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Suitability of Conventional 1D Noise Subspace Algorithms for DOA Estimation using Large Arrays at Millimeter Wave Band

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Abstract

Array signal processing has attracted the interest of the scientific community for the past several decades. An Array of sensor elements (be it microphones, hydrophones, antenna elements, piezoelectric sensors) achieves better performance than a single element would. Antenna arrays are made up of antenna elements which can be arranged in a variety of configurations (with respect to the geometry, inter-element spacing, etc.). The Direction of Arrival (DOA) estimation is a signal processing technique that can be used at a receiving array to find the directions of the incoming signals that impinge on the antenna array. Beamforming is a technique that can be used to focus the transmit energy towards or to collect the received energy from certain desired directions. A smart antenna system is one which can perform DOA estimation as well as beamforming. The correctness of DOA estimation algorithms is a major contributing factor in the performance of smart antenna systems. Since beamforming is a key enabler for Millimeter Wave (mmWave) and fifth-generation cellular (5G); DOA estimation assumes much importance in future wireless communications. MmWave allows the use of large arrays owing to the small wavelengths. In this paper, we have studied the suitability of DOA estimation algorithms using Eigen decomposition methods which include the Pisarenko Harmonic Decomposition (PHD), Multiple Signal Classification (MUSIC), Modified MUSIC and Root-MUSIC at 30 GHz, which is a proposed frequency in 5G. MATLAB simulations throw light upon the various factors affecting DOA estimation accuracy and it is found that the above conventional methods are very much suited for the mmWave frequencies.

Keywords: Array Signal Processing, Angle of Arrival (AOA), Beamforming, Direction of Arrival Estimation (DOA), Millimeter Wave (mmWave), MUSIC, Large arrays, Uniform Linear Array (ULA).

INTRODUCTION

Array processing was traditionally applied in fields such as Radar and Sonar. Array processing involves the use of an array of sensors to provide better directional properties during signal transmission and/or reception. The limitations of a single element in providing the required directivity, gain and beamwidth can be overcome by the use of an array of sensors as it helps in focusing the energy to specific directions by suitable phasing of its elements [1]. Though array processing has many applications in different fields, it was the last two decades that have witnessed the applicability and suitability of antenna arrays for mobile communications, opening up a new area of research, namely, the smart antennas [2]. This array of antennas is instrumental in increasing the accuracy and resolution of the achieved results. In order to attain better accuracy, more antenna elements are required [3].

Some well-known algorithms for direction of arrival estimation include, MUltiple SIgnal Classification (MUSIC), Root MUSIC, Minimum Variance Distortion less Response (MVDR), Pisarenko Harmonic Decomposition (PHD), Estimation of Signal Parameter via Rotational Invariance Technique (ESPRIT), Maximum Likelihood (ML) techniques [3].

The PHD was introduced in 1973 and is considered as the first Eigen decomposition based spectral estimation method which makes use of the noise subspace [4]. From the time MUSIC was first introduced in 1986 [5], several modifications were made to it [6]. Applications and comparisons of DOA estimation algorithms have been widely studied [7], [8]. A particular variant of MUSIC which is suitable for coherent sources was also proposed [9]. A method for coherent signal classification was proposed in [10], [11]. MUSIC is a computationally intensive algorithm which searches for peaks in the spatial spectrum. Root MUSIC is a particular technique where the peak searching operation is replaced by a computational technique where the roots of a polynomial are evaluated and there is a one-to-one relation between the roots and the desired DOAs [12]. MUSIC and ESPRIT are well-known algorithms that are based on the Eigen decomposition of the array covariance matrix. ESPRIT uses arrays with peculiar geometry. This algorithm exploits the rotational invariance in the signal subspace created from two sub arrays (doublets which are linearly displaced) resulting from the main array [13]. The MUSIC algorithm is the most conventional and widely accepted estimation approach employed for both uniform and non-uniform arrays. Hardware implementations of DOA estimation algorithms have also been studied extensively [14].

