International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 20 (2018) pp. 14531-14539

A Hierarchical Structured Queuing System with Feedback and Chances of Two Revisits of Customer to Any of the Three Servers

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Abstract

A queuing model has been developed for a system having three servers wherein a customer may revisit to any of the servers. The revisit of the customer is limited to maximum twice. Customer may require the services of one or all the servers. If he/she requires the service of more than one server, then it is done in hierarchical order i.e. lower level (Server 1) to higher levels (Server 2 and then Server 3, if required). After getting the service from any of the servers, the customer may revisit to any other server following the hierarchy or may leave the system at any stage depending upon his/her satisfaction. Whenever, a customer revisits, the probability of leaving the server does not remain same as that was for leaving that server on his/her previous visit. The steady-state equations have been derived for finding mean queue length using generating function technique and the graphical analysis of the model is done thereafter.

Keywords: Queuing system, Three servers, Hierarchical order, Feedback, Chances of Two Revisits, Expected Queue Length.

INTRODUCTION

Feedback queuing systems may be observed in our daily life where customer may revisits to the same server or some other server, if any, for service. Many a time it is observed that customer may not found satisfied after getting service and he/she have to revisit the system more than once. A lot of work has been done on multi-server and feedback queuing models by a good number of researchers such as Luo and Tang (2011), Chassioti et.al. (2013), Wei et al. (2013), Morozov (2014), Yanfeng and Christos (2015), Sreekumari et.al. (2016), Bouchentouf and Yahiaoui (2017), Antonioli et al. (2018). Service in hierarchical order in a feedback queuing system is also worth considering as the same may be observed in many practical situations and hence Kumar and Taneja (2017) have considered a feedback queuing system with three servers in hierarchical order wherein a customer may revisit atmost once to the system in hierarchical order. However, the possibility of moving backward/forward to the preceeding/succeeding servers more than once may also be observed in many practical situations such as administration, hospitals, manufacturing etc.

Keeping the above in view, a queuing model with provision of service in hierarchical order by three servers and chances of two revisits has been developed in the present paper.

First of all, a customer arrives at first server from outside the system. He/she may leave the system after getting service from first server or go to second server for further service. After getting the service from second server, he/she may leave the system or go to the lower level server or higher level server depending upon his/her satisfication. Then from third server he/she may leave the system or go back to any of the previously visited servers as per his/her satisfication level. If he/she is not satisfied by the service then he/she may revisit any server atmost twice but following the hierarchy.

The steady-state equations have been derived to find the mean queue length using generating function technique. Numerical results have also been obtained.

NOTATION

λ = mean Arrival rate

μ_1 = mean service rate of 1st server

μ_2 = mean service rate of 2nd server

μ_3 = mean service rate of 3rd server

n = the number of times a customer may be serviced

n_1, n_2, n_3 = be the no. of customers at 1st, 2nd and 3rd server at any time t

a^i = probability of customer leaving 1st server ith time i = 1, 2, 3

b^i = probability of customer leaving 2nd server ith time where i = 1, 2, 3

c^i = probability of customer leaving 3rd server ith time, i = 1, 2, 3

p_1^i = probability of customer going outside the system from 1st server ith time, i = 1, 2, 3

p_{12}^i = probability of customer going from 1st server to 2nd server ith time where i = 1, 2, 3

p_2^i = probability of customer going inside the system from second server ith time, i = 1, 2, 3

p_{21}^i = probability of customer going from second server to 1st server ith time, i = 1, 2, 3

p_{23}^i = probability of customer going from second server to 3rd server ith time, i = 1, 2, 3

p_3^i = probability of customer going outside the system from third server ith time, i = 1, 2, 3

p_{32}^i = probability of customer going from third server to second server ith time; i = 1, 2, 3

p_{31}^i = probability of customer going from third server ith time; i = 1, 2, 3

$$A_1 = \sum_{i=1}^{n} a^i p_{12}^i,\ A = \sum_{i=1}^{n} a^i p_1^i,\ B = \sum_{i=1}^{n} b^i p_2^i,$$

$$B_1 = \sum_{i=1}^{n} b^i p_{23}^i,\ B_2 = \sum_{i=1}^{n-1} b^i p_{21}^i,\ C = \sum_{i=1}^{n} c^i p_3^i,$$

$$C_1 = \sum_{i=1}^{n-1} c^i p_{31}^i,\ C_2 = \sum_{i=1}^{n-1} c^i p_{32}^i$$

such that A + A_1 = 1, B + B_1 + B_2 = 1, and C + C_1 + C_2 = 1.

FORMULATION OF PROBLEM

The queue network consists of three service channels in hierarchical order i.e. lower level (Server 1) to higher levels (Server 2 and then server 3) if required. It is assumed that customer arrives at first server from outside the system with mean rate λ and according to a poison process and then goes to second and third server. The situation has been shown by the following sate transition diagram:

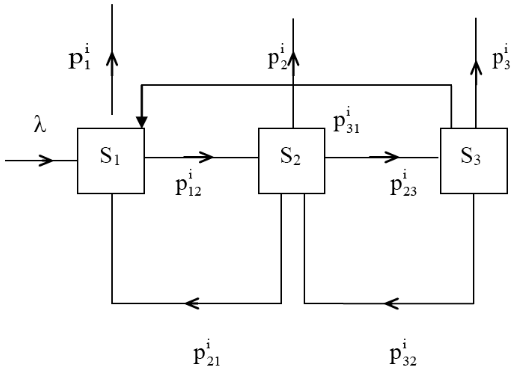
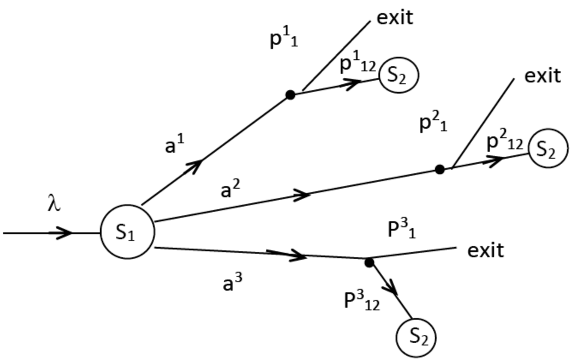
