

Profit and Reliability Analysis of a Ceramic Tile Production System Considering Various Subsystem Failures

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ABSTRACT

The present paper discusses a reliability model developed for a ceramic tile production system which consists mainly of five subsystems viz. Ball Mill, Spray Dryer, Hydraulic Press, Glaze Line and Kiln that works in series configuration. Apart from these subsystems, Silo is a subsystem to store in-process material for manufacturing tiles. The working of the system also depends on the minimum storage capacity of the Silo. If raw material stored in the Silo get consumed during the repair/replacement of the failed Ball Mill and Spray Dryer subsystems, the system fails, otherwise the system remains operational. Various measures of the system effectiveness have been obtained using Markov process and regenerative point technique. The profit incurred to the system has also been computed. The conclusions for the system in respect of its reliability, availability and profit have been drawn through graphical analysis.

Keywords: Ceramic tile production system, stochastic model, reliability, availability, profit, Markov process and regenerative point technique.

INTRODUCTION

Market competition among industries in the past few years has intensified. The industries in order to remain in competition need to have accurate and advanced systems with latest technologies. Besides the industries have to attain a high-level of performance by maintaining high quality, low manufacturing cost and high customer satisfaction. From the early 1950's, reliability engineering of the systems including system modeling and analysis has been emphasized to investigate and enhance performance of the systems. Infact reliability models have been proved to be useful to support and drive industries in improving the reliability and performances of the production and logistic systems. Various industrial systems/ processes have been investigated for the purpose by the researchers considering some realistic and practical situations. Some other relevant factors were also taken into account while carrying out analyses of the systems. For instance, Britiney (1974) studied complex systems with dependent subsystem failures. Kumar et al. (2001) dealt with a two-unit redundant system with degradation and replacement. Gupta and Shivakar (2003) analyzed cloth weaving system. Gupta and Varshney (2004) investigated gas leakage detection system in an industrial workshop. Davoli et al. (2010) had given stochastic simulation approach for production

scheduling and investment planning in the tile industry. Sharma et al. (2011) discussed the reliability improvement of modern aircraft engine through failure modes and effects analysis of rotor support system. Kumar and Kapoor (2012) analyzed a base transceiver system considering hardware/software faults and congestion of calls. Kumar and Rani (2013) discussed water process system having two types of redundant subsystems. Kumar et al (2015) investigated performance of a system having bath-tub curve shaped failure pattern with provision of two types of replacement. Jia et al. (2016) dealt with reliability analysis of repairable multistate two-unit series systems when repair time can be neglected. Infact many other systems that are being used in the industries need to be analyzed with regards to their reliability and cost-benefit. As in the literature of system modeling, analysis of a ceramic tile production system with these aspects has not been reported, the present paper is an attempt in this direction.

A ceramic tile production system consists mainly of five subsystems viz. Ball Mill, Spray Dryer, Hydraulic Press, Glaze Line and Kiln that works in series configuration. In the production process, the raw materials such as silica, sand, quartz, flint, silicates etc. are dissolved in water in the Ball Mill (say, subsystem A) and the resulting suspension is known as 'slip'. The slip is then pumped, sprayed and dried by a stream of hot gases inside the Spray-Dryer (say, subsystem B). The dried powder is stored in a subsystem termed as Silos. Then a machine, called Hydraulic Press (say, subsystem C) exerts pressure on the dried powder to straightened and mold tiles. After pressing process, the tiles are glazed in the Glaze Line (say, subsystem D), i.e. the tiles are covered with one or more layers of glaze. This whole process gives the tiles a series of technical and aesthetic features including color, gloss and surface texture. Thereafter, the glazed tiles are sent to the Kiln (say, subsystem E) wherein through firing glazes over the ceramic tiles are made a permanent decoration. During the firing stage, the key variables in the press cycle are the firing time and temperature and the kiln atmosphere, which depend on the composition of the raw materials and the type of product required. Thus, the reliability and cost-benefit of the process/ system plays a very important role to the concerned tile industry.

Keeping above practical situation in view, the present paper discuss a reliability model developed for a ceramic tile production system consisting of five subsystems that works in series configuration, namely A, B, C, D, E. Apart from these, a subsystem Silo is considered that is there to store 'Slip', an in-process material. The failure of the system due to the

subsystems A and B depends upon the quantity of Slip in the Silo while failures due to other subsystems are independent of the quantity of Slip. If slip stored in the Silo is consumed during the repair/ replacement of the failed subsystems A and B then the system goes to failure state otherwise remains operational. The replacement of any component of a subsystem has been considered as a repair. Other assumptions of the model taken are as under:

1. Faults are self- announcing.
2. The system is as good as new after each repair.
3. Switching is perfect and instantaneous.
4. The failure time distributions are assumed exponential

while other time distributions are taken general.

5. All the random variables are mutually independent.

Various measures of the system effectiveness have been obtained using Markov process and regenerative point techniques. The profit incurred to the system has also been computed. In the last the conclusions regarding reliability, availability and profit of the system have been drawn plotting several graphs.

