International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 16 (2018) pp. 12638-12643

A Study on Fuzzy α-Minimum Edge Wighted Spanning Tree with Cut Property Algorithm

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INTRODUCTION

The minimum spanning tree problem (Graham and Hell 1985) is one of the most typical problems in combinatorial optimization. It has many applications in communication network (Chiang, Liu, and Huang 2007), statistical cluster analysis (Gower and Ross 1969), image processing (Osteen and Lin 1974), etc. For instance, in network routing protocols, the minimum cost spanning tree is one of the most effective methods to broadcast the messages from a source node to a set of destinations.

When the edge weights associated to a graph are assumed to be crisp numbers, the minimum spanning tree problem can be solved in polynomial time by some well-known algorithms such as the Kruskal algorithm (Kruskal 1956) and the Prim algorithm (Prim 1957).

However, the edge weights are not always deterministic (nor crisp) in real applications. For example, the links in a communication network may be affected by collisions, congestions, interferences or some other factors. Considering non-deterministic factors may be encountered in a minimum spanning tree problem, Ishii et al. (1981) and Ishii and Matsutomi (1995) first discussed the problem with random edge weights, called the stochastic spanning tree problem and presented a polynomial time algorithm to solve it when the parameters of probability distributions of the edge weights are unknown. Torkestani and Meybodi (2012) designed a learning automata-based heuristic algorithm which significantly decreases the rate of unnecessary samples to solve the stochastic spanning tree problem with unknown probability distributions of weights. Furthermore, Dhamdhere, Ravi, and Singh (2005) and Swamy and Shmoys (2006) discussed the two stage stochastic minimum spanning tree problems and Torkestani (2012) considered the degree-constrained minimum spanning tree problem on a stochastic graph.

In this paper, we make a further study of the minimum spanning tree problem with fuzzy edge weights. We propose the concept of fuzzy α-minimum spanning trees based on the credibility measure defined by Liu and Liu (2002), and then discuss this problem under different conditions. For the general case in which the edge weights are general fuzzy numbers, a hybrid intelligent algorithm is designed to solve it. Besides, the scenarios with triangular and trapezoidal fuzzy edge weights are discussed separately.

Definition 1: (α-Weight of Spanning Tree) Let $\bar{G}= (V, E, \xi)$ denote a connected graph, where ξ is a fuzzy vector consisting of the fuzzy edge weights. For a predetermined confidence level $\alpha \in (0, 1]$, the α-weight of a spanning tree T, denoted by $W_\alpha(T, \xi)$, is defined as

$$W_\alpha(T, \xi)= \min\{W|\mathrm{Cr}\{W(T, \xi) \leq W\} \geq \alpha\}.$$

On the basis of Definition 3, we present the following conception of fuzzy α-minimum spanning tree for graphs with fuzzy edge weights.

Example, in the communication network graph, when the communication costs among centres are assumed as fuzzy variables, and the decision-maker sets a confidence level α and hopes to minimize the α-weight of spanning tree, finding the minimum cost spanning tree in this fuzzy network graph is a fuzzy α-minimum spanning tree problem. In the following sections, the solutions of the fuzzy α-minimum spanning tree problem are divided into three cases to discuss from the particular to the general Membership function

