International Journal of Applied Engineering Research ISSN 0973-4562 Volume 12, Number 5 (2017) pp. 773-778

An Application of Wavelet Transform on the Dynamic Effect of Curvature Changing in a Cam Profile

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Abstract

In this paper we study numerically the motion of the follower near a characteristic point of a cam. An analytical formulation of the cam profile has been proposed, in terms of a function depending on a given angular parameter. Therefore we investigate the smoothness of the relative trajectory by applying the wavelet analysis. Our proposed procedure proves that the wavelet analysis can be utilized for studying the dynamic response of a mechanical system, which depends on the regularity of the profile of the cam.

Keywords: Wavelet analysis, numerical analysis, cam profile, cam motion.

INTRODUCTION

A cam is mechanical element, which is used to transmit a desired motion to another mechanical element by direct contact. Specifically, the purpose of the cam is the transmission of power, motion or information. Usually, a cam is composed of three different parts: a driving element called itself cam, a driven element called follower and a fixed frame. Cam mechanisms are usually used in most modern applications, especially in automatic machines and instruments, internal combustion engines and control systems. Generally, the design of cam profile is based on well note simple regular curves such as circles, parabolas cycloids, sinusoidal or trapezoidal curves, polynomial functions and Fourier series curves. In the recent literature, many studies have been addressed to circular-arc cams [1-2] have studied the motion equation of an equivalent system model of an automative valve train.

On the other side, the Continuous Wavelet Transformation (CWT) represents a time-scale analysis of the smoothness of a signal or, more in general, of a time series or a curve profile. The Wavelet analysis, unlike the Fourier one, is very useful when one analyzes and decompose signal with a not constant frequency. Let us consider the simple case in which we want to find the Fourier expansion of a signal, defined from 0 to 2, that assumes a linear form from 0 to 1 and it is sinusoidal from 1 to 2. In this case, in order to obtain an appraisable approximation of the signal, we must evaluate many coefficients of the Fourier expansion.

Qualitatively, the difference between the usual sine wave and a wavelet can be described from the localization property: the sine wave is localized in frequency domain, but not in time domain, while a wavelet is localized both in the frequency and time domain. Furthermore, the duration of its maximum oscillation is relatively small. One can regard a wavelet is a shape of wave of limited duration and zero moments of a given order. The choice of a wavelet and of signal decomposition level depends on the shape of signals and on the experience of the analyst.

For its versatility, the wavelet analysis is diffused in many fields, such as Acoustics, Electrodynamics [3], Finance [4], Medicine and Statistics [5], Robotics [6-9], Mechanics [10-11] and advanced signal processing [12-16].

In this paper, we study the smoothness (such as the regularity of the induced motions) of a determined cam profile, which is composed by subsets of circular arcs. Our study is performed by applying the Continuous Wavelet Transformation. We concentrate our analysis on the point where the profile of the cam changes.

MATERIAL AND METHODS

Referring to Figure 1, a cam profile can be composed by the following curves. The first two curves are the circle Γ_α, ($\alpha \in \{1, 2\}$), whose radius and center are, respectively, ρ_α and C_α. The third and the four circle, named respectively Γ_3 and Γ_4, are centered on the cam rotation axis O; their radiuses are, respectively, r and $r + h_1$. If one assumes a fixed frame OXY, three characteristic points can be identified: A, which joins Γ_2 with Γ_3; F, which is the point joining Γ_1 with Γ_2; D which joins Γ_1 with Γ_4. In these points, the relative circles have the same tangential vector [17-18].

Referring to system in Figure 1, let us consider the motion of the cam around its center O, having constant angular velocity a. If we suppose that the follower is in A" at time 0, in A at time $t_1 > 0$ and in $F(t)$ at $t > t_1$, then $A\hat{O}F = a(t - t_1) = : \hat{a}$. Therefore one can deduce that

$$\rho_2{}^2 = (\rho_2 - r)^2 + \\ + d(O,F)^2 - 2(\rho_2 - r)d(O,F)\cdot\cos(\pi - \hat{a}) =$$

from which it follows

$$d(t) = dist(O, F(t)) = \\ = (r - \rho_2)\cdot\cos\hat{a} + \sqrt{(\rho_2 - r)^2\cos^2\hat{a} - r^2 + 2\rho_2 r}$$