STATES OF THE SYSTEM

S_i	ith state of the system ; $i=1, 2, \dots, 7.$
$A/\bar{A}/\bar{A}_r$	the subsystem A is operative/ failed/ under repair
$B/\bar{B}/\bar{B}_r$	the subsystem B is operative /failed/ under repair
$C/\bar{C}/\bar{C}_r$	the subsystem C is operative/ failed/ under repair
$D/\bar{D}/\bar{D}_r$	the subsystem D is operative/ failed/ under repair
$E/\bar{E}/\bar{E}_r$	the subsystem E is operative/ failed/ under repair

NOTATIONS

$\lambda_1/\lambda_2/\lambda_3/\lambda_4/\lambda_5$	constant failure rate of subsystems A/B/C/D/E, respectively
X	dust storage level in Silo at a time
x_0	required level of dust for the operation of the system
t^*	time duration in which dust storage reduced to required level x_0
p_1	probability that the dust storage is more than the required level i.e. $P(X \geq x_0)$
q_1	probability that the dust storage is less than the required level i.e. $P(X < x_0)$
p_2	probability that the repair is done prior to the dust storage becomes less than required level x_0
q_2	probability that the repair is not done prior to the dust storage becomes less than required level x_0 .
$g_{a_1}(t)/G_{a_1}(t)$	p.d.f /c.d.f of repair time for subsystem A when repair is done before the dust storage consumed
$g_{a_2}(t)/G_{a_2}(t)$	p.d.f /c.d.f of repair time for subsystem A when repair is done after the dust storage consumed
$g_{b_1}(t)/G_{b_1}(t)$	p.d.f /c.d.f of repair time for subsystem B when repair is done before the dust storage consumed
$g_{b_2}(t)/G_{b_2}(t)$	p.d.f /c.d.f of repair time for subsystem B when repair is done after the dust storage consumed
$g_c(t)/G_c(t)$	p.d.f /c.d.f of repair time for subsystem C
$g_d(t)/G_d(t)$	p.d.f /c.d.f of repair time for subsystem D
$g_e(t)/G_e(t)$	p.d.f/c.d.f of repair time for subsystem E

TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

A state-transition diagram in fig. 1 shows various states of transitions of the system. The epochs of entry into states 0, 1, 2, 3, 4, 5, 6 and 7 are regeneration points and thus these are regenerative states. The states 2, 4, 5, 6 and 7 are failed states.

The transition probabilities are given by

$$\begin{aligned}
 q_{01}(t) &= p_1 \lambda_1 e^{-(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)t} & q_{02}(t) &= q_1 \lambda_1 e^{-(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)t} \\
 q_{03}(t) &= p_1 \lambda_2 e^{-(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)t} & q_{04}(t) &= q_1 \lambda_2 e^{-(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)t} \\
 q_{05}(t) &= \lambda_3 e^{-(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)t} & q_{06}(t) &= \lambda_4 e^{-(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)t} \\
 q_{07}(t) &= \lambda_5 e^{-(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)t} & q_{12}(t) &= q_2 g_{a_1}(t) \\
 q_{34}(t) &= q_2 g_{b_1}(t) & q_{10}(t) &= p_2 g_{a_1}(t) \\
 q_{30}(t) &= p_2 g_{b_1}(t) & q_{20}(t) &= g_{a_2}(t) \\
 q_{40}(t) &= g_{b_2}(t) & q_{50}(t) &= g_c(t) \\
 q_{60}(t) &= g_d(t) & q_{70}(t) &= g_e(t)
 \end{aligned}$$

The non-zero elements $p_{ij} = \lim_{s \rightarrow 0} q_{ij}^*(s) = \lim_{s \rightarrow 0} Q_{ij}^{**}(s)$ have been obtained as under:

$$\begin{aligned}
 p_{01} &= \frac{p_1 \lambda_1}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5} & p_{02} &= \frac{q_1 \lambda_1}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5} \\
 p_{03} &= \frac{p_1 \lambda_2}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5} & p_{04} &= \frac{q_1 \lambda_2}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5} \\
 p_{05} &= \frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5} & p_{06} &= \frac{\lambda_4}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5} \\
 p_{07} &= \frac{\lambda_5}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5} & p_{10} &= p_2 g_{a_1}^*(0) \\
 p_{30} &= p_2 g_{b_1}^*(0) & p_{12} &= q_2 g_{a_1}^*(0) \\
 p_{34} &= q_2 g_{b_1}^*(0) & p_{20} = p_{40} = p_{50} = p_{60} = p_{70} &= 1
 \end{aligned}$$

Clearly, it can be verified that

$$p_{01} + p_{02} + p_{03} + p_{04} + p_{05} + p_{06} + p_{07} = 1; \quad p_{10} + p_{12} = 1; \quad p_{30} + p_{34} = 1$$