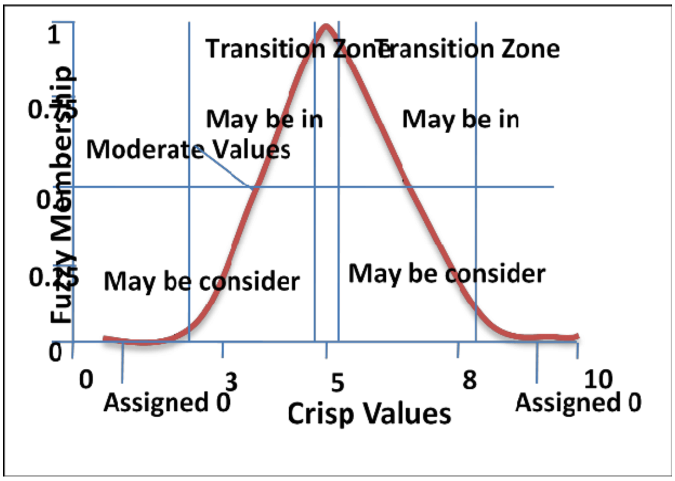


Figure 1.1. Membership Function

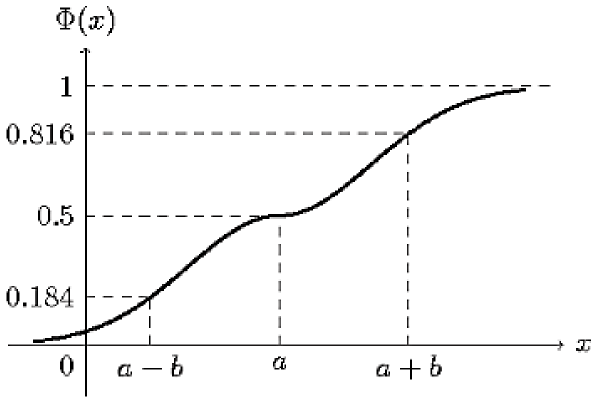


Figure 1.2. Credibility Distribution

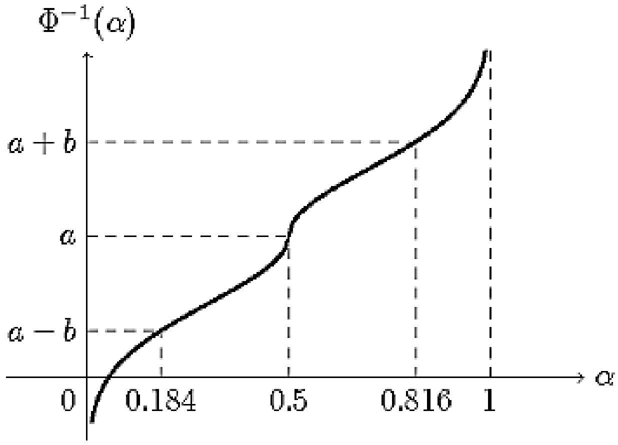


Figure 1.3. Inverse Credibility Distribution

Theorem 1. An arc *(a, b)* is a fuzzy bridge of G :(σ , μ) if and only if *(a, b)* is in every maximum spanning tree of G.

Proof: Let *(a,* b) be a fuzzy bridge of G. Then arc *(a,* b) is the unique strongest

a -b path and hence is in every maximum spanning tree of G.

Conversely, let *(a, b)* be in every maximum spanning tree *T* of G and assume that

(a, b) is not a fuzzy bridge. Then *(a, b)* is a weakest arc of some cycle in G and μ^{∞} *(a, b)* > μ *(a, b)),* which implies that *(a, b)* is in no maximum spanning tree of G.

Theorem 2. A node w is a fuzzy cutnode of G :$(\sigma, \mu\ )$ if and only if *w* is an internal node of every maximum spanning tree of G.

Proof: Let w be a fuzzy cutnode of G. Then there exist *a, b* distinct from c such that c is on every strongest *a* - b path. Now each maximum spanning tree of G contains unique strongest a - b path and hence w is an internal node of each maximum spanning tree of G.

Conversely, let w be an internal node of every maximum spanning tree. Let *T* be a maximum spanning tree and let *(a, c)* and *(c, b)* be arcs in *T.* Note that the path *a,* c, b is a strongest a -b path in *T.* If possible assume that w is not a fuzzy cutnode. Then between every pair of nodes a, *b* there exist at least one strongest *a* - b path not containing c. Consider one such *a* - b path *P* which clearly contain arcs not in *T.* Now, without loss of generality, let $\mu^{\infty}(a, b) = \mu\ (a, c)$ in *T.* Then arcs in *P* have strength ~ ≥ $\mu(a, c)$. Removal of *(a, c).* and adding *P* in T will result in another maximum spanning tree of G for which w is an end node, which contradicts our assumption.

Remark 1. It follows from Theorem 2. that the end nodes of a maximum spanning tree *T* of G are not fuzzy cutnodes of G. This results in the following corollary.

Corollary. Every fuzzy graph has at least two nodes which are not fuzzy cutnodes of G.

Definition 2. FUZZY α-MINIMUM SPANNING TREE.

A fuzzy α −minimum edge-weighted graph is a graph where we associate weights or costs with each edge. A fuzzy α-minimum spanning tree (FMST) of an fuzzy α-minimum edge-weighted graph is a fuzzy α-minimum spanning tree whose weight (the sum of the fuzzy α-minimum weights of its edges) is no larger than the weight of any other fuzzy α-minimum spanning tree.