Two-dimensional DOA estimation has assumed much importance recently [15], [16]. Also, research on DOA estimation and beamforming for large arrays in the context of 5G cellular networks is ongoing [17]. A large array becomes practically possible at higher operating frequencies since more elements can be accommodated in a given space owing to the smaller wavelengths. In line with the ongoing research trend and the aforementioned facts, we have evaluated standard algorithms such as PHD, MUSIC, modified MUSIC and Root MUSIC for DOA estimation using large uniform linear arrays. We next explain the mathematical signal model, the algorithms, the methodology followed, the results obtained and finally conclude the paper with a discussion and future directions.

SIGNAL MODEL AND MATHEMATICS INVOLVED

A uniform linear array of *M* elements is considered. *D (D<M)* narrowband source signals impinge on the considered array. All these *M* elements are placed in a linear fashion and are equidistant. Inter-element spacing is specified to be half of the given wavelength. The uniform linear array structure of *M* elements with an inter-element spacing of *d* is shown in Fig. 1.

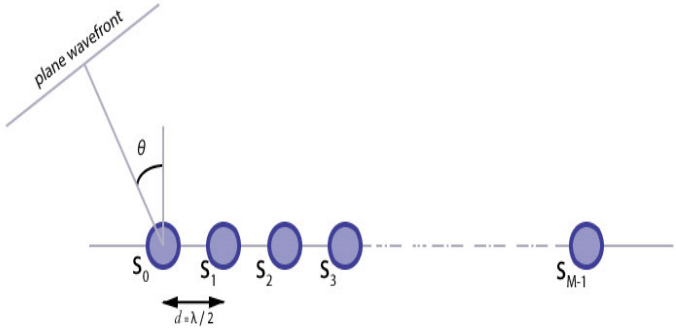

Figure 1. Uniform Linear Array with *M* elements.

Since the source is assumed to be in the far field of the array, only plane waves reach the array. Each received signal **y**(t) has a zero-mean additive white Gaussian noise (AWGN) random process. The received signal **y**(t) can be represented as

$$\mathbf{y}(t) = \mathbf{A}\,\mathbf{s}(t) + \mathbf{n}(t) \quad (1)$$

The matrix form representation of the array steering vector is given as follows,

$$\mathbf{A} = [\mathbf{a}(\theta_1)\ \ \mathbf{a}(\theta_2)\ \ \mathbf{a}(\theta_3) \ldots \mathbf{a}(\theta_D)] \quad (2)$$

$$\boldsymbol{a}(\theta_k) = [1\ \ \exp(-j2\pi d \sin\theta_k/\lambda)\ \ldots\ \exp(-j(M-1)2\pi d \sin\theta_k/\lambda)]^T \quad (3)$$

where, **s**(t)=Incident complex signal vector at time *t*

n(t)=Zero mean with variance σ_n^2, noise vector at each array element

$\mathbf{a}(\theta_i)$ = M element array steering vector for angle of arrival θ_i
A= *M X D* matrix of steering vectors $\mathbf{a}(\theta_i)$

Therefore, the i^{th} source signal arrives at an angle θ_i, and is captured by the *M* antenna elements. The incoming signals are known to be time varying and thus we make our calculations depending upon time snapshots of the arriving signals.

The array correlation matrix is given by,

$$\begin{aligned}\mathbf{R}_{yy} &= E[\mathbf{y}.\mathbf{y}^H] = E[(\mathbf{As}+\mathbf{n})(\mathbf{s}^H\mathbf{A}^H + \mathbf{n}^H)] \\ &= \mathbf{A}\,E[\mathbf{s}.\mathbf{s}^H]\,\mathbf{A}^H + E[\mathbf{n}.\mathbf{n}^H] \\ &= \mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H + \mathbf{R}_{nn} \\ &= \mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H + \sigma_n^2\mathbf{I} \end{aligned} \quad (4)$$

where, $\mathbf{R}_{yy}$ = *M X M* Array correlation Matrix
$\boldsymbol{R}_{ss}$ = *D X D* Source Correlation Matrix
$\boldsymbol{R}_{nn}$= *M X M* Noise Correlation Matrix
I = *M X M* Identity matrix

In case when the source signals are uncorrelated, $\mathbf{R}_{ss}$ is by default a diagonal matrix. The $\mathbf{R}_{ss}$ becomes non-singular when the signals are partially correlated and is found to be singular when coherent signals are there because rows become the linear combination of each other.