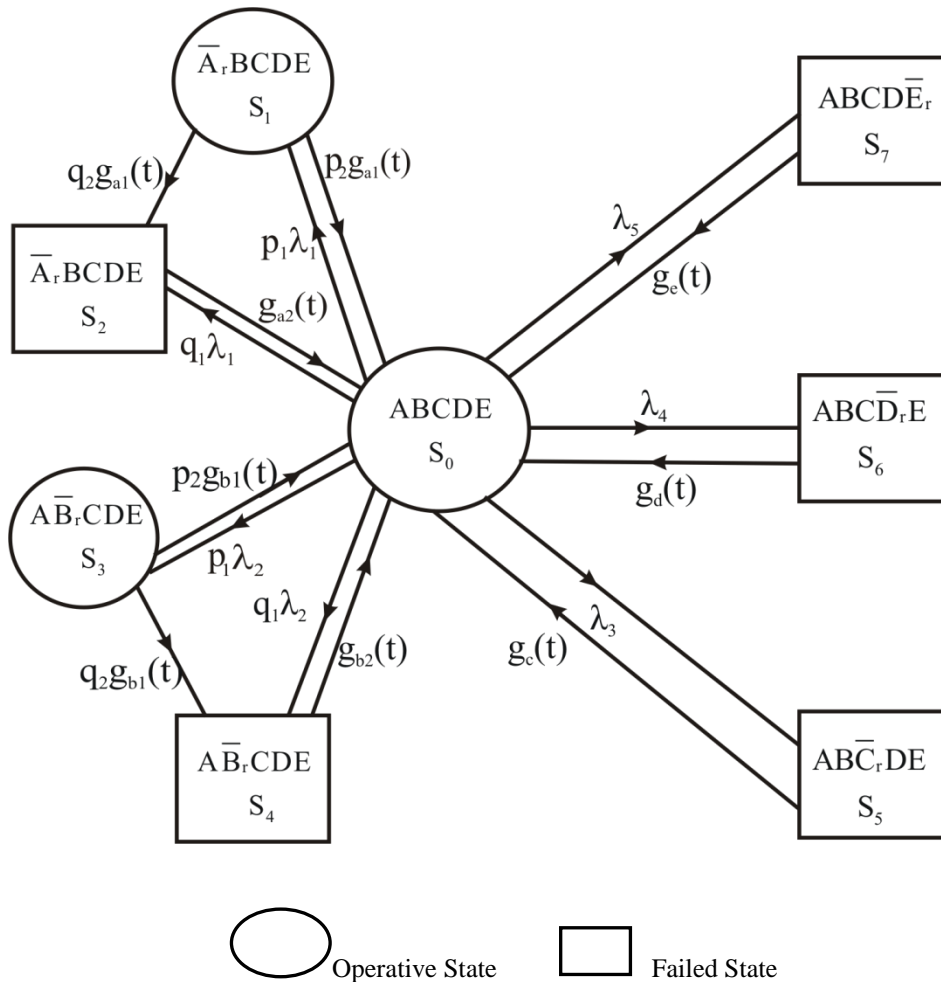


Figure 1. State Transition Diagram

Mean sojourn time μ_i in the i th state is the expected first passage time taken by the i th state before transiting to any other state and is given by

$$\mu_i = \int_0^{\infty} \Pr(T_i > t) dt, \text{ where } T_i \text{ is life time of the subsystem.}$$

Thus following have been obtained:

$$\begin{aligned} \mu_0 &= \frac{1}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5} ; & \mu_1 &= -g_{a1}^*(0) ; & \mu_2 &= -g_{a2}^*(0) ; \\ \mu_3 &= -g_{b1}^*(0) ; & \mu_4 &= -g_{b2}^*(0) ; & \mu_5 &= -g_c^*(0) ; \\ \mu_6 &= -g_d^*(0) ; & \mu_7 &= -g_e^*(0) \end{aligned}$$

The unconditional mean time taken by the system to transit for any state j when it is counted from the epoch of entrance into the state i , is mathematically stated as:

$$m_{ij} = \int_0^{\infty} t q_{ij}(t) dt = -q_{ij}^*(0).$$

Thus, we have

$$\begin{aligned} m_{01} &= \frac{p_1 \lambda_1}{(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)^2} ; & m_{02} &= \frac{q_1 \lambda_1}{(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)^2} ; \\ m_{03} &= \frac{p_1 \lambda_2}{(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)^2} ; & m_{04} &= \frac{q_1 \lambda_2}{(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)^2} ; \\ m_{05} &= \frac{\lambda_3}{(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)^2} ; & m_{06} &= \frac{\lambda_4}{(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)^2} ; \\ m_{07} &= \frac{\lambda_5}{(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)^2} ; & m_{10} &= -p_2 g_{a1}^*(0) ; m_{12} = -q_2 g_{a1}^*(0) ; \\ m_{30} &= -p_2 g_{b1}^*(0) ; & m_{34} &= -q_2 g_{b1}^*(0) \end{aligned}$$

It is clear that

$$\begin{aligned} m_{01} + m_{02} + m_{03} + m_{04} + m_{05} + m_{06} + m_{07} &= \mu_0 ; \\ m_{10} + m_{12} &= \mu_1 ; & m_{20} &= \mu_2 ; & m_{30} + m_{34} &= \mu_3 ; \\ m_{40} &= \mu_4 ; & m_{50} &= \mu_5 ; & m_{60} &= \mu_6 ; \\ m_{70} &= \mu_7 \end{aligned}$$

OTHER MEASURES OF SYSTEM EFFECTIVENESS

Using the probabilistic arguments for regenerative process, several recursive relations for various other measures of the system effectiveness have been obtained and on solving them using Laplace/Laplace-Stieltjes transforms, we get the following results:

$$\text{Mean time to system failure } (T_0) = \frac{N}{D}$$

$$\text{Availability of the system } (AV_0) = \frac{N_1}{D_1}$$

$$\text{Expected number of repairs of the subsystem A } (RA_0) = \frac{N_2}{D_1}$$

$$\text{Expected number of repairs of the subsystem B } (RB_0) = \frac{N_3}{D_1}$$

$$\text{Expected number of repairs of the subsystem C } (RC_0) = \frac{N_4}{D_1}$$

$$\text{Expected number of repairs of the subsystem D } (RD_0) = \frac{N_5}{D_1}$$