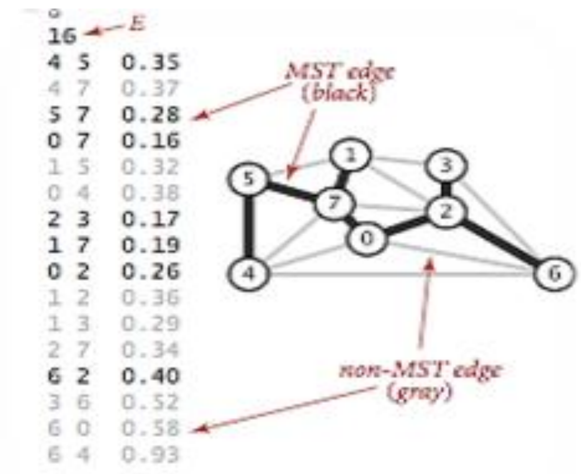


Figure 1.4. Fuzzy α −minimum edge weighted graph

ASSUMPTIONS.

To streamline the presentation, we adopt the following conventions:

- The graph is connected. The fuzzy α- minimum spanning-tree condition in our definition implies that the graph must be connected for an FMST to exist. If a graph is not connected, we can adapt our algorithms to compute the FMSTs of each of its connected components, collectively known as a fuzzy α-minimum spanning forest.
- The edge weights are not necessarily distances. Geometric intuition is sometimes beneficial, but the edge weights can be arbitrary.

- The edge weights may be zero or negative. If the edge weights are all positive, it suffices to define the FMST as the subgraph with fuzzy α- minimal total weight that connects all the vertices.
- The edge weights are all different. If edges can have equal weights, the fuzzy α- minimum spanning tree may not be unique. Making this assumption simplifies some of our proofs, but all of our algorithms work properly even in the presence of equal weights.

PRINCIPLES.

We recall two of the defining properties of a tree:

- Adding an edge that connects two vertices in a tree creates a unique cycle.
- Removing an edge from a tree breaks it into two separate subtrees.

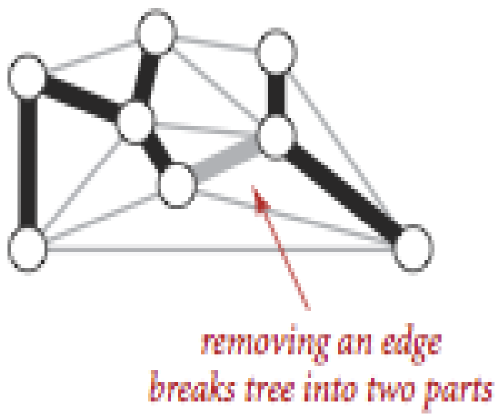


Figure 1.5. Fuzzy α-minimum removing an edge

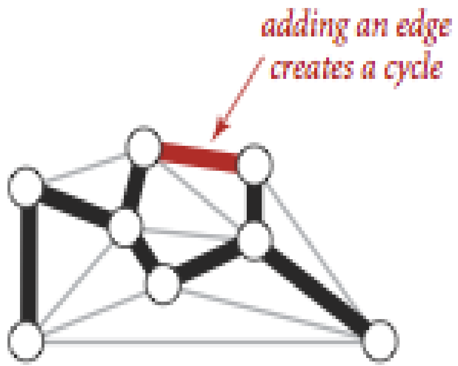


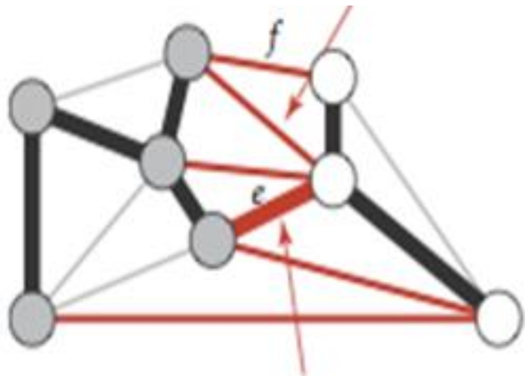
Figure 1.6. Fuzzy α-minimum adding an edge

A *cut* of a graph is a partition of its vertices into two disjoint sets. A *crossing edge* is an edge that connects a vertex in one set with a vertex in the other. We recall for simplicity, we assume all edge weights are distinct. Under this assumption, the FMST is unique. Define cut and cycle. The following properties lead to a number of FMST algorithms.