We next explain few of the popular pseudospectra solutions for super resolution DOA estimation.

THE PISARENKO HARMONIC DECOMPOSITION

The Pisarenko Harmonic Decomposition (PHD) is one of the earliest Eigen subspace methods and is based on the minimum mean squared error (MMSE) approach [14]. Out of the *M* Eigen vectors obtained, the Eigen vector which corresponds to the MMSE will be the smallest one and the only Eigen vector in the noise subspace.

The pseudospectrum for PHD is given by

$$P_{PHD}(\theta) = \frac{1}{\left|\mathbf{a}^H(\theta)\mathbf{e}_1\right|^2} \quad (5)$$

Where, $\mathbf{e}_1$ is the eigenvector associated with the smallest eigenvalue λ_{min}, the only member in the noise subspace.

THE MUSIC ALGORITHM

MUSIC algorithm employs a simple methodology where the array correlation matrix $R_{\mathbf{yy}}$ is determined to further decompose it into Eigen vectors. These vectors are subsequently sorted in descending order. The first *D* Eigen values are assigned to the signal subspace, while the remaining smaller *M-D* Eigen values are allotted to the noise subspace. Since this algorithm makes use of the noise Eigen vectors subspace, it is often referred to as a subspace method.

The eigenvectors are then decomposed into noise and signal subspace $\mathbf{E}_N$ and $\mathbf{E}_S$. For uncorrelated signals, the smallest eigenvalues are equal to the variance of the noise. We can then construct the *M X (M - D)* dimensional subspace spanned by the noise eigenvectors such that

$$\mathbf{E}_N = [\mathbf{e}_1\ \mathbf{e}_2\ \mathbf{e}_3 \text{ ---- } \mathbf{e}_{M-D}] \quad (6)$$

Noise subspace and array steering vectors are orthogonal to each other. Because of this orthogonality, the denominator term in the Equation 7 is equal to zero and as a result, sharp peaks are obtained at the DOAs and the MUSIC pseudospectrum is given by

$$P_{MUSIC}(\theta)=\frac{1}{\left|\mathbf{a}^{H}(\theta)\mathbf{E_N}\mathbf{E_N}^{H}\mathbf{a}(\theta)\right|} \qquad (7)$$

THE MODIFIED MUSIC ALGORITHM

MUSIC algorithm achieves high resolution in DOA estimation only when the signals being incident on the sensor array are non-coherent. For coherent sources, MUSIC does not perform well.

To improve the results for MUSIC algorithm, authors in [9] have introduced an exchange transition matrix 'J' so that the new received signal vector **z** is given as:

$$\mathbf{z} = \mathbf{J}\mathbf{y}^{T} \qquad (8)$$

$$J=\begin{bmatrix}0 & 0 & \dots & 1\\ 0 & 0 & \dots & 0\\ \dots & \dots & \dots & \dots\\ 1 & 0 & \dots & 0\end{bmatrix}, \qquad (9)$$

Where **J** is the complex conjugate of the initial signal matrix received.

$$\mathbf{R}_{zz} = E\,[\mathbf{z}.\mathbf{z}^{H}] = \mathbf{J}\,\mathbf{R}_{yy}\,\mathbf{J}^{T} \qquad (10)$$

Now the new re-construction matrix can be obtained by adding $\mathbf{R}_{yy}$ and $\mathbf{R}_{zz}$. Let the new reconstruction matrix be **R**.

$$\mathbf{R}= \mathbf{R}_{yy} + \mathbf{R}_{zz} \qquad (11)$$

As per the matrix principle **R**, $\mathbf{R}_{yy}$ and $\mathbf{R}_{zz}$ will have the same noise subspace. The new correlation matrix **R** is now used to estimate the eigenvalues and eigenvectors. These obtained eigenvectors are further decomposed into the new noise and signal subspace. The obtained subspace is used for the pseudospectrum evaluation.

