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k-Super Harmonic Mean Labeling of Some *H*-GRAPHS

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Abstract:

Harmonic mean labeling was introduced by Sandhya et al. We extend this notion to k-super harmonic mean labeling. In this paper we investigate k-super harmonic mean labeling of some H-graphs.

Keywords: harmonic mean labeling, k-harmonic mean labeling, super harmonic mean graph, k-super harmonic mean graph.

INTRODUCTION

By a graph $G = (V(G), E(G))$ with p vertices and q edges we mean a simple, connected and undirected graph. In this paper a brief summary of definitions and other information is given in order to maintain compactness. The term not defined here are used in the sense of Harary [3].

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. A useful survey on graph labeling by Gallian (2016) can be found in [1]. Somasundaram and Pondraj [11] have introduced the notion of mean labeling of graphs. Sandhya and David Raj introduced super harmonic mean labeling in [8]. k-super harmonic mean labeling was introduced by M. Tamilselvi and N. Revathi in [13]. In this paper we investigate k-super harmonic mean labeling of some H-graphs.

PRELIMINARIES

Definition 2.1

Let G be a (p, q) graph and $f: V(G) \to \{1, 2, ..., p + q\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$, then f is called super harmonic mean labeling if $f(V) \cup \{f^*(e); e \in E(G)\} = \{1, 2, ..., p + q\}$. A graph that admits a super harmonic mean labeling is called super harmonic mean graph.

Definition 2.2

Let G be a (p, q) graph and $f: V(G) \to \{k, k + 1, ..., k + p + q - 1\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$, then f is called k-super harmonic mean labeling if $f(V) \cup \{ f^*(e); e \in E(G)\} = \{k, k + 1, ..., k + p + q - 1\}$. A graph that admits a k-super harmonic mean labeling is called k-super harmonic mean graph.

Definition 2.3

The H-graph of a path P_n is the graph obtained from two copies of P_n with vertices $u_1, u_2, ..., u_n$ and $v_1, v_2, ..., v_n$ joining the vertices $u_{\frac{n+1}{2}}$ and $v_{\frac{n+1}{2}}$ if n is odd and the vertices $u_{\frac{n+1}{2}}$ and $v_{\frac{n}{2}}$ if n is even.

Definition 2.4

If G has order n, the corona of G with H, $G \odot H$ is the graph obtained by taking one copy of G and n copies of H and joining the i^{th} vertex of G with an edge to every vertex in the i^{th} copy of H.

Theorem 2.1

The H-graph G is k-super harmonic mean graph for all odd $n \geq 5$ and k.

Proof

Let $V(G) = \{u_i, v_i; 1 \leq i \leq n\}$

$$E(G) = \{u_iu_{i+1}, v_iv_{i+1}; 1 \leq i \leq n-1\} \cup \left\{u_{\frac{n+1}{2}}v_{\frac{n+1}{2}}\right\}$$

Define a function $f: V(G) \to \{k, k + 1, ..., k + 4n - 1\}$ by

$$f(u_i) = \begin{cases} k+4i-4 & \text{if } 1 \leq i \leq \frac{n+1}{2} \\ k+4i-3 & \text{if } \frac{n+3}{2} \leq i \leq n \end{cases}$$

For $k \leq 2$,

$$f(v_i) = \begin{cases} k+2 & \text{if } i = 1 \\ k+4i-3 & \text{if } 2 \leq i \leq \frac{n-1}{2} \\ k+2n+1 & \text{if } \frac{n+1}{2} \\ k+4i-2 & \text{if } \frac{n+3}{2} \leq i \leq n \end{cases}$$

For $k \geq 3$,

$$f(v_i) = \begin{cases} k+4i-3 & \text{if } 1 \le i \le \dfrac{n-1}{2} \\ k+4i-2 & \text{if } \dfrac{n+1}{2} \le i \le n \end{cases}$$

