid were introduced, dissipated member-by-member must be determined for the oscillation period for the plate beam of b width, L length, H height and with N number of plies that equals to the width:

$$\begin{cases} \Delta W_L = b \int_{x=0}^L \left[2 \int_{z=0}^{H/2} \frac{1}{2} \psi_L \sigma_L \varepsilon_L dz \right] dx, \\ \Delta W_T = b \int_{x=0}^L \left[2 \int_{z=0}^{H/2} \frac{1}{2} \psi_T \sigma_T \varepsilon_T dz \right] dx, \\ \Delta W_{LT} = b \int_{x=0}^L \left[2 \int_{z=0}^{H/2} \frac{1}{2} \psi_{LT} \sigma_{LT} \varepsilon_{LT} dz \right] dx. \end{cases}$$

For small beam oscillations, dissipation factors do not depend on the amplitude of such oscillations and can be written as:

$$\begin{cases} \Delta W_L = b \cdot \psi_L \int_{x=0}^L \left[\int_{z=0}^{H/2} \sigma_L \varepsilon_L dz \right] dx, \\ \Delta W_T = b \cdot \psi_T \int_{x=0}^L \left[\int_{z=0}^{H/2} \sigma_T \varepsilon_T dz \right] dx, \\ \Delta W_{LT} = b \cdot \psi_{LT} \int_{x=0}^L \left[\int_{z=0}^{H/2} \sigma_{LT} \varepsilon_{LT} dz \right] dx. \end{cases}$$

When stresses and deformations in the direction the composite is laid are substituted, the equation reads:

$$\Delta W_L = b \cdot \psi_L \int_{z=0}^{H/2} F_L z^2 dz \cdot \int_{x=0}^L M_x^2 dx, \quad (8)$$

where

$$F_L = m^2 \left[Q_{11}^k D_{11}^{-1} + Q_{12}^k D_{12}^{-1} + Q_{16}^k D_{16}^{-1} \right] \left[m^2 D_{11}^{-1} + n^2 D_{12}^{-1} + mn D_{16}^{-1} \right].$$

For the symmetrical composite with its plies of equal width and height of the upper and lower planes, which are located higher than the medial plane, for $k = 1 \dots N/2$:

$$\begin{cases} \bar{z} = h_k = \left(\frac{H}{2} / \frac{N}{2} \right) k = \frac{H}{N} k, \\ \underline{z} = h_{k-1} = \left(\frac{H}{2} / \frac{N}{2} \right) (k-1) = \frac{H}{N} (k-1). \end{cases}$$

When heights are substituted in (8) and the integral is decomposed according to laminate weight:

$$\Delta W_L = b \cdot \psi_L \cdot F_L \left[\sum_{k=1}^{N/2} \frac{z^3}{3} \right]_{\frac{H}{N}}^{k \frac{H}{N}} \cdot \int_{x=0}^L M_x^2 dx$$

Thus:

$$\Delta W_L = b \cdot \psi_L \cdot F_L \cdot \frac{H^3}{3N^3} \cdot \left[\sum_{k=1}^{N/2} k^3 - (k-1)^3 \right] \cdot \int_{x=0}^L M_x^2 dx$$

Stored potential deformation energy from the moment M_x :

$$W = \frac{b}{2} \int_{x=0}^L M_x \cdot \kappa_x dx = \frac{b}{2} D_{11}^{-1} \int_{x=0}^L M_x^2 dx.$$

Finally, the first member of the sum of total dissipation factor Ψ (1):

$$\Psi_L = \Delta W_L / W = \psi_L \cdot \frac{F_L}{D_{11}^{-1}} \cdot \frac{2H^3}{3N^3} \cdot \left[\sum_{k=1}^{N/2} k^3 - (k-1)^3 \right].$$

Similarly, the second and the third member of the sum of total dissipation factor Ψ :

$$\Psi_i = \Delta W_i / W = \psi_i \cdot \frac{F_i}{D_{11}^{-1}} \cdot \frac{2H^3}{3N^3} \cdot \left[\sum_{k=1}^{N/2} k^3 - (k-1)^3 \right], i = T, LT.$$