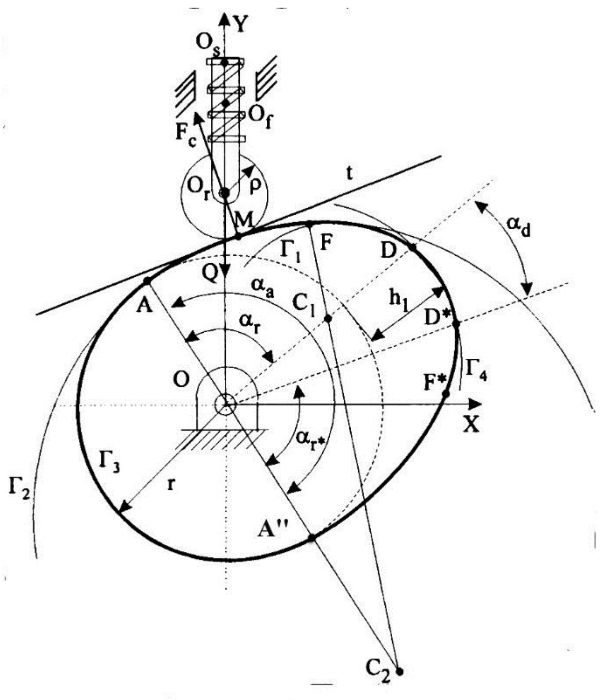

Figure 1: A roller follower two circular-arc cam

Note that $d(t_1) = r$. This last equation gives the motion equation of the follower. Moreover, at time $t = t_1$ (i.e. at point A), the velocity of the follower is zero.

The dynamical system of Figure 2, represents an example of motion, transmitted by a cam. It is a mass m fixed to a point P and to the axis y, such that $y(t) = d(t) - r$. Furthermore the dynamics of the system is described by the following d.e.:

$$m_r\ddot{x}(t) = -k_r\left(x(t) - d - (d(t) - r)\right) \quad (1)$$
$$s.\ t. \quad \dot{x}(t_1) = 0 \quad x(t) = d$$

for any $t \in [t_0, t_1]$, where $d = \text{dist}(m_r, O)$.

In our study we used MatLab package and the Toolbox Wavelet ver. 6.5. In particular, the wavelets used in this paper are those proposed by [19]. She constructed a series of mather wavelets (indexed by N and denoted by db$N$) with each mother in the series having regularity proportional to N. Each Daubechies' wavelet are compactly supported in the time domain. Typically wavelets of class m_r are specifically constructed so that some properties are verified. In order to study the smoothness of a cam profile, we utilize the (continuous) wavelet transform.

A mother wavelet ψ is a function of zero h-th moment:

$$\int_{-\infty}^{+\infty} x^h \psi(x)dx = 0 \quad , \quad h \in \mathbf{N}.$$

From this definition, it follows that, if ψ is a wavelet whose all moments are zero, also the function ψ_{ik} is a wavelet, where

$$\psi_{jk}(x) = 2^{j/2}\psi(2^j x - k)$$

In fact, we have

$$\int_{-\infty}^{+\infty} 2^{j/2} x^h \psi(2^j x - k)dx =$$
$$= 2^{j/2}\int_{-\infty}^{+\infty} \frac{1}{2^j}\left(\frac{y+k}{2^j}\right)^h \psi(y)dy =$$
$$= \frac{2^{j/2}}{2^{j(h+1)}}\int_{-\infty}^{+\infty}(y+k)^h\psi(y)dy =$$
$$= \frac{2^{j/2}}{2^{j(h+1)}}\sum_{m=0}^{h}\binom{h}{m}k^{h-m}\int_{-\infty}^{+\infty} y^m\psi(y)dy = 0. \quad (2)$$

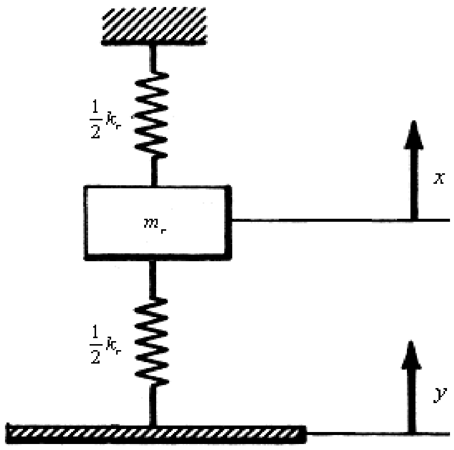

Figure 2: Illustration of a dynamical system.