THE ROOT MUSIC ALGORITHM

The above-mentioned algorithms lie in the category of spectral based estimation, where the pseudospectrum of the respective algorithm is plotted and the peaks estimate the respective angle of arrival. ROOT MUSIC is model based estimation where the DOA is calculated directly by formulating mathematical equations as given in [3].

Consider the MUSIC pseudospectrum given in Equation 7,

$$P_{MUSIC}(\theta)=\frac{1}{\left|\mathbf{a}^{H}(\theta)\mathbf{E_N}\mathbf{E_N}^{H}\mathbf{a}(\theta)\right|}$$

The denominator in the above expression can be simplified by considering a matrix $C = \mathbf{E}_N E_N^H$, which is a Hermitian matrix.

Root MUSIC spectrum can be given by,

$$P_{ROOT-MUSIC}(\theta)=\frac{1}{\left|\mathbf{a}^{H}(\theta)\mathbf{C}\mathbf{a}(\theta)\right|} \qquad (12)$$

The denominator argument in equation 12 can be written as

$$\mathbf{a}^{H}(\theta)\mathbf{C}\mathbf{a}(\theta)=\sum_{m=1}^{M}\sum_{n=1}^{M}e^{\frac{-j2\pi d(m-1)\sin\theta}{\lambda}}C_{mn}e^{\frac{j2\pi d(n-1)\sin\theta}{\lambda}}$$

$$=\sum_{l=-M+1}^{M-1}C_l e^{\frac{j2\pi dl\sin\theta}{\lambda}} \qquad (13)$$

This can be further simplified as

$$D(z)=\sum_{-M+1}^{M-1}C_l z^l \qquad (14)$$

Where, $$z=e^{-\frac{j2\pi d\sin\theta}{\lambda}} \qquad (15)$$

Roots of D (z) have to be evaluated. There will be 2(*M-1*) roots. Roots which lie closer to the unit circle will be considered for DOA estimation.

$$z_i=\left|z_i\right|e^{j\arg(z_i)} \qquad (16)$$

Exact zeros in D(z) exist when the root magnitudes $\left|z_i\right|$=1, Angle of Arrival (AOA) can be calculated by

$$\theta_i=-\sin^{-1}\left(\frac{\lambda}{2\pi d}\arg(z_i)\right) \qquad (17)$$

METHODOLOGY

In this study, a uniform linear array (ULA) of M elements is considered. Four narrowband sources are assumed at 30°, 45°, 60° and 75° with respect to the array. For all the simulations, we have considered a ULA with inter-element spacing of $d = \lambda/2$. The frequency of operation is taken to be 30 GHz and hence the wavelength comes out to be 0.01 m. The selected frequency belongs to the millimeter wave (mmWave) band which is being considered for 5G cellular networks. The above parameters are followed for all the algorithms mentioned in this paper. We evaluate four existing algorithms, namely, the Pisarenko Harmonic Decomposition, the MUltiple SIgnal

Classification (MUSIC), the root-MUSIC and a variant of the MUSIC called as modified MUSIC. All these algorithms are based on the Eigen decomposition where the noise subspace is considered for calculations. MATLAB R2015a was used for all the simulations on a personal computer with i3 processor with 4 GB RAM.

The effect of different parameters (such as the array size, number of snapshots, inter-element spacing, correlation between the sources, and angular separation between the sources) on the output of the DOA estimation was studied for the case of the MUSIC algorithm.

RESULTS

As discussed in the methodology, simulations were done by considering each algorithm. Different parameters such as the array size, number of snapshots, angular separation between the sources etc., were varied and the results were noted.

The Pisarenko Harmonic Decomposition

The array size was considered as M=32 elements. Four sources were considered at 30°, 45°, 60° and 75° as mentioned earlier and the following result was obtained.