F_T and F_{LT} according to (6) and (7):

$$\begin{cases} F_T = n^2 \left[Q_{12}^k D_{11}^{-1} + Q_{22}^k D_{12}^{-1} + Q_{26}^k D_{16}^{-1} \right] \left[n^2 D_{11}^{-1} + m^2 D_{12}^{-1} - mn D_{16}^{-1} \right], \\ F_{LT} = mn \left[Q_{11}^k D_{11}^{-1} + Q_{12}^k D_{12}^{-1} + Q_{16}^k D_{16}^{-1} \right] \left[2mn D_{11}^{-1} - 2mn D_{12}^{-1} - (m^2 - n^2) D_{16}^{-1} \right]. \end{cases}$$

Thus, dissipation factor for oscillations along the central line of the laminate beam is as follows, when oscillations in other plate planes are negligibly small:

$$\Psi_{bx} = \Psi = \Psi_L + \Psi_T + \Psi_{LT}.$$

As long as total dissipation factor equals to doubled damping decrement [5,14]:

$$\Psi_{bx} = 2\delta,$$

then, the equation is made for the damping decrements for each type of laying pattern φ and each beam unit, which were calculated in experiments:

$$\Psi_{bx}(\varphi) - 2\delta(\varphi) = 0. \quad (9)$$

Finally, combined equations (9) for each beam unit were solved, using the least square method and the following substitutions:

$$(\Psi_{bx}(\varphi) - 2\delta(\varphi))^2 \rightarrow \min. \quad (10)$$

This research further deals with determination of natural frequencies.

Total behavior differential equation of an undamped beam for longitudinal oscillations against zero forces in the plate plain is [15]:

$$\frac{\partial^2 M_x}{\partial x^2} + p = \rho_s \frac{\partial^2 w_0}{\partial t^2},$$

where ρ_s is mass per cross-sectional unit area p is normal pressure applied to the beam, and w_0 is beam deflection.

According to (3),

$$\kappa_x = -\frac{\partial^2 w_0}{\partial x^2} = D_{11}^{-1} M_x,$$

natural oscillations equation is:

$$\rho_s \frac{\partial^2 w_0}{\partial t^2} + k_s \frac{\partial^4 w_0}{\partial x^4} = 0,$$

where k_s determines rigidity per cross-sectional unit area:

$$k_s = \frac{1}{D_{11}^{-1}}.$$

Total transverse oscillations equation is:

$$\frac{\partial^2 w_0}{\partial t^2} + \omega_0^2 L^4 \frac{\partial^4 w_0}{\partial x^4} = 0, \quad (11)$$

thus, circular frequency of the undamped beam of H height is:

$$\omega_0 = \frac{1}{L^2} \sqrt{\frac{k_s}{\rho_s}} = \frac{1}{L^2} \sqrt{\frac{1}{\rho_s D_{11}^{-1}}} = \frac{1}{L^2} \sqrt{\frac{E_x^b H^3}{12 \rho_s}},$$

where E_x^b is an effective flexural modulus of elasticity:

$$E_x^b = \frac{12}{H^3 D_{11}^{-1}} \quad (12)$$

Deflection for the i th natural mode for a certain beam point is presented in the form that is separated in time and coordinates:

$$w_0(x, t) = w_i(x)(A \cos \omega_i t + B \sin \omega_i t), \quad (13)$$

where $w_i(x)$ is the form of the i th natural mode and ω is its circular frequency.

Substitution of (13) into (11) produces:

$$\frac{\partial^4 X_i}{\partial x^4} - \frac{1}{L^4} \frac{a_i^2}{\omega_0^2} X_i = 0, \quad (14)$$

and this equation can be generally written as:

$$X_i = w_i(x) = \cos \lambda_i \frac{x}{L} + \cosh \lambda_i \frac{x}{L} + \gamma_i \left(\sin \lambda_i \frac{x}{L} + \sinh \lambda_i \frac{x}{L} \right). \quad (15)$$

Thus, taking into account (14), natural frequencies of the i th mode are:

$$\omega_i = \lambda_i^2 \omega_0. \quad (16)$$

Parameters λ_i and γ_i of equation (15) are determined by boundary conditions of beam fixation. For free fixation of beam ends, λ_i and γ_i are identical to the values obtained for complete fixation of both ends. Except for additional roots for free fixation, which are associated with beam moving and rotating as a whole. For the first natural mode that is associated with beam oscillations in the transverse plane, $\lambda = 4.73$ and $\gamma = -0.9825$. The first mode is illustrated in Figure 2.



Figure 2. The first natural mode for free fixation of beam ends.