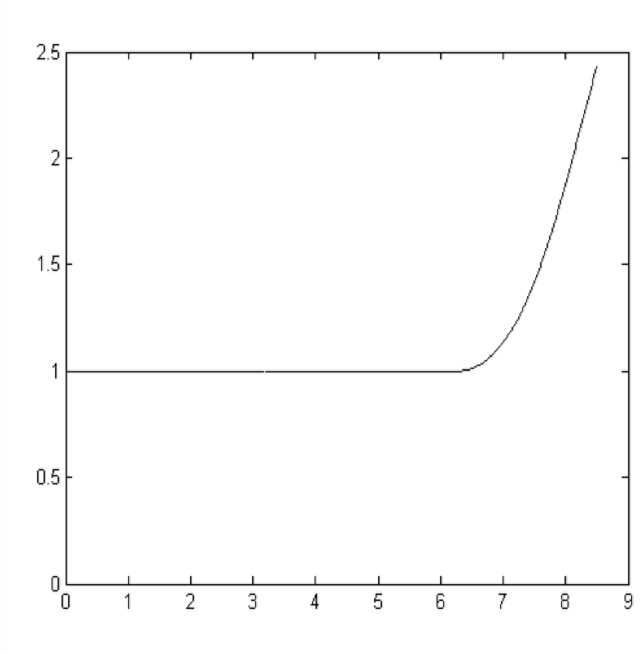

Figure 3: Trajectory of the Point M. The parameter values used here are: a = 2, r = 1, ρ2 = 2 and kr = 0.6m.

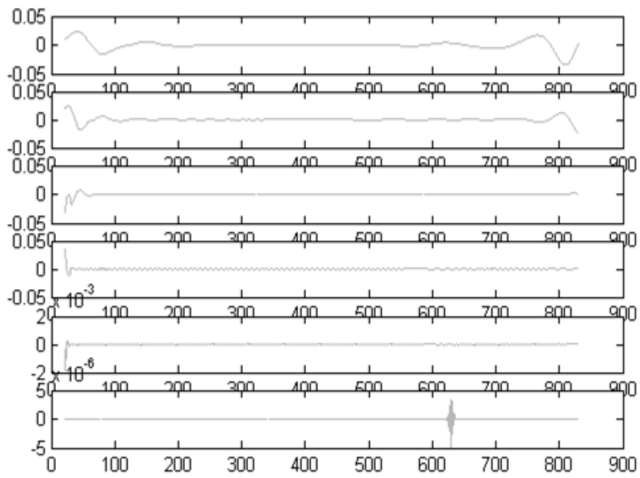

Figure 4: Detail of the trajectory of M, obtained by setting ψ = db7

Moreover, consider a wavelet ψ and a function φ such that {{ φ_{j_0k} }, {ψ_{jk}}, $k \in \mathbf{Z}$, $j = 0, , 2,\ldots$} is a complete orthonormal system. By Parseval theorem, for every s ∈ $L_2(\mathbf{R})$, it follows that

$$s(t) = \sum_k a_{j_0k}\varphi_{j_0k}(t) + \sum_{j=j_0}^{j_1}\sum_k d_{jk}\psi_{jk}(t)$$
.

The decomposition of a signal $s(t)$ by wavelet (i.e., the CWT) is represented by the following detail function coefficients:

$$d_{jk} = \int_{-\infty}^{+\infty} s(\tau)\cdot\frac{1}{\sqrt{2^j}}\psi\left(\frac{\tau-k}{2^j}\right)d\tau \quad (3)$$

and by the approximating scaling coefficients

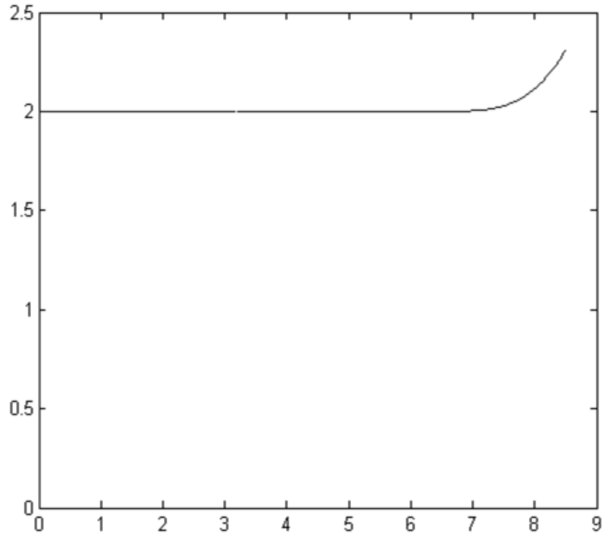

Figure 5: Trajectory of the system. The parameter values used here are: kr = 0.6mr, a = 2, r = 1, ρ2 = 2 and d = 2.

$$a_{j_0k} = \int_{-\infty}^{+\infty} s(\tau)\cdot\psi(\tau-k)d\tau \quad (4)$$

Note that d_{jk} can be regarded, for any j, as a function of k. Consequently, it is constant if the signal $s(t)$ is a smooth function, having considered that a wavelet has zero moments. To show the above mentioned property, it is sufficient to expand the signal in Taylor's series.