Then the induced edge labels are

For $k \le 2$,

$$f^*(u_i u_{i+1}) = \begin{cases} k+1 & \text{if } i = 1 \\ k+4i-2 & \text{if } 2 \le i \le \dfrac{n-1}{2} \\ k+2n & \text{if } i = \dfrac{n+1}{2} \\ k+4i-1 & \text{if } \dfrac{n+3}{2} \le i \le n-1 \end{cases}$$

For $k \ge 3$,

$$f^*(u_i u_{i+1}) = \begin{cases} k+4i-2 & \text{if } 1 \le i \le \dfrac{n-1}{2} \\ k+4i-1 & \text{if } \dfrac{n+1}{2} \le i \le n-1 \end{cases}$$

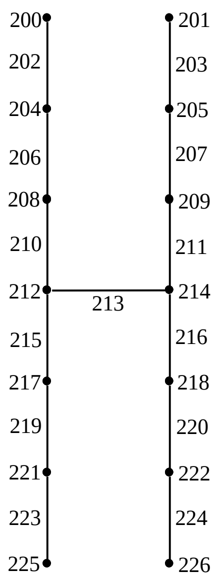
For all k,

$$f^*(v_i v_{i+1}) = \begin{cases} k+4i-1 & \text{if } 1 \le i \le \dfrac{n-1}{2} \\ k+4i & \text{if } \dfrac{n+1}{2} \le i \le n-1 \end{cases}$$

$$f^*\left(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right) = k + 2n - 1$$

Thus $f(V) \cup \{ f^*(e) ; e \in E(G)\} = \{k, k+1, ..., k+4n-1\}$. Hence H-graph G is k-super harmonic mean graph for all odd $n \ge 5$.

Example 2.1



200-super harmonic mean labeling of *H*-graph of P_9

Theorem 2.2

The H-graph G is k-super harmonic mean graph for all even $n \ge 6$ and $k \ge 2$.

Proof

Let $V(G) = \{u_i, v_i; 1 \le i \le n\}$

$$E(G) = \{u_i u_{i+1}; v_i v_{i+1}; 1 \le i \le n-1\} \cup \left\{u_{\frac{n+1}{2}} v_{\frac{n}{2}}\right\}$$

Define $f : V(G) \to \{k, k+1, ..., k+4n-1\}$ by

$$f(u_i) = \begin{cases} k+4i-4 & 1 \le i \le \dfrac{n}{2} \\ k+4i-3 & \dfrac{n}{2}+1 \le i \le n \end{cases}$$

$$f(v_i) = \begin{cases} k+2 & \text{if } i = 1 \,\&\, k = 2 \\ k+4i-3 & \text{if } 2 \le i \le \dfrac{n}{2} \,\&\, k = 2 \\ k+4i-3 & \text{if } 1 \le i \le \dfrac{n}{2} \,\&\, k \ge 3 \\ k+4i-2 & \text{if } \dfrac{n}{2}+1 \le i \le n \end{cases}$$

Then the induced edge labels are

$$f^*(u_i u_{i+1}) = \begin{cases} k+1 & \text{if } i = 1 \,\&\, k = 2 \\ k+4i-2 & \text{if } 2 \le i \le \dfrac{n}{2} \,\&\, k = 2 \\ k+4i-2 & \text{if } 1 \le i \le \dfrac{n}{2} \,\&\, k \ge 3 \\ k+4i-1 & \text{if } \dfrac{n}{2}+1 \le n-1 \end{cases}$$

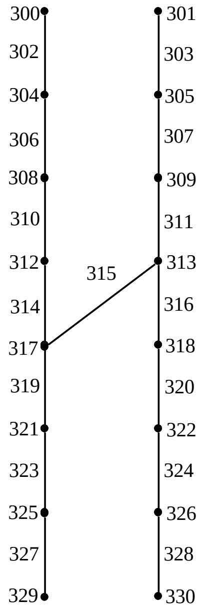
$$f^*(v_i v_{i+1}) = \begin{cases} k+4i-1 & 1 \le i \le \dfrac{n}{2}-1 \\ k+4i & \dfrac{n}{2} \le i \le n-1 \end{cases}$$

$$f^*\left(u_{\frac{n}{2}+1} v_{\frac{n}{2}}\right) = k + 2n - 1$$

Thus $f(V) \cup \{ f^*(e) ; e \in E(G)\} = \{k, k+1, ..., k+4n-1\}$.