THE EXPERIMENT

Samples that were used for the experiments were plate beams with length of $L = 0.4$ m, width $b = 0.020$ m and height $H = 0.00132$ m. All samples comprised four unidirectional layers of the investigated material - carbon fiber based on the unilateral carbon fabric SAATITEXINDUSTRIAUC350 and epoxy compound Etal-InjectSL (M), made by vacuum infusion. Sample material density was $\rho = 1,500$ kg/m³.

Structure of the investigated samples was as follows:

- $[\pm\varphi]S$ for $\varphi = 20^\circ, 40^\circ, 50^\circ, 70^\circ$;
- $[0^\circ]4T$;
- $[90^\circ]4T$;

- $[0^\circ/90^\circ]S$;
- $[90^\circ/0^\circ]S$.

Three sample beams of each structure were presented for the experiments.

Parameters of the unidirectional layer (ply) that were pre-calculated are as follows:

- $E_L = 103.5$ GPa,
- $E_T = 6.7$ GPa,
- $G_{LT} = 5.5$ GPa,
- $\nu_{LT} = 0.27$.

Tests were done at the composite laboratory of the R&D Institute for Special Engineering of the Bauman Moscow State Technical University in accordance with a certain technical assignment for R&D Experimental identification of Experimental identification of elastic and dissipative parameters of carbon fiber. Tests were done in accordance with technique *UDKh-PO-LAKOM-12/15* on a test unit, based on the VEDS-10A vibration table [6].

The experiment was done as follows.

The sample was suspended horizontally and hinge-fastened at its medial surface in nodes of the first mode of longitudinal flexural oscillations of the free fixing condition, see Figure 2 and Figure 3.

Dynamic test unit layout is given in Figure 4, and photo is given in Figure 5.

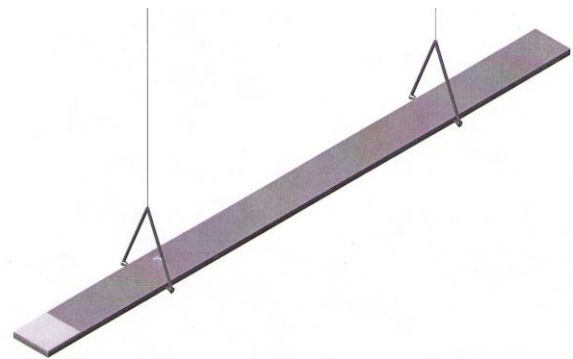


Figure 3. Suspending a sample with a foil tape attached to record displacements

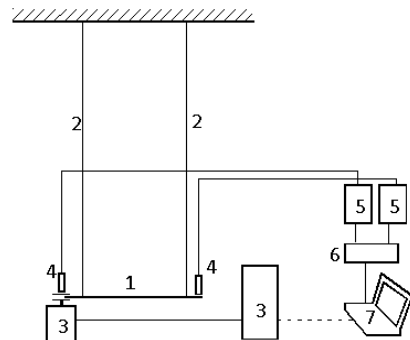


Figure 4. Dynamic test unit layout.

- 1 - PMK sample;
- 2- suspension threads;
- 3 - VEDS-10A vibration table;
- 4 - non-contacting vibration displacement gages;
- 5 - VVV-302 eddy current vibration sensors;
- 6 - LA2-USB a/d converter;
- 7 - PC.



Figure 5. General view of a dynamic image test unit

Vibration displacements of vibrator fan plates were given in a certain frequency range, and oscillations were transmitted to the sample via the acoustic medium. Resonant frequency f of the first mode of flexural oscillations was determined according to the maximum amplitude of sample oscillations. Its value is determined according to the spectrum, using Fourier series transformation.

The vibrator was turned off from the control panel after a certain period of time of oscillations at the resonant frequency (single-frequency oscillations), and vibrations of the fans and the sample damped.

Oscillation damping image was stored in the PC memory and shown on the display as a process record, see Figure 6.

Damping parameter was determined according to sample vibration displacement gage readings on the vibro-record section that corresponds to complete movement stop of driving plates, see Figure 6, in the area that is highlighted with a white frame.