CUT PROPERTY

Given any cut in an edge-weighted graph (with all edge weights distinct), the crossing edge of fuzzy α- minimum weight is in the FMST of the graph.

Crossing edges separating gray from white vertices are drawn in red



Fuzzy α −minimum weight crossing edge must be in FMST

Figure 1.7. Fuzzy α-minimum cut property

The cut property is the basis for the algorithms that we consider for the FMST problem. Specifically, they are special cases of the *greedy algorithm*.

GREEDY FMST ALGORITHM

The following method colours black all edges in the FMST of any connected fuzzy α- minimum edge-weighted graph with B vertices: Starting with all edges collared gray, find a cut with no black edges, colour its fuzzy α- minimum-weight edge black, and continue until B-1 edges have been collared black.

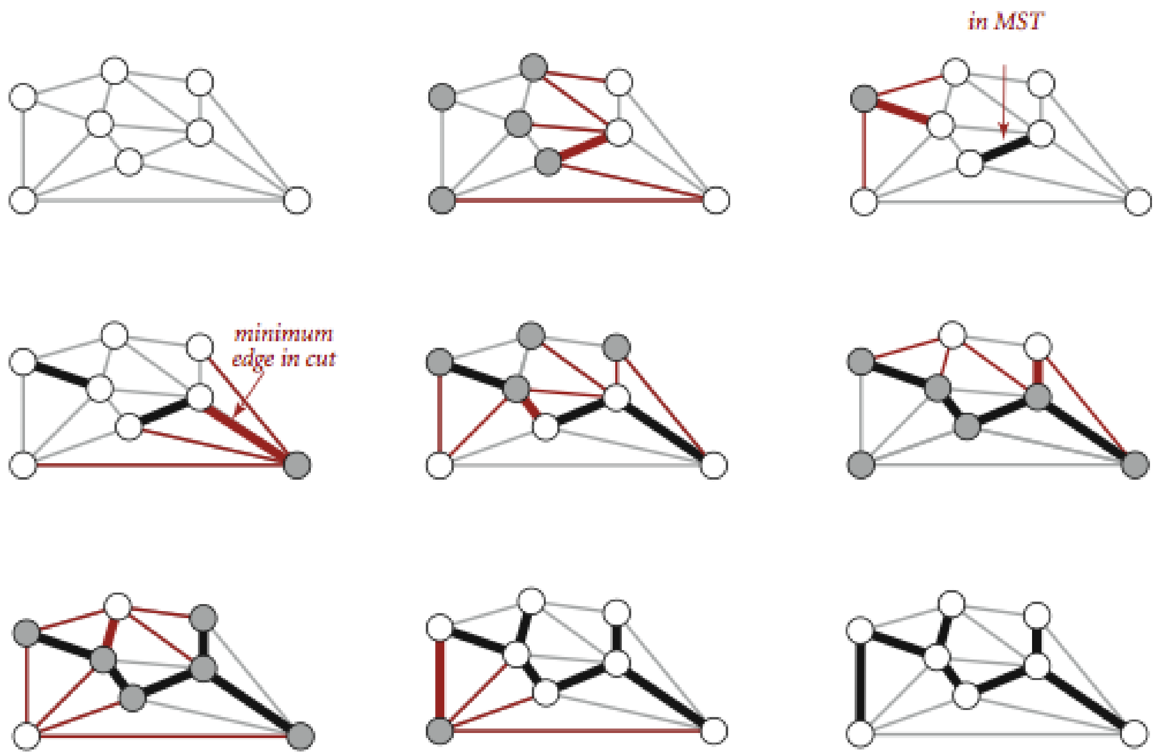


Figure 1.8. Fuzzy α-minimum GREEDY ALGORITHM

CLUSTERING BASED ON MINIMAL SPANNING TREE

The use of the fuzzy α- minimal spanning tree in the clustering methods was initially proposed by Zahn. Fig.1.9. depicts a fuzzy $\alpha-$minimal spanning tree, on which points are distributed into three clusters. The objects belonging to different clusters are marked with different dot notations.