$$\text{Expected number of repairs of the subsystem E } (RE_0) = \frac{N_6}{D_1}$$

$$\text{Expected number of visits by the repairman } (V_0) = \frac{N_7}{D_1},$$

where

$$N = \mu_0 + p_{01}\mu_1 + p_{03}\mu_3 ; \quad D = 1 - p_{01}p_{10} - p_{03}p_{30} ;$$

$$N_1 = \mu_0 + p_{01}\mu_1 + p_{03}\mu_3 ; \quad N_2 = p_{01} + p_{02} + p_{01}p_{12} ;$$

$$N_3 = p_{03} + p_{04} + p_{03}p_{34} ; \quad N_4 = p_{05} ;$$

$$N_5 = p_{06} ; \quad N_6 = p_{07} ;$$

$$N_7 = p_{01} + p_{02} + p_{03} + p_{04} + p_{05} + p_{06} + p_{07}$$

and

$$D_1 = \mu_0 + p_{01}\mu_1 + (p_{01}p_{12} + p_{02})\mu_2 + p_{03}\mu_3 + (p_{03}p_{34} + p_{04})\mu_4 + p_{05}\mu_5 + p_{06}\mu_6 + p_{07}\mu_7$$

PROFIT ANALYSIS OF THE SYSTEM

The expected profit incurred to the system is given as

$$P = C_0AV_0 - C_1A_0 - C_2B_0 - C_3C_0 - C_4D_0 - C_5E_0 - C_6V_0,$$

where

C_0 = revenue per unit up time of the system

C_1 = cost per unit repair of the subsystem A

C_2 = cost per unit repair of the subsystem B

C_3 = cost per unit repair of the subsystem C

C_4 = cost per unit repair of the subsystem D

C_5 = cost per unit repair of the subsystem E

C_6 = cost per visit of the repairman

GRAPHICAL INTERPRETATIONS AND CONCLUSION

The following particular case has been considered for the graphical analysis purpose:

$$\begin{aligned} g_{a_1}(t) &= \beta_{a_1} e^{-\beta_{a_1} t} ; & g_{a_2}(t) &= \beta_{a_2} e^{-\beta_{a_2} t} ; & g_{b_1}(t) &= \beta_{b_1} e^{-\beta_{b_1} t} ; \\ g_{b_2}(t) &= \beta_{b_2} e^{-\beta_{b_2} t} ; & g_c(t) &= \beta_c e^{-\beta_c t} ; & g_d(t) &= \beta_d e^{-\beta_d t} ; \\ g_e(t) &= \beta_e e^{-\beta_e t} \end{aligned}$$

Various graphs have been plotted for mean time to system failure, availability and profit of the system for different values of failure rates ($\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$), repair rates ($\beta_{a_1}, \beta_{a_2}, \beta_{b_1}, \beta_{b_2}, \beta_c, \beta_d, \beta_e$), probabilities (p_1, p_2, q_1, q_2) and costs ($C_0, C_1, C_2, C_3, C_4, C_5, C_6$), respectively.

Following interpretations and conclusion have been made from the graphs:

Fig. 2 shows the behavior of mean time to system failure (T_0) with respect to failure rate λ_1 for different values of probability p_2 . It can be concluded from the graph that mean time to system failure decreases with the increase in the values of failure rate of the subsystem A and has higher values for higher values of probability that the repair is done prior to the dust storage becomes low when other parameters are kept fixed.

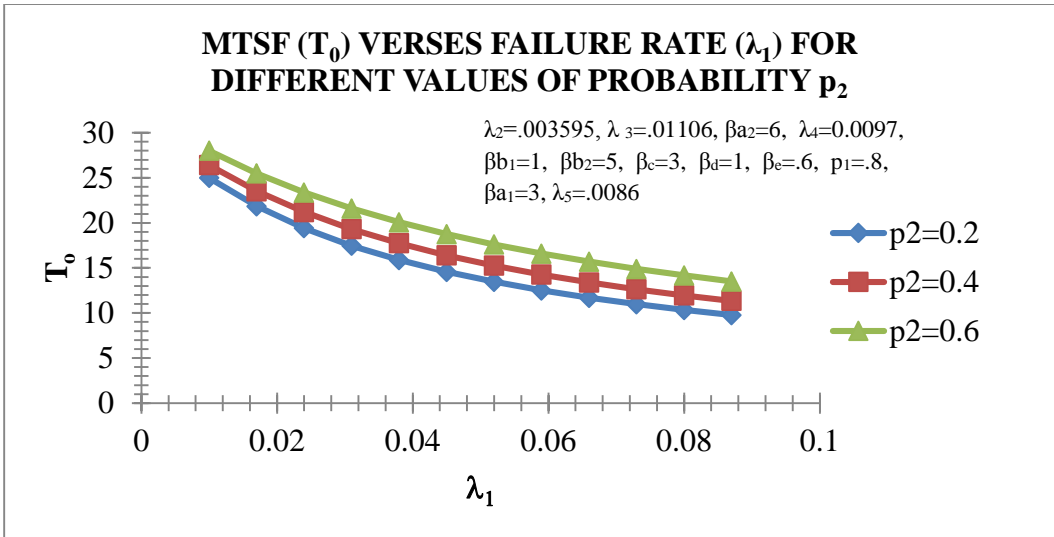


Figure 2.