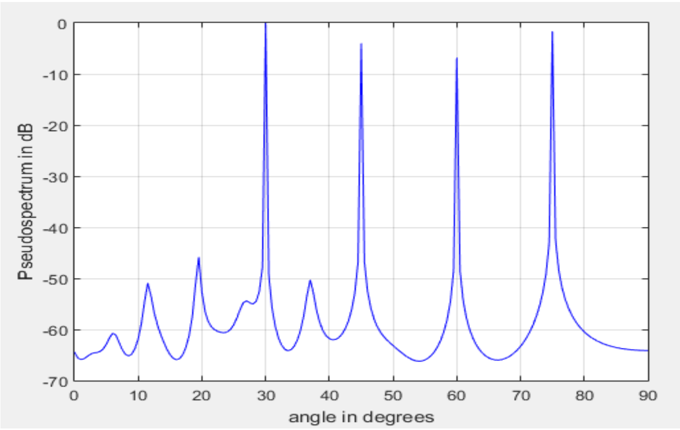

Figure 2. PHD pseudospectrum for θ=30°, 45°, 60° and 75°.

The pseudospectrum for PHD algorithm is plotted against DOA in Fig. 2. The peaks give an indication of the angles at which the noise subspace Eigen vector is orthogonal to the signal and hence the DOA.

The MUSIC

MUSIC pseudospectrum is plotted for an array consisting of 32 elements, with four narrowband sources present at 30°, 45°, 60° and 75° and shown in Fig. 3.

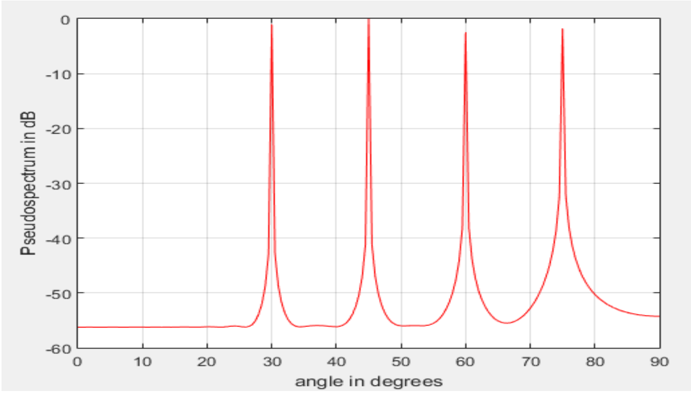

Figure 3. MUSIC pseudospectrum for θ=30°, 45°, 60° and 75°.

Fig. 3 clearly depicts the peaks at angles 30°, 45°, 60° and 75° respectively. The estimation accuracy of MUSIC depends upon certain factors which are explained below.

FACTORS AFFECTING THE DOA ESTIMATION ACCURACY

Number of Array Elements or the Array Size

M is the number of elements present in the considered array. More the number of elements better will be the resolution of

DOA estimation. Large arrays are possible owing to the frequency chosen.

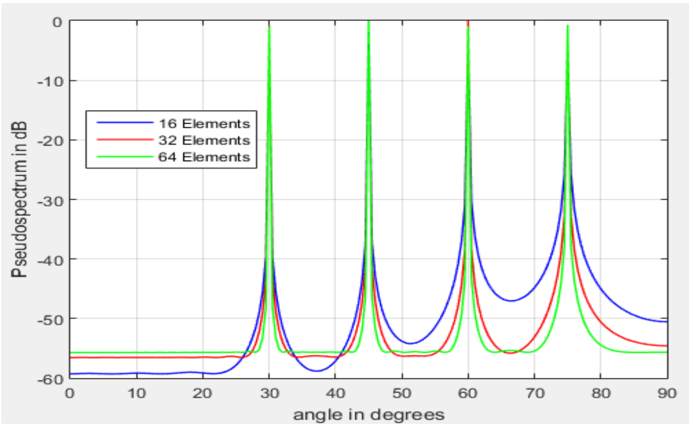

Figure 4. MUSIC Algorithm for 16, 32 and 64 elements

The result on the effect of the array size (number of elements) is shown in Fig. 4. The green lines represent the estimated DOAs for 64 elements.