An example of wavelets is given by Daubechies' family {db$N$, $N = 1, 2, \ldots$} (see [4]). It is

$$supp\ \varphi \subseteq [0, 2N-1] \qquad supp\ \psi \subseteq [0, 2N-1]$$

and

$$\int_{-\infty}^{+\infty} x^h\psi(x)dx = 0, \quad h = 0, 1, \ldots, N-1.$$

Moreover, there is the following smoothness property: for any $N > 2$, the D2N wavelets verify

$$\varphi, \psi \in H^{\lambda N}, \quad 0.1936 \le \lambda \le 0.2075,$$

where $H^{\lambda N}$ is the Hölder smoothness class with parameter λ.

RESULTS

Let us consider the trajectory of point M, moving on

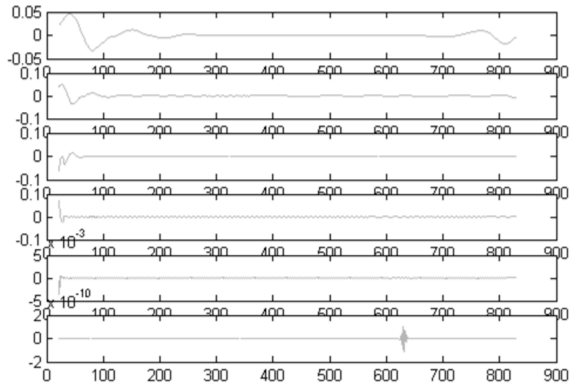

Figure 6: Detail of the motion of the system for ρ2 = 1.8, ψ = db7, $k_r = 0.6m_r$, a = 2, r = 1, d = 2.

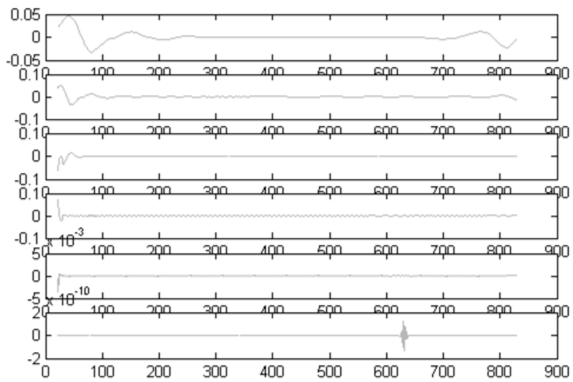

Figure 7: Detail of the motion of the system for ρ2 = 2, ψ = db7, $k_r = 0.6m_r$, a = 2, r = 1, d = 2.

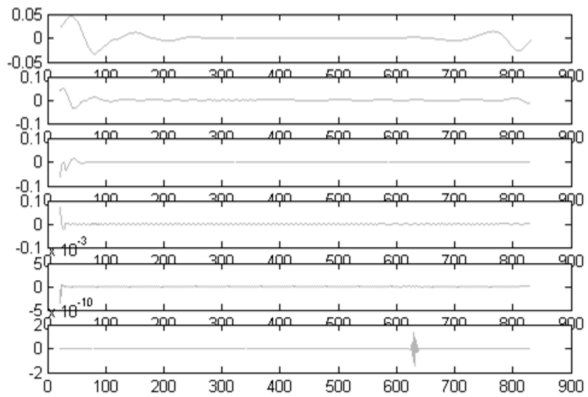

Figure 8: Detail of the motion of the system for ρ2 = 2.2, ψ = db7, $k_r = 0.6m_r$, a = 2, r = 1, d = 2.

The cam trajectory $d(t)$ is shown in Figure 3. The details coefficients of the wavelet transform for $d(t)$, are plotted in Figure 4: the irregularity of the trajectory is revealed at point whose abscissa is 2π. In fact, this point corresponds to the instant in which the point M comes away from Γ_3 and reaches Γ_2, touching the point A. Note that the spike (i.e., the anomaly) is registered only at the highest resolution level utilized, that is j = 6.