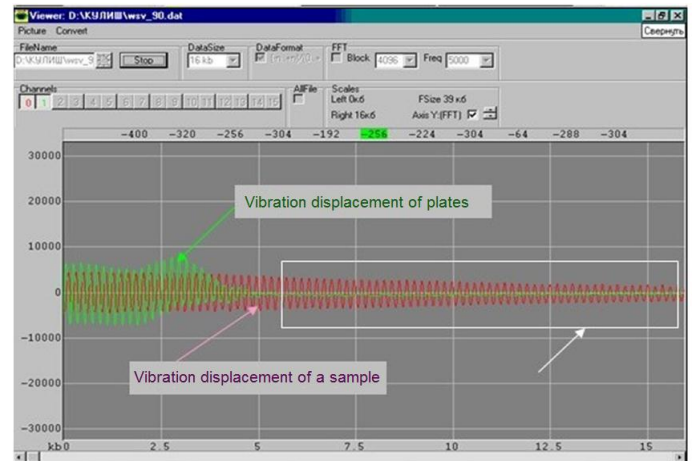


Figure 6. Screen shot for fixation of vibration displacements and damping parameter analysis area

Maximum values exclusively were selected to calculate the damping decrement from the record of natural sample oscillations. Maximum sequence with view to period numbers and their approximating dependence are presented in Figure 7. This diagram is provided for the second sample with laying pattern $[\pm 20^\circ]_S$.

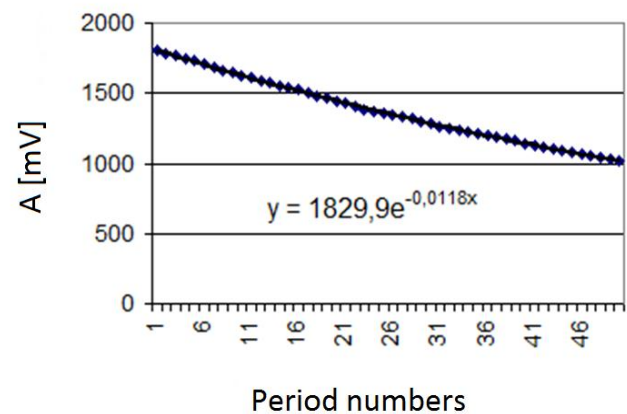


Figure 7. Amplitude maximums of natural oscillations with laying pattern $[\pm 20^\circ]_S$ with frequency of 49.8 Hz with view to period number and approximating dependence

In order to determine logarithmic damping decrement of samples with varying laying patterns, calculated amplitude and time dependences were used. In particular, the average logarithmic damping decrement was determined, using three

recorded (see Figure 7) natural oscillations of samples with laying pattern $[\pm 20^\circ]_S$ for flexural oscillations $\delta = 1.19 \pm 0.04 \%$. Table 1 presents calculated values first natural frequencies and logarithmic damping decrements for samples with varying laying patterns.

Table 1. First frequency of natural oscillations of samples of varying laying patterns and logarithmic damping decrements

Laying pattern	f_1 [Hz]	δ [%]
$[0^\circ]_4T$	73.2	0.47 ± 0.06
$[\pm 20^\circ]_S$	49.8	1.19 ± 0.04
$[\pm 40^\circ]_S$	29.3	1.92 ± 0.10
$[\pm 50^\circ]_S$	25.4	2.19 ± 0.07
$[\pm 70^\circ]_S$	20.5	2.61 ± 0.38
$[90^\circ]_4T$	19.5	2.19 ± 0.07
$[0^\circ/90^\circ]_S$	68.4	0.46 ± 0.06
$[90^\circ/0^\circ]_S$	32.2	1.13 ± 0.23

DATA ANALYSIS

First, frequencies that were obtained in the experiments will be compared with theoretical frequencies and then with frequency values produced by the DPSP (EM).

Table 2 represents theoretical first natural frequencies of a freely fixed beam that were calculated according to (16).

Table 2. First frequency of natural oscillations of samples of varying laying patterns and effective moduli of elasticity tension/ compression and flexion (12)

Laying pattern	$\omega_1 / (2\pi)$ [Hz]	E_x [GPa]	E_x^b [GPa]
$[0^\circ]_4T$	71.0	103.5	103.5
$[\pm 20^\circ]_S$	51.1	71.8	53.6
$[\pm 40^\circ]_S$	31.7	24.0	20.6
$[\pm 50^\circ]_S$	25.5	14.6	13.4
$[\pm 70^\circ]_S$	19.5	8.0	7.8
$[90^\circ]_4T$	18.1	6.7	6.7
$[0^\circ/90^\circ]_S$	66.8	55.3	91.7
$[90^\circ/0^\circ]_S$	30.3	55.3	18.9

A beam model was developed for calculations in the DPSP (EM), using flat square finite elements with side dimensions of $\Delta x = 0.002$ m, $\Delta y = 0.0025$ m, see Figure 8.