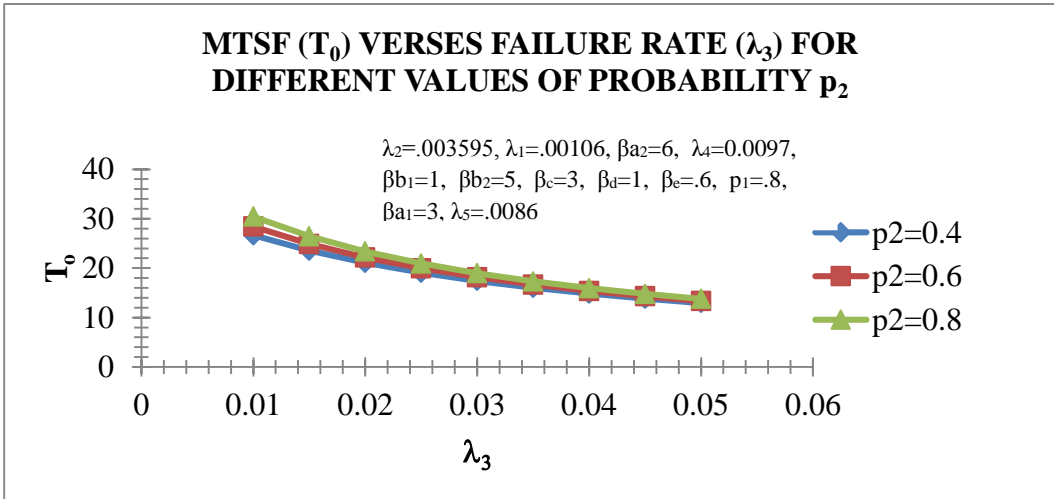


Figure 3.

The pattern in fig. 3 indicates the behavior of mean time to system failure (T_0) with respect to failure rate λ_3 for different values of probability p_2 . It can be concluded from the graph that mean time to system failure decreases with the increase in

the values of failure rate of the subsystem C and has higher values for higher values of probability that the repair is dust storage is less than the required level when other parameters are fixed.

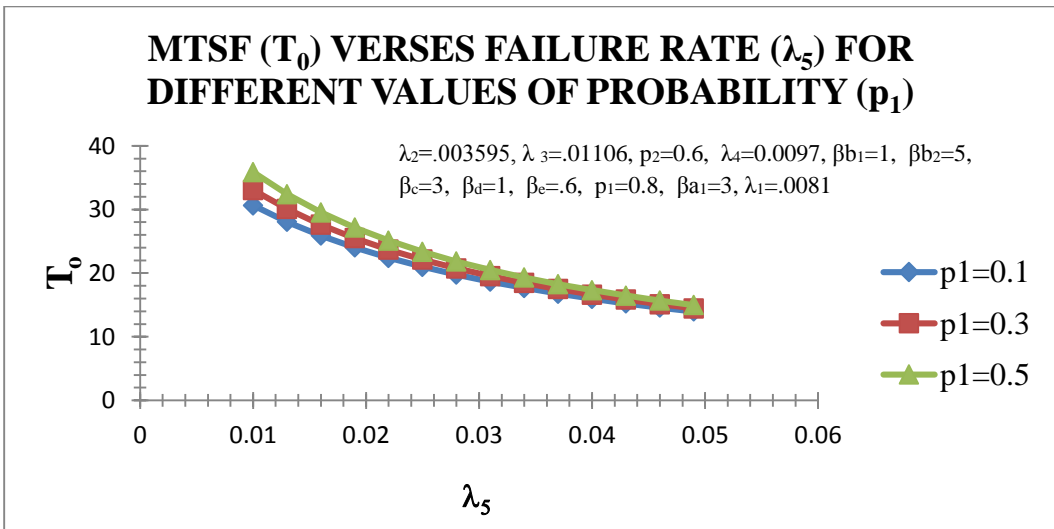


Figure 4.

Fig. 4 depicts the behavior of mean time to system failure (T_0) with respect to failure rate (λ_5) for different values of probability (p_1). It is evident from the graph that mean time to system failure decreases with the increase in the values of failure rate of subsystem C and has higher values for higher values of probability that the dust storage is greater than the required level when other parameters remains fixed.

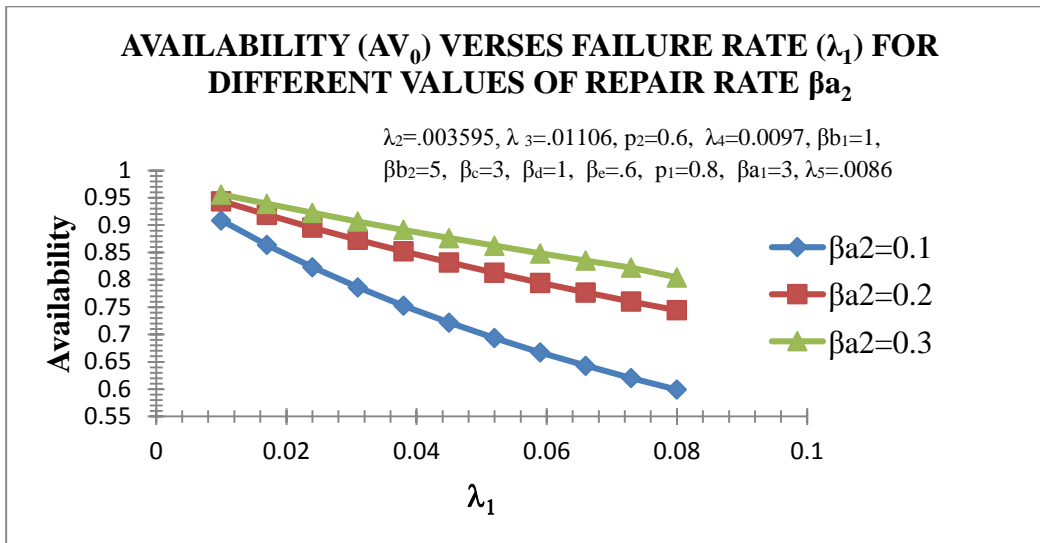


Figure 5.