A fuzzy α- minimal spanning tree is a weighted connected graph, where the sum of the weights is minimal. In a *G=(B,E)* graph an element of *E*, called edge, is $e_{i,j}=(b_i, b_j)$, where $b_i, b_j \in V$ (vertices). There is a *w* weight function is defined, which function determines a $c_{i,j}$ weight for each $e_{i,j}$ edge. Creating the fuzzy α-minimal spanning tree means, that we are searching the *G'=(B,E')* connected subgraph of *G*, where *E'*⊂*E* and the cost is minimum. The cost is computed in the following way:

$$\sum_{e\in E'} we$$

Where *w(e)* denotes the weight of the e∈*E* edge. In a *G* graph, where the number of the vertices is *N*, MST has exactly *N-1* edges. The major advantage of the clustering with using MST is that while the complete graph including *N* vertices has exactly $\binom{N}{2}$ edges, in the MST we can find only *N-1* edges. So the answering the possible most exciting question, namely which edge is the best choice for the elimination, becomes less expensive. A fuzzy α- minimal spanning tree can be efficiently computed in $O(N^2)$ time using either cut property algorithm.

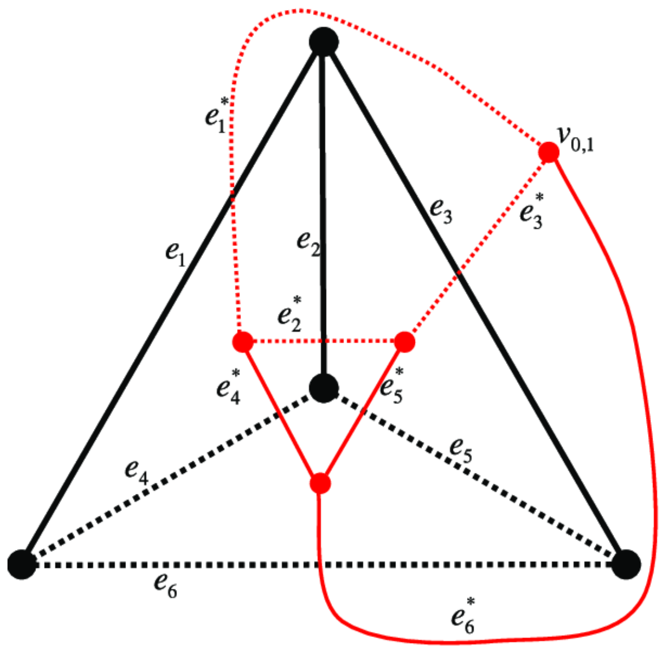


Figure 1.9. Example of a fuzzy α- minimal spanning tree

A fuzzy α- minimal spanning tree can be used in clustering in the following way: let $X=\{x_1, x_2 \dots . x_N\}$ be a set of the data with *N* distinct objects which we want to distribute in different clusters. x_i Denotes the i^{th} object, which consists *n* measured variables, grouped into an *n*-dimensional column vector $x_i = (x_{1,i}, x_{2,i} \dots . x_{N,i})\,T,\ x_i \in R_N$. Let $d_{i,j} = d\,(x_i, x_j)$ be the distance defined between any x_i and x_j. This distance can be computedin different ways (e.g. Euclidean distance, Manhattandistance, Mahalanobis distance, mutual neighbour distance, etc.). Removing edges from the FMST leads to a collection of connected subgraph of *G*, which can be considered as clusters. Using FMST for clustering we are interested in finding the *inconsistent edges*, which lead to the best clustering result.

Clustering by fuzzy α −minimal spanning tree can be viewed as ahierarchical clustering algorithm which follows the divisive approach. Using this method firstly we construct a linked structure of the objects, and then the clusters are recursively divided into sub clusters. Elimination of k edges from a fuzzy α- minimal spanning tree results in *k+1* disconnected sub trees. Denote δ the length of the deleted edge, and letB_1, B_2 be the sets of the points in the resulting two clusters. In the set of clusters we can state that there are no pairs of points (x_1, x_2),

$x_1 \in B_1$, $x_2 \in B_2$, such that $d(x_1, x_2) < \delta$.

The identification of the inconsistent edges causes problems in the FMST clustering algorithms. There exist numerous ways to divide clusters successively, but there is not a suitable choice for all cases. In special cases the elimination is carried out in one step. In these cases a global parameter is used, which determines the edges to be removed from the FMST. When this elimination is repeated, we must determine a terminating criterion, when the running of the algorithm is finished, and the current trees can be seen as clusters. Determination of the terminating criterion is also a difficult challenge. The methods which use recursive cutting define some possible terminating criteria. In the next paragraphs we will overview some well-known cutting conditions and terminating criteria, then we introduce our suggestions for using the fuzzy α- minimal spanning tree for clustering with new cutting criteria.