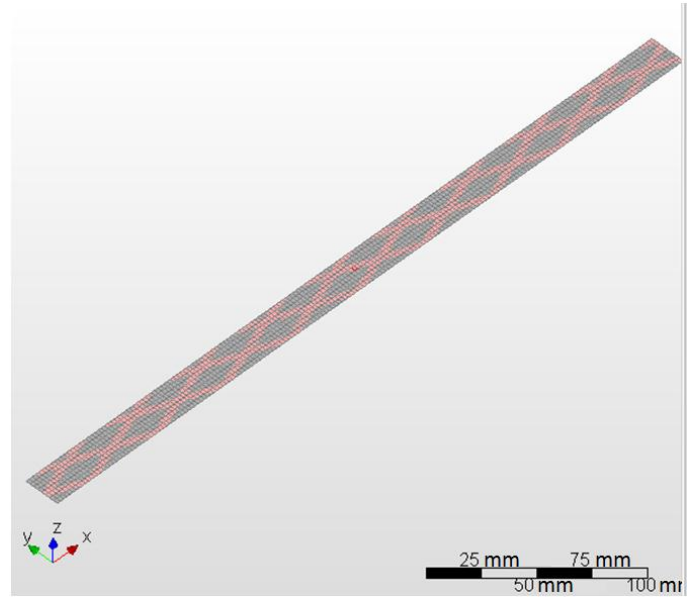


Figure 8. Model of a freely fixed beam with laying pattern of $[\pm 20^\circ]_S$

Table 3 provides first natural frequencies of freely fixed beams of varying laying patterns that were obtained in theory, in experiments, and using the DPSP (EM).

Table 3. First natural frequency of natural oscillations of freely fixed samples of varying laying patterns that were calculated in theory, in experiments and in the DPSP (EM).

Laying pattern	$\omega_1^{\text{theor}} / (2\pi)$ [Hz]	f_1^{exp} [Hz]	$f_1^{\text{DPSP (EM)}}$ [Hz]
$[0^\circ]_4T$	71.0	73.2	70.6
$[\pm 20^\circ]_S$	51.1	49.8	50.8
$[\pm 40^\circ]_S$	31.7	29.3	31.6
$[\pm 50^\circ]_S$	25.5	25.4	25.4
$[\pm 70^\circ]_S$	19.5	20.5	19.4
$[90^\circ]_4T$	18.1	19.5	18.0
$[0^\circ/90^\circ]_S$	66.8	68.4	66.3
$[90^\circ/0^\circ]_S$	30.3	32.2	30.1

Frequency data that were calculated theoretically and computed, using the DPSP (EM), differ by not more than 1%. However, the discrepancy between values that were obtained in the experiments and using the DPSP (EM), is up to 8.3% for laying pattern $[90^\circ]_4T$. Such error can be explained by different thickness of real and model images that are used in the calculation.

If absolute frequency values are taken, the discrepancy of the calculation and the experiment does not exceed 3 Hz, which can be considered a good result.

The first mode of the freely fixed laying patten sample $[\pm 20^\circ]_S$ is illustrated in Figure 9.

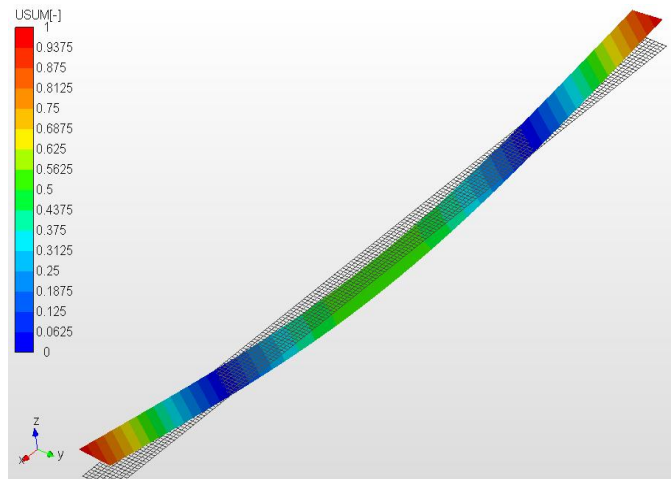


Figure 9. The first mode of a freely fixed laying patten sample $[\pm 20^\circ]_S$

According to Figure 7, zero displacement lines are not perpendicular to the beam central line X, which is closely associated with non-zero elements of the constraint matrix of momenta and curvature that are responsible for twisting. As it turned out, the position of the zero displacement line of the freely fixed beams differs most from the perpendicular to axis x just for the laying pattern sample $[\pm 20^\circ]_S$. Discrepancy of zero displacement points for such laying pattern along axis x for width $b = 0.020$ m was 0.0123 m.