The curves in the fig.5 reveal the behavior of availability (AV_0) with respect to failure rate λ_1 for different values of repair rate β_{a2} . It can be concluded from the graph that availability decreases with the increase in the values of failure rate of subsystem A and has higher values for higher values of repair rate of subsystem A when repair is done after the dust storage.

Fig. 6 gives the behavior of availability (AV_0) with respect to failure rate λ_2 for different values of repair rate β_{a2} . The graph indicates that availability decreases with the increase in the values of failure rate of subsystem B and has higher values for higher values of repair rate of subsystem A when other parameters remain fixed.

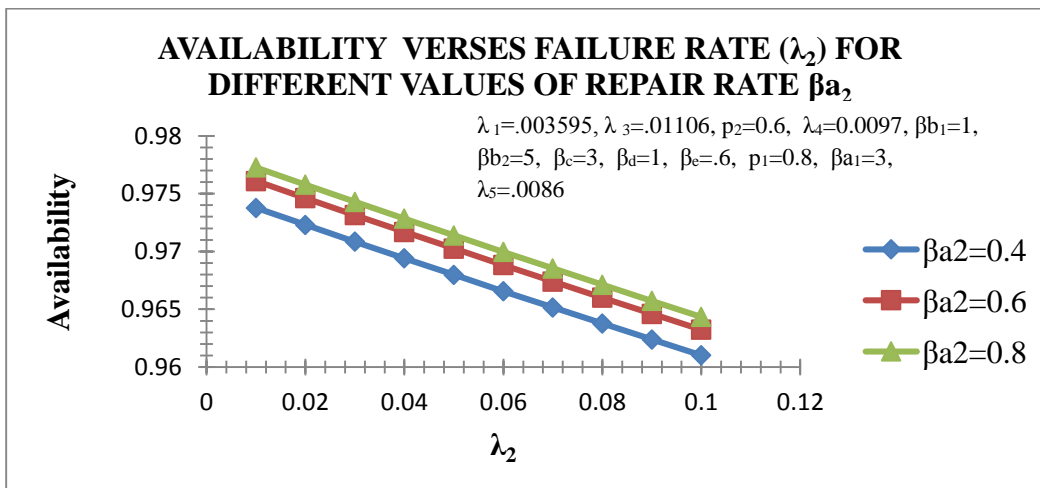


Figure 6.

The pattern in fig. 7 shows the behavior of availability with respect to failure rate λ_3 for different values of repair rate β_c . It can be concluded that availability decreases with the increase in the values of failure rate of subsystem C and has higher values for higher values of repair rate of subsystem C when other parameters remain fixed.

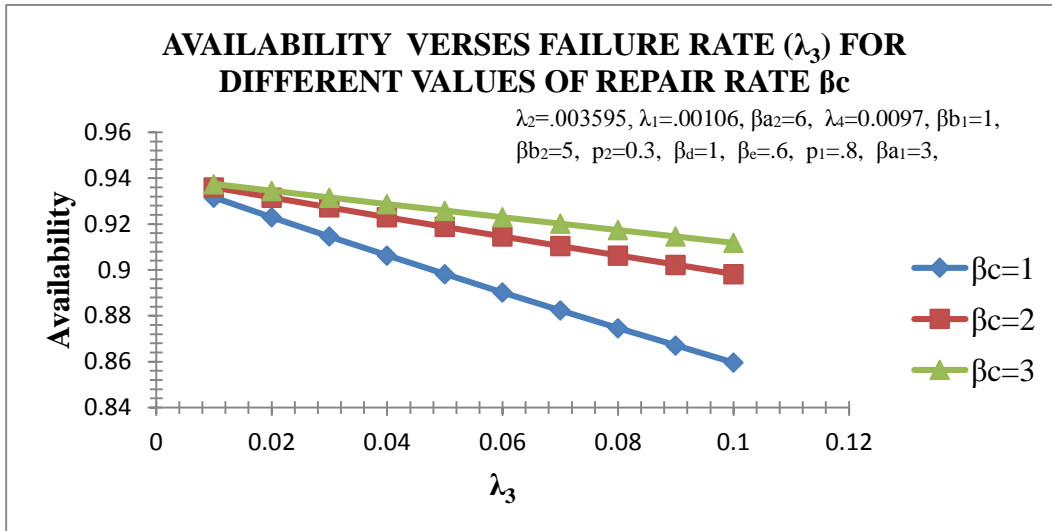


Figure 7.