***Criterion-1*:** The simplest way to delete edges from the fuzzy α- minimal spanning tree is based on the distance between the vertices. By deleting the longest edge in each iteration step we get a nested sequence of subgraphs. As other hierarchical methods, this approach also requires a terminating condition. Several ways are known to stop the algorithms, for example the user can define the number of clusters, or we can give a threshold value on the length also.

Zahn suggested a global threshold value, δ which considers the distribution of the data in the feature space. In this δ threshold is based on the average weight (distances) of the FMST :

$$\delta = \frac{\lambda}{N-1} \sum_{e \in E'} w(e)$$

Where a user is defined parameter. Of course, λ can be defined in several manners.

***Criterion-2*:** Long edges of the FMST do not indicate cluster separation always. When the hidden clusters show different densities, the recursive cutting of the longest edges does not results the expected cluster scheme. Solving this problem Zahn proposed also another idea to detect the hidden separations in the data. Zahn's suggestion is based on the distance of the separated sub trees. He suggested that an edge is inconsistent if its length is at least *f* times as long as the average of the length of nearby edges. The input parameter *f* must be adjusted by the user. To determine which edges are “nearby” is another question. It can be determined by the user, or we can say, that point $\boldsymbol{xi}$ is nearby point of x_j if point $\boldsymbol{xi}$ is connected to the point x_j by a path in a fuzzy α- minimal spanning tree containing k or fewer edges. This method has the advantage of determining clusters which have different distances separating one another. Another use of the FMST based on this criterion is to find dense clusters embedded in a sparse set of points. All that has to be done is to remove all edges longer than some predetermined length in order to extract clusters which are closer than the specified length to each other. If the length is chosen accordingly, the dense clusters are extracted from a sparse set of points easily. The drawback of this method is that the influence of the user is significant at the selection of the *f* and *k* parameters.

CONCLUSIONS

In this paper, the inverse spanning tree problem with fuzzy $\alpha -$ edge weights is investigated. Based on the notion of fuzzy α -minimum spanning tree defined in the fuzzy inverse spanning tree problem is modeled as a fuzzy programming model with a set of chance constraints. When the fuzzy parameters are assumed to be independent fuzzy variables with regular credibility distributions, the proposed fuzzy programming model can be reformulated as a traditional linear programming problem with some specified inverse credibility distributions in it. As an illustration, a LAN reconstruction problem is given to show the performance of the proposed models. However, in this paper, we only focus on the case when the fuzzy α -edge weights have regular credibility distributions. As a future work on this topic, an effective algorithm should be designed for solving model for general cases.

REFERENCES

[1] L.S. Bershtein, T.A. Dziouba (2000). Allocation of maximal bipartite part from fuzzy graph. In *Proceedings of European Symposium on Intelligent Techniques ESIT'2000*, pages 45-47,full version on CD, Aachen, Germany, 2000.

[2] N. Christofides (1975). Graph theory. An algorithmic approach.*London, Academic press.*

[3] G. Frank, I. Frish (1978). Networks, connection and flows. Moscow, translation from English.

[4] H.J.Zimmermann (1991).Fuzzy Set Theory and Its Applications (2nd edition). *Boston/Dordrecht/London, Kluwer Academic Publishers.*

[5] Berge C., 1989, Hypergraphs, Amsterdam, North-Holland.

[6] Bershtein L.S.,Dziouba T.A.,1998,Construction of the optimal bipartite part in the fuzzy graph, Proceedings of IT&SE.98,Yalta, Ukraine.

[7] Dubois, D., Prade H., 1980, Fuzzy Sets and Systems: Theory and Applications, Academic Press, London.

[8] Kaufmann, Arnold, 1977, Introduction a la theorie des sous-ensemlesflous, Masson, Paris, France.

[9] Volkmann L., 1991, Graphen und Digraphen: eineEinfuehrung in die Graphentheorie, Wien; New York: Springer.

[10] Zadeh, Lotfi, 1975, Fuzzy sets and their application to cognitive and decision, Academic Press, New York, USA.