The task of finding dissipation factors of the unidirectional carbon fiber layer will be discussed next.

For this purpose, the research was done in two steps.

At the first steps, factors ψ_L and ψ_T were determined. Experimental findings for the structures that comprise exclusively longitudinal and transverse layers (structures $[0^\circ]_4T$, $[90^\circ]_4T$, $[0^\circ/90^\circ]_S$ and $[90^\circ/0^\circ]_S$) were used for this purpose. Function (10) was minimized with varying damping values in the range of ψ_L 0.80 to 1.10% and of ψ_T 3.00 to 6.0%, which is typical for the carbon fiber ply.

To make sure that varying parameters found ensure minimum of function (10), global grid search was done with a step of 0.01% for both variables.

Dissipation factors of the unidirectional carbon fiber layer that were identified this way are: $\psi_L = 0.98\%$, $\psi_T = 4.44\%$.

At the second step, values of ψ_L and ψ_T obtained along with given values of E_1 , E_2 , G_{12} and ν_{12} were used to identify factor ψ_{LT} ; at the same time, experimental values of dissipation factors of cross-reinforced structures $[\pm\phi]_S$ were analyzed.

Target function (10) was the sum of squared deviations of calculated dissipation factors for the structures $[\pm 20^\circ]_S$, $[\pm 40^\circ]_S$, $[\pm 50^\circ]_S$ and $[\pm 70^\circ]_S$ from the corresponding experimental data. Final dissipation factor of the unidirectional carbon fiber layer in shear was $\psi_{LT} = 5.02\%$.

Using these values, factors of the matrix of elastic damping parameters of the ply were obtained [5,17]:

- $P_{11} = 1.019$ GPa,
- $P_{12} = 0.04925$ GPa,
- $P_{22} = 0.2962$ GPa,
- $P_{66} = 0.2761$ GPa.

Calculation of forced oscillations can be done further, where natural oscillations and damping will be simulated with damping parameters determined.

In order to neglect displacement and turning of the item as a whole, hinge support was done in the points with zero displacement of the first mode for the simulation of forced oscillations of the beam model.

Calculation can be done in the DPSP (EM) using forth integration or expansion in natural modes [18]. For the first case, elastic damping parameters of finite structural members must be given, based on elastic damping parameters of the ply and laminate laying pattern. For the second case, damping decrement of the whole structure and number of modes that are used for the calculation must be given.

Implementation of the second approach in the DPSP (EM) is shown further. To disturb the balance of the model, a distributed force was applied at the center of the beam half the first period of natural oscillations, and such force equaled to zero in any further time markings. Such model of forced oscillations is illustrated in Figure 10.

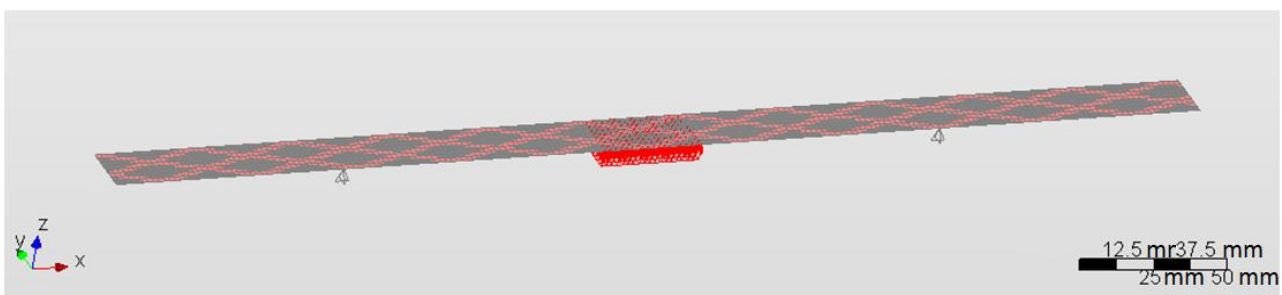


Figure 10. Model of forced oscillations of a sample with laying pattern $[\pm 20^\circ]_S$

Calculation settings for forced oscillations according to the first natural mode for some laying pattern sample $[\pm 20^\circ]_S$ are given in Figure 11.