Hence H-graph G is k-super harmonic mean graph for all even $n \ge 6$.

Example 2.2



300-super harmonic mean labeling of *H*-graph of P_8

Theorem 2.3

For a *H*-graph G, $G \odot K_1$ is k-super harmonic mean graph for all odd $n \geq 5$ and $k \geq 3$.

Proof

Let $V(G \odot K_1) = \{u_i, v_i; 1 \leq i \leq n\} \cup \{u_i', v_i'; 1 \leq i \leq n\}$

$E(G \odot K_1) = \{u_iu_{i+1}, v_iv_{i+1}; 1 \leq i \leq n-1\} \cup \{u_iu_i', v_iv_i'; 1 \leq i \leq n\} \cup \left\{u_{\frac{n+1}{2}}v_{\frac{n+1}{2}}\right\}$

Define a function $f: V(G \odot K_1) \rightarrow \{k, k+1, ..., k+8n-2\}$ by

$$f(u_i) = \begin{cases} k+8i-6 & \text{if } 1 \leq i \leq \frac{n+1}{2} \\ k+8i-5 & \text{if } \frac{n+3}{2} \leq i \leq n \end{cases}$$

$$f(v_i) = \begin{cases} k+8i-5 & \text{if } 1 \leq i \leq \frac{n-1}{2} \\ k+8i-4 & \text{if } \frac{n+1}{2} \leq i \leq n \end{cases}$$

$$f(u_i') = \begin{cases} k+8i-8 & \text{if } 1 \leq i \leq \frac{n+1}{2} \\ k+8i-7 & \text{if } \frac{n+3}{2} \leq i \leq n \end{cases}$$

$$f(v_i') = \begin{cases} k+7 & \text{if } i = 1 \;\&\; k \leq 10 \\ k+15 & \text{if } i = 2 \;\&\; k \leq 10 \\ k+8i-3 & \text{if } 3 \leq i \leq \frac{n-1}{2} \;\&\; k \leq 10 \;\&\; n \neq 5 \\ k+8i-3 & \text{if } 1 \leq i \leq \frac{n-1}{2} \;\&\; k \geq 11 \\ k+4n+3 & \text{if } i = \frac{n+1}{2} \;\forall k \\ k+8i-2 & \text{if } \frac{n+3}{2} \leq i \leq n \;\forall k \end{cases}$$

Then the induced edge labels are:

$$f^*(u_iu_i') = \begin{cases} k+8i-7 & \text{if } 1 \leq i \leq \frac{n+1}{2} \\ k+8i-6 & \text{if } \frac{n+3}{2} \leq i \leq n \end{cases}$$

$$f^*(v_iv_i') = \begin{cases} k+8i-4 & \text{if } 1 \leq i \leq \frac{n-1}{2} \\ k+8i-3 & \text{if } \frac{n+1}{2} \leq i \leq n \end{cases}$$

$$f^*(u_iu_{i+1}) = \begin{cases} k+5 & \text{if } i = 1 \;\&\; k \leq 10 \\ k+13 & \text{if } i = 2 \;\&\; k \leq 10 \\ k+8i-2 & \text{if } 3 \leq i \leq \frac{n+1}{2} \;\&\; k \leq 10 \\ k+8i-2 & \text{if } 1 \leq i \leq \frac{n+1}{2} \;\&\; k \geq 11 \\ k+8i-1 & \text{if } \frac{n+3}{2} \leq i \leq n-1 \;\forall k \end{cases}$$