Number of Snapshots

Snapshots can be simply referred to be the number of samples in time domain while in frequency domain it directs to the number of time sub-segments in DFT.

The following result in Fig. 5 is obtained for two cases, one for 200 numbers of snapshots and another for 1000 snapshots.

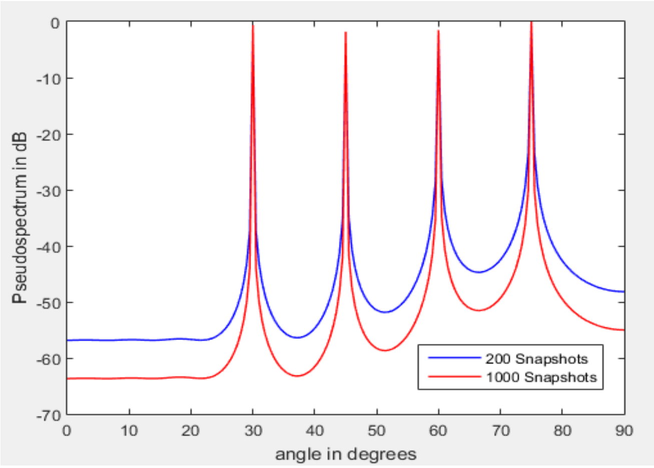

Figure 5. MUSIC Pseudo spectrum for varying snapshots

Effect of Element Spacing

An array with 32 elements is considered and inter-element spacing is kept to be λ/6, λ/4 and λ/2 respectively, keeping all the other parameters constant.

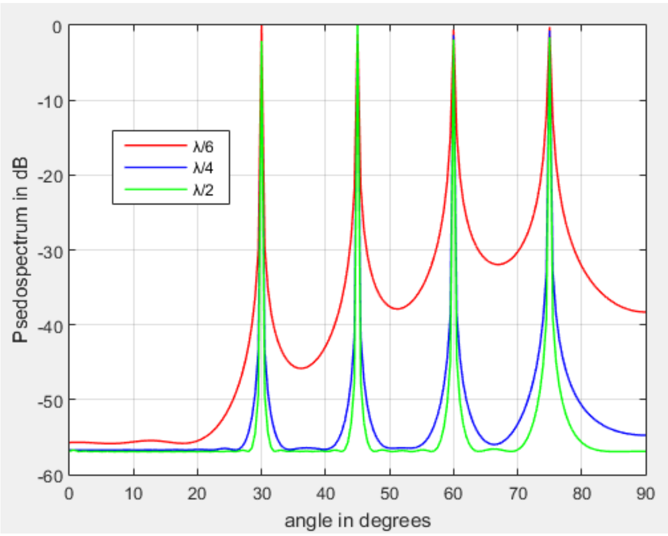

Figure 6. MUSIC Pseudospectrum for d = λ/6, λ/4 and λ/2

The pseudo spectrum thus obtained is shown in the above figure. The red, blue and green lines correspond to λ/6, λ/4 and λ/2 element spacing respectively.

Effect of angular separation between signals

The following results presented in Fig. 7 are obtained when the incident angles are 30°, 35°, 40° and 45°, while for the second case it is 30°, 45°, 60° and 75°.