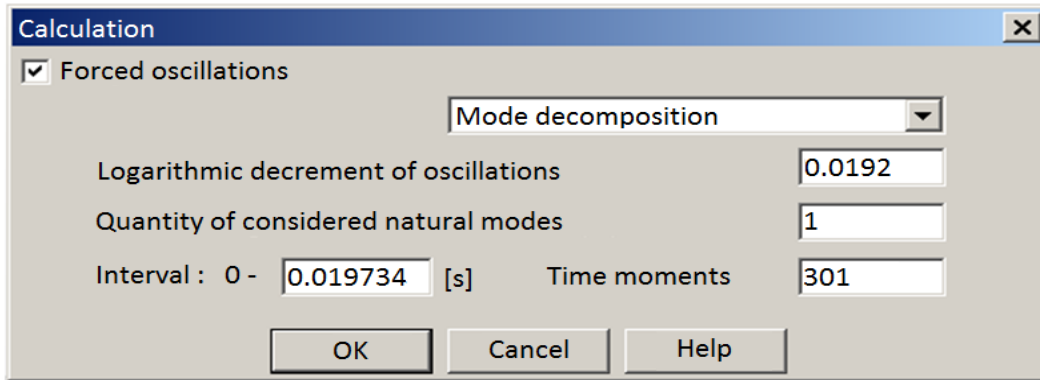


Figure 11. Forced oscillations calculation task

Displacements of the point at the end of the laying pattern sample $[\pm 20^\circ]_S$ that were obtained in calculation of forced oscillations are shown in Figure 12.

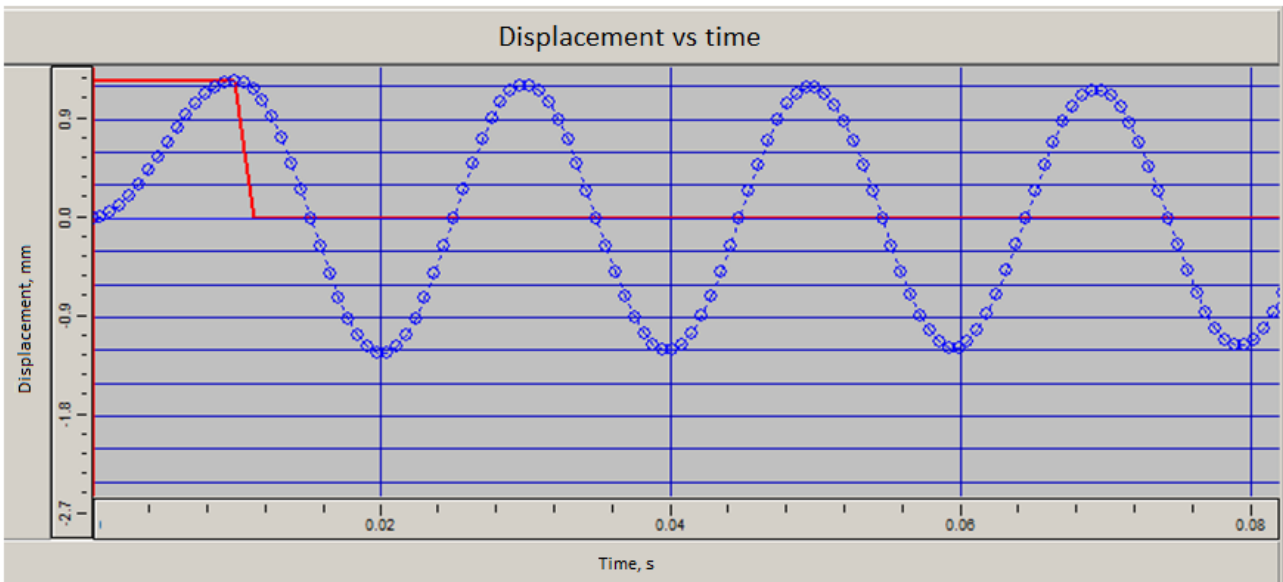


Figure 12. Displacement of the point at the end of the laying pattern sample $[\pm 20^\circ]_S$ in forced oscillations.

In order to calculate behavior of a laminate structure by forth integration, a damping matrix must be given instead of a logarithmic damping decrement of the method according to natural modes. In order to simulate beam oscillations at the first mode, initial displacement conditions as displacement of the first natural mode must be given, see Figure 9.