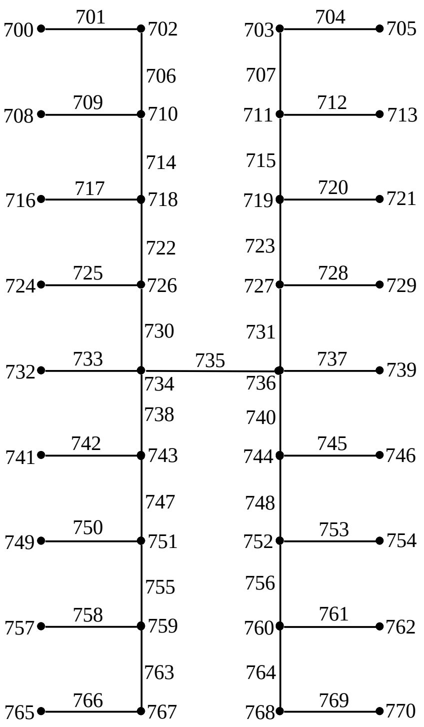
$$f^*(v_iv_{i+1}) = \begin{cases} k+6 & \text{if } i = 1 \;\&\; k \leq 10 \\ k+14 & \text{if } i = 2 \;\&\; k \leq 10 \\ k+8i-1 & \text{if } 3 \leq i \leq \frac{n-1}{2} \;\&\; k \leq 10 \\ k+8i-1 & \text{if } 1 \leq i \leq \frac{n-1}{2} \;\&\; k \geq 11 \\ k+8i & \text{if } \frac{n+1}{2} \leq i \leq n-1 \;\forall k \end{cases}$$

$$f^*\left(u_{\frac{n+1}{2}}v_{\frac{n+1}{2}}\right) = k + 4n - 1$$

Hence $f(V) \cup \{f^*(e); e \in E(G)\} = \{k, k+1, ..., k+8n-2\}$.

Thus for a *H*-graph G, $G \odot K_1$ is k-super harmonic mean graph for all odd $n \geq 5$.

Example 2.3



700-super harmonic mean labeling of *H*-graph of $G \odot K_1$

Theorem 2.4

For a *H*-graph G, $G \odot K_1$ is *k*-super harmonic mean graph for all even $n \geq 6$ and $k \geq 3$.

Proof

Let $V(G \odot K_1) = \{u_i, v_i;\ 1 \leq i \leq n\} \cup \{u_i', v_i'; 1 \leq i \leq n\}$

$E(G \odot K_1) = \{u_iu_{i+1},\ v_iv_{i+1};\ 1 \leq i \leq n-1\} \cup \{u_iu_i', v_iv_i'; 1 \leq i \leq n\} \cup \left\{u_{\frac{n+1}{2}}v_{\frac{n}{2}}\right\}$

Define a function $f : V(G \odot K_1) \to \{k, k+1, ..., k+8n-2\}$ by

$$f(u_i) = \begin{cases} k+8i-6 & \text{if } 1 \leq i \leq \frac{n}{2} \\ k+4n+3 & \text{if } i = \frac{n}{2}+1 \\ k+4n+12 & \text{if } i = \frac{n}{2}+2 \\ k+8i-5 & \text{if } \frac{n}{2}+3 \leq i \leq n \end{cases}$$

$$f(v_i) = \begin{cases} k+8i-5 & \text{if } 1 \leq i \leq \frac{n}{2} \\ k+4n+6 & \text{if } i = \frac{n}{2}+1 \\ k+4n+11 & \text{if } i = \frac{n}{2}+2 \\ k+8i-4 & \text{if } \frac{n}{2}+3 \leq i \leq n \end{cases}$$

$$f(u_i') = \begin{cases} k+8i-8 & \text{if } 1 \leq i \leq \frac{n}{2} \\ k+8i-7 & \text{if } \frac{n}{2}+1 \leq i \leq n \end{cases}$$