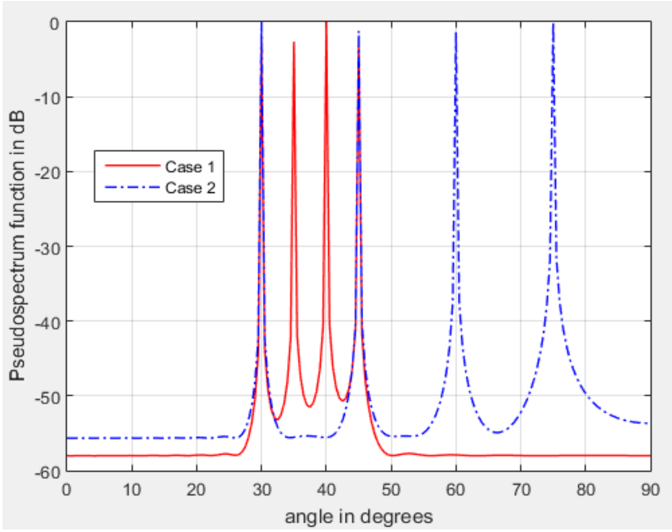

Figure 7. MUSIC pseudospectrum for varying angular separation between sources

Effect of coherent sources

The MUSIC pseudospectrum for four coherent sources is shown in Fig. 8.

Modified MUSIC

A modified MUSIC algorithm proposed in [15] is said to overcome the problem of correlated sources in MUSIC. The modified MUSIC algorithm is simulated and the pseudo spectrum is obtained as shown below in Fig. 9.

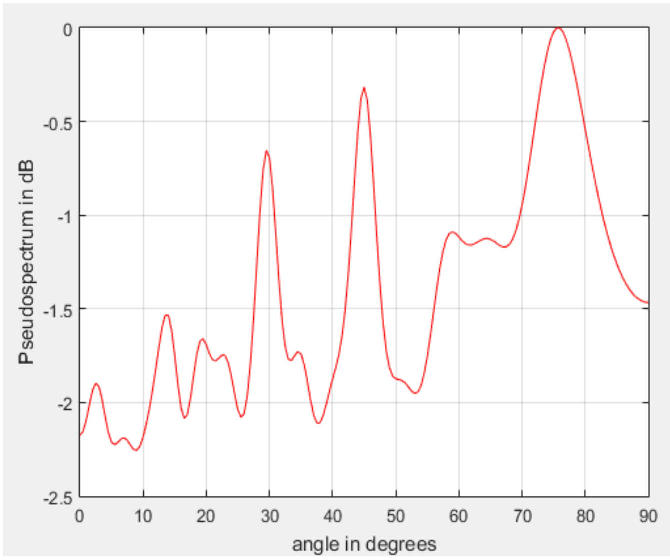

Figure 8. MUSIC pseudospectrum for four coherent sources

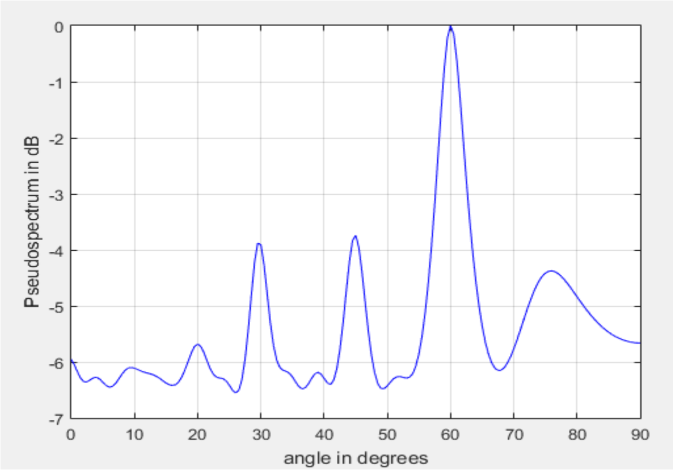

Figure 9. Modified-MUSIC pseudo spectrum

Root MUSIC

Fig. 10 shows the Z-plane, where the roots of D(z) given in Equation 14 are plotted. Four roots which are closer to the unit circle are extracted corresponding to the four angles of arrival.

The roots from the above figure which are closer to the unit circle are (-0.9998 - 0.0821i), (-0.9999 - 0.4778i), (-0.9998 - 0.0477i) and (-1.074 - 0.654i). Corresponding angles are calculated from Equation 17. The angles obtained using this method are in close resemblance with the true DOAs.