Fig. 8 depicts the behaviour of profit of the system with respect to failure rate λ_2 of the system for different values of revenue per unit up of the system C_0 . It can be concluded from the graph that the profit increases with the increase in the values of revenue per unit uptime and has lower values for higher values of failure rate of subsystem B. Further, it can be observed that for $C_0 = \text{Rs. } 200$, the profit is $< \text{or } = \text{ or } >$ according as λ_2 is $> \text{ or } = \text{ or } <$ 0.4173. Hence in this case for the system to be profitable, the failure rate (λ_2) should be less than 0.4173. Similarly, for $C_0 = \text{Rs. } 250$ and $C_0 = \text{Rs. } 300$, for the system to be profitable, the failure rate of subsystem B should be less than 0.5401 and 0.6669, respectively.

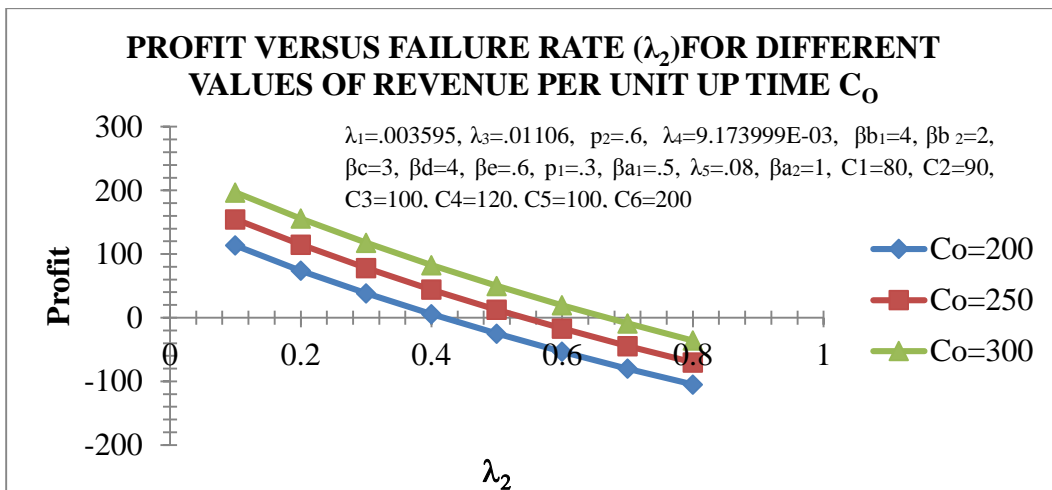


Figure 8.

The curves in the fig. 9 show the behavior of the profit with respect to failure rate λ_4 of the system for the different values of repair rate β_c . It is concluded from the graph that profit decreases with the increase in the failure rate of subsystem D and attain higher values for higher values of repair rate of subsystem C.

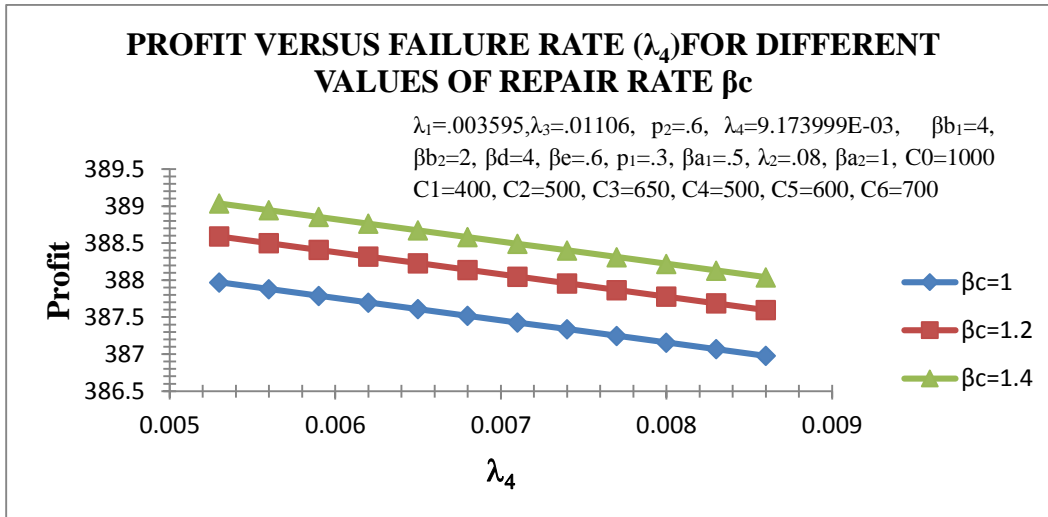


Figure 9.

Fig. 10 reveals the behaviour of profit of the system with respect to failure rate λ_5 of the system for different values of revenue per unit up of the system C_0 . From the graph, it is noticed that profit increases with the increase in the values of C_0 and has lower values for higher values of λ_5 . For $C_0 = \text{Rs. } 400$, the profit is $<$ or $=$ or $>$ according as λ_5 is $>$ or $=$ or $<$ 0.2089. Hence in this case for the system to be profitable, the failure rate (λ_5) should be less than 0.2089. Similarly, for $C_0 = \text{Rs. } 600$ and $C_0 = \text{Rs. } 800$ for the system to be profitable the failure rate (λ_5) should be less than 0.3642 and 0.5184, respectively.