$$f(v_i') = \begin{cases} k+7 & \text{if } i = 1 \,\&\, k \leq 10 \\ k+15 & \text{if } i = 2 \,\&\, k \leq 10 \\ k+8i-3 & \text{if } 3 \leq i \leq \frac{n}{2} \,\&\, k \leq 10 \\ k+8i-3 & \text{if } 1 \leq i \leq \frac{n}{2} \,\&\, k \geq 11 \\ k+4n+4 & \text{if } i = \frac{n}{2}+1\ \forall k \\ k+8i-2 & \text{if } \frac{n}{2}+2 \leq i \leq n\ \forall k \end{cases}$$

Then the induced edge labels are:

$$f^*(u_iu_i') = \begin{cases} k+8i-7 & \text{if } 1 \leq i \leq \frac{n}{2} \\ k+8i-6 & \text{if } \frac{n}{2}+1 \leq i \leq n \end{cases}$$

$$f^*(v_iv_i') = \begin{cases} k+8i-4 & \text{if } 1 \leq i \leq \frac{n}{2} \\ k+8i-3 & \text{if } \frac{n}{2}+1 \leq i \leq n \end{cases}$$

$$f^*(u_iu_{i+1}) = \begin{cases} k+5 & \text{if } i = 1 \,\&\, k \leq 10 \\ k+13 & \text{if } i = 2 \,\&\, k \leq 10 \\ k+8i-2 & \text{if } 3 \leq i \leq \frac{n}{2}-1 \,\&\, k \leq 10 \\ k+8i-2 & \text{if } 1 \leq i \leq \frac{n}{2} \,\&\, k \geq 11 \\ k+8i-1 & \text{if } \frac{n}{2}+1 \leq i \leq n \end{cases}$$

$f^+(v_iv_{i+1}) =$

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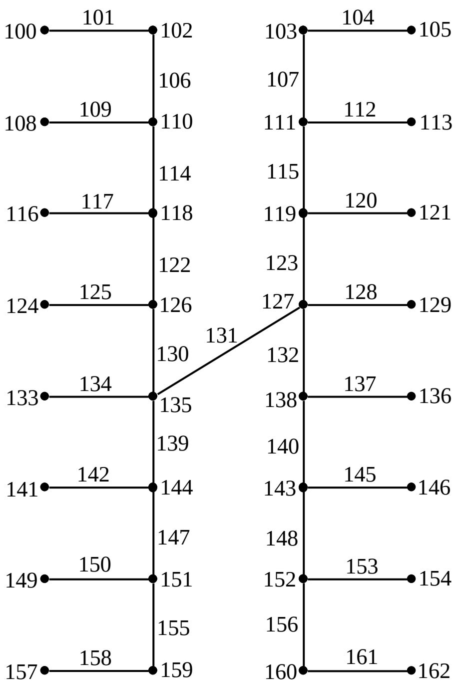
$$\begin{cases} k+6 & \text{if } i=1 \,\&\, k \le 10 \\ k+14 & \text{if } i=1 \,\&\, k \le 10 \\ k+8i-1 & \text{if } 3 \le i \le \dfrac{n}{2}-1 \,\&\, k \le 10 \,\&\, n \ne 6 \\ k+8i-1 & \text{if } 1 \le i \le \dfrac{n}{2}-1 \,\&\, k \ge 11 \\ k+8i & \text{if } \dfrac{n}{2} \le i \le n \end{cases}$$

$$f^*\left(u_{\frac{n+1}{2}} v_{\frac{n}{2}}\right) = k + 4n - 1$$

Thus $f(V) \cup \{ f^*(e) ; e \in E(G)\} = \{k, k+1, ..., k+8n-2\}$.

Hence for a H-graph, $G \odot K_1$ is k-super harmonic mean graph for all even $n \ge 6$.

Example 2.4



100-super harmonic mean labeling of *H*-graph of $G \odot K_1$

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