Since root MUSIC is a model based estimation, the DOAs are calculated directly based on the location of the roots unlike the other estimation techniques where the peaks depict the DOAs.

Table 1 shows the true and estimated DOAs along with the Root Mean Square Error (RMSE) for three cases where the array has 8, 16 and 32 elements. As stated above, if the array consists of *M* antenna elements then 2(*M-1)* roots are present. 14, 30 and 62 roots will be observed for 8, 16 and 32 elements respectively.

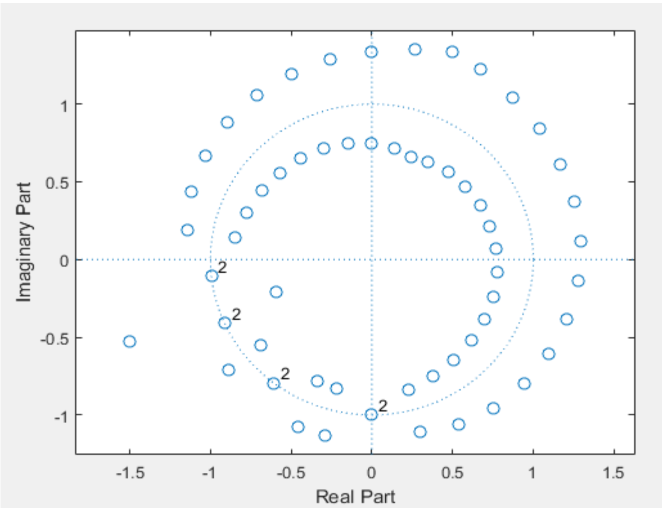

Figure 10. Roots of D (z) plotted in the z-plane

Table 1 Root MUSIC – True and estimated DOA and RMSE

M	Actual DOA in deg	Estimated DOA in deg	Error in deg	Average Error in deg	RMSE
8	30°	29.9999°	0.0001	0.0721	0.0221
	45°	45.0012°	0.0012		
	60°	59.8721°	0.1279		
	75°	75.1582°	0.1578		
16	30°	30.0001°	0.0001	0.01625	0.0067
	45°	44.9970°	0.0030		
	60°	60.0014°	0.0014		
	75°	75.0020°	0.0020		
32	30°	29.9990°	0.0010	0.0008	0.0004
	45°	44.9993°	0.0007		
	60°	59.9992°	0.0008		
	75°	74.9993°	0.0007		

DISCUSSION AND CONCLUSION

In this study, it was found that the Pisarenko harmonic decomposition has additional peaks apart from the four angles of arrival. The pseudo spectrum of MUSIC has peaks that give accurate estimation of the DOAs. It was also noted that as the array size increases, the resolution of the array increases and we obtain sharper peaks at the DOAs considered. Similar sharper peaks were obtained when the number of snapshots was increased from 200 to 1000. It was also noted that the optimum array response was obtained when the inter-element spacing was half of the wavelength when compared to the case when the inter-element spacing is $1/6^{th}$ and $1/4^{th}$ of the wavelength; and that the Modified MUSIC performs well

under the case of coherent signal sources when compared to conventional MUSIC. Root MUSIC gives the DOAs directly in terms of the angles. The RMSE was computed for 8, 16 and 32 elements and was the least for a 32-element array.

Since 2D and 3D beamforming is a key enabler for 5G communication networks, 1D and 2D DOA estimation assume immense prominence. The correct operation of smart antennas depends on the accuracy of the DOA estimation algorithms. Based on the simulations done at 30 GHz considering large sized arrays, the authors conclude that the existing methods such as PHD, MUSIC, modified MUSIC and root MUSIC are very well suited even for the case of large or massive arrays proposed for 5G telecom in the mmWave band.

FUTURE SCOPE

In future, this work can be extended to randomly oriented sources, non-uniform arrays. New methods such as compressive sensing and configurations such as co-prime arrays, nested arrays could be studied along with other algorithms. The algorithms may be tested in real-time on modern day Digital Signal Processors or Field Programmable Gate Arrays.

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