In order to calculate structural damping decrement according to displacements, maximum adjacent oscillation periods must be used. Damping decrements that were obtained in the experimental simulation in the DPSP (EM) coincide with an error less than 8%. Damping decrement was affected in particular by the integration step and fallback of maximums.

Accordingly, if density, elastic and elastic damping parameters of the laminate plate are known, finite element

simulation of behavior of composite structures can be done. This refers to calculation of natural modes and frequencies and calculation of structural behavior in the near-resonant mode over time.

CONCLUSION

Thus and so, the determination technique of elastic dissipative parameters of a laminate layer, using a rather limited set of symmetrical composite samples exhibiting different laying patterns, which was presented in this research, allows for discovering necessary values of damping parameters of a ply. Elastic parameters of the ply may also be clarified, using their natural frequencies.

As a result, a laminate can be designed with the required dissipation parameters and damping of a finished composite structure can be calculated, which proves relevant for vibration and acoustical effects, and self-oscillations (such as wing flutter) can also be simulated. DPSP (EM) serves as an element of computer-aided engineering (CAE) analysis in this case, and introduction of this locally designed software ensures that defense industrial companies are import-independent.

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REFERENCES.

- [1] Zinoviev P.A., Ermakov Y.N. Energy dissipation in vibrations of fiber composite elements (review). Central R&D Institute for scientific and technical information, Moscow, 1989, 76 p.
- [2] Bert C. W. Composite materials: a survey of damping capacity of fiber reinforced composites. Damping applications for vibration control.– ASME AMD-38, 1980, 53-63.
- [3] Gibson R.F. Dynamic mechanical properties of advanced composite materials and structures: A review. Shock & Vibration Digest., 19, 7, 1987, 13-22.
- [4] Zinoviev P.A., Ermakov Y.N. Energy dissipation in composite materials. Lancaster (USA): Technomicpublishing Co., 1994, 246 p.
- [5] Zinov'ev P.A., Smerdov A.A., and Kulish G.G.. Experimental investigation of elasto-dissipative characteristics of carbon-fiber-reinforced plastics. Mechanics of Composite Materials, 39, 5, 2003, 393-398.
- [6] Zinoviev P.A. Structural, thermoelastic and dissipative behavior of composites. In Composite materials: guide. Edited by V.V. Vasiliev, Yu.M. Tarnopolski. Mashinostroyenie: Moscow, 1990. pp. 232-267.
- [7] Adams R.D., Bacon D.G.C. Effect of fiber orientation and laminate geometry on the dynamic properties of CFRP. Journal of Composite Materials, 1973: p. 402-408.
- [8] Crane R.M., Gillespie J.J.W. Characterization of the vibration damping loss factor of glass and graphite fibre composites. Composite Science and Technology. 40, 1991, 355–375.
- [9] Smerdov A.A. Developing design methods for composite materials and rocket and space structures. PhD thesis. Moscow, 2008, 410 p.
- [10] Aflutov N.A., Zinoviev P.A., Popov B.G. Calculating multilayer plates and sheathes made of composite materials. Mashinostroyenie: Moscow, 1984.– 264 p.
- [11] Zinoviev P.A., Smerdov A.A. Optimum design of composite materials. Moscow: Bauman Moscow State Technical University Press, 2006.- 103 p.
- [12] Biderman V.L. The theory of mechanical vibrations. Moscow: Vysshaya Shkola, 1980. 408 p.
- [13] Adams RD, Maheri MR. Dynamic flexural properties of anisotropic fibrous composite beams. Composites science and technology, 50, 1994, 497-514.
- [14] Maheri M.R., Adams R.D. Vibration properties of structural FRP composites. JSME International Journal. Series A. 42(3), 1999, 307-320.
- [15] Bensahal D., Amrane M.N., Chabane F., Belahssen O., Benramache S.. Structural damage detection by using finite element method as function of length. Engineering Journal, 17(4), 2013, 111-123.
- [16] Smerdov A.A., Kulish G.G., Gusev S.A., Rostovtsev M.Yu., Sokolov S.B. Elastic and dissipative behavior of carbon fiber made with the use of RTM-technology. Composite Materials Structures. 2016, 2, 21-25
- [17] Rikards R., Chate A., Korjakin A. Vibration and damping analysis of laminated composite plates by the finite element method. Engineering Computations, 17(1), 1995, 61-74.
- [18] Zenkevich O. Finite element method in engineering. Moscow: Mir, 1975, 543 p.