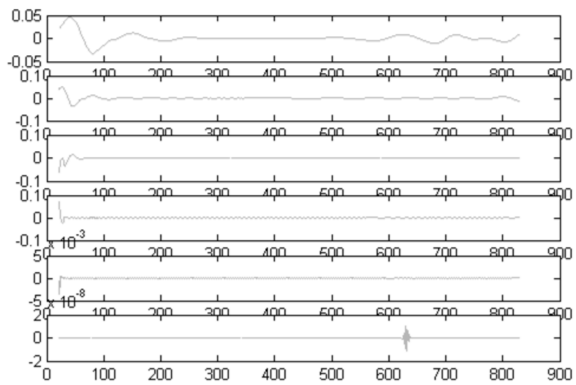

Figure 9: Detail of the motion of the system for kr=50 mr, ψ = db7, a = 2, r = 1, d = 2 and ρ2 = 2

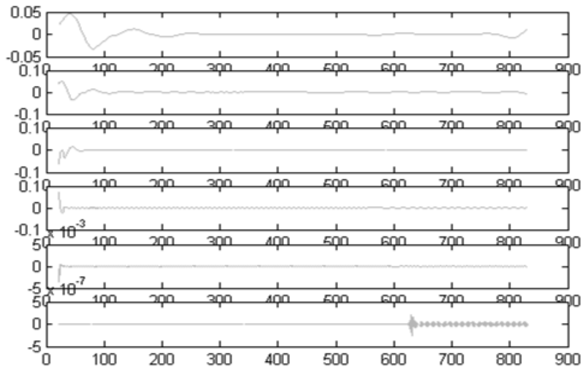

Figure 10: Detail of the motion of the system for kr=1000 mr, ψ = db7, a = 2, r = 1, d = 2 and ρ2 = 2

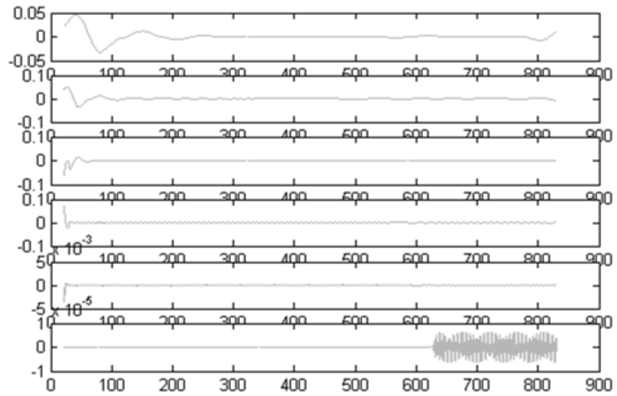

Figure 11: Detail of the motion of the system for kr=10000 mr, ψ = db7, a = 2, r = 1, d = 2 and ρ2 = 2

Therefore, it has been studied the solution of the dynamical system (1). It has been solved, by expanding $d(t)$ in Taylor's series: in this way, the differential equation assumes a well note form. To study the regularity of the solution, we have used again the wavelet family db7. In fact, its second moment is zero; therefore, following the discussion in paragraph 4, it is useful to analyze discontinuities of the second derivative of a signal. In fact, we can observe that, by construction, the investigated dynamical system is at least C^1. We have also noted that, for the sake of our calculations, the detail of $d(t)$, illustrated in Figure 4, coincides with the detail of its Taylor's expansion.

Finally, Figure 5 shows the trajectory of the system and Figure 6-11 illustrate its detail. Also in this case the anomaly is revealed at level $j = 6$. Moreover, also if it is not evident from graphical inspection, the oscillations of the CWT are increasing as ρ_2 or k_r increase. Note that, when k_r is large enough, the oscillations, of the system's detail, are major than the oscillation of the detail of $d(t)$.

Besides, in this case, the anomaly is different from that, which derives from $d(t)$. It is formed from many spikes that derives from the oscillatory component of the motion.

CONCLUSIONS

A methodology of evaluating the dynamic response, of a mechanical system, caused by a cam profile, was presented. It concurs to optimize the cam profile both in terms of functional aspects and its lifetime, due to a minor dynamic stress. Some critical points of cam profile were investigated (for various ρ_2 and k_r) for testing the CWT methodology. The objective, in the future, is to implement a more complex model in order to study the real dynamic response, which will be carried out from a prototype, test-bed at laboratory of Department of Industrial Engineering in Naples.

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