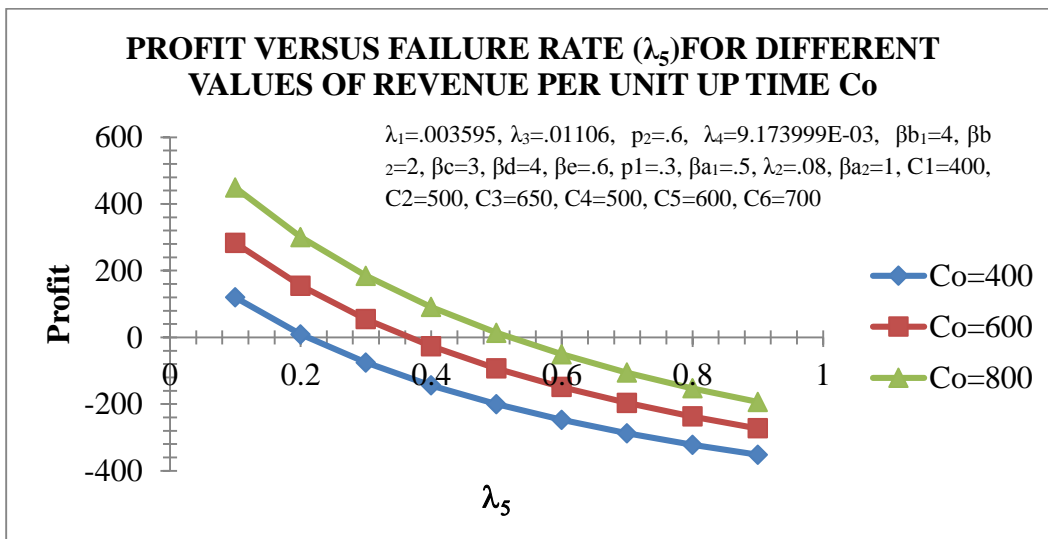


Figure 10

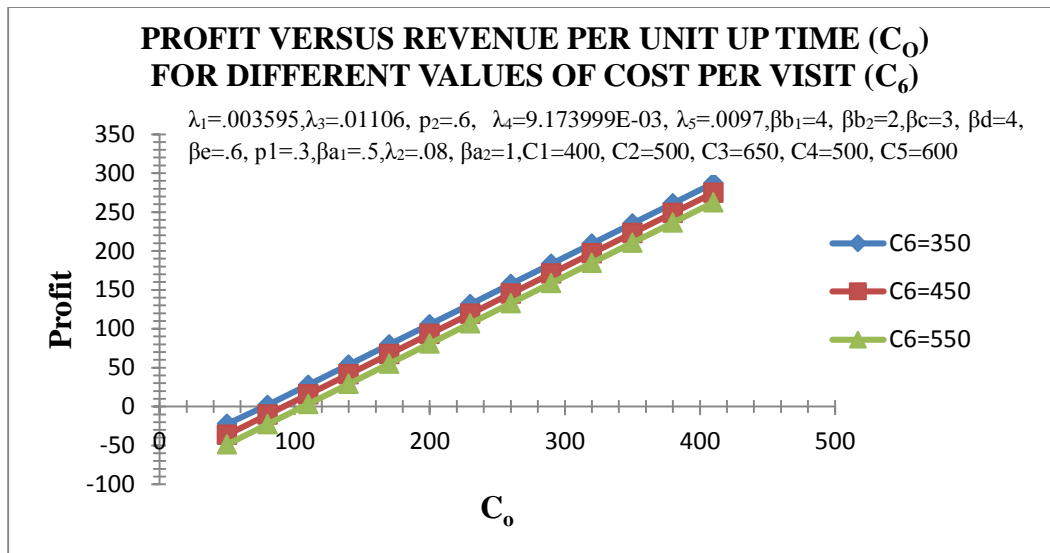


Figure 11

The behavior of profit with respect to revenue per unit up time of the system C_0 for different values of cost per visit of the repairman C_6 is shown in fig. 11. It can be concluded from the graph that the profit increases with the increase in the values of C_0 and has lower values for higher values of C_6 . It is evident from the graph that for $C_6=$ Rs. 350, the profit is positive or zero or negative according as $C_0 >$ or $=$ or $<$ 78.2938. Hence in this case, for the system to be profitable, the revenue per unit up time should be fixed greater than Rs. 78.2938. Similarly for $C_6=$ Rs. 450 and $C_5=$ Rs. 550 the system to be profitable, the revenue per unit up time of the system should be greater than Rs. 93.0612 and Rs. 108.4336, respectively.

CONCLUSION

The expressions for various measures of system effectiveness for the ceramic tile production system were obtained developing a reliability model considering its five subsystems viz. Ball Mill, Spray Dryer, Hydraulic Press, Glaze Line and Kiln that works in series configuration. The working of the system also depends on the minimum storage capacity of the Silo, a subsystem to store in-process material for manufacturing tiles. From the graphical analyses, it has been concluded that mean time to system failure, availability and profit of the system decreases with the increase in the values of failure rates of its subsystems viz. Ball mill, Spray Dryer, Hydraulic press, Glaze line and Kiln. Further, the mean time to system failure, availability and profit of the system has higher values for higher values of repair rates of the subsystems when other parameters remain fixed. Also profit incurred to the system increases with the increase in the values of revenue per unit up time of the system and has lower values for higher values of cost per visit of the repairman. The cut-off points for the failure rates of the subsystems and various costs associated with the system may be obtained to take crucial decision about its designing and production. This would also help the stakeholders to enhance the system's

productivity with minimum costs apart from making future planning about the similar industrial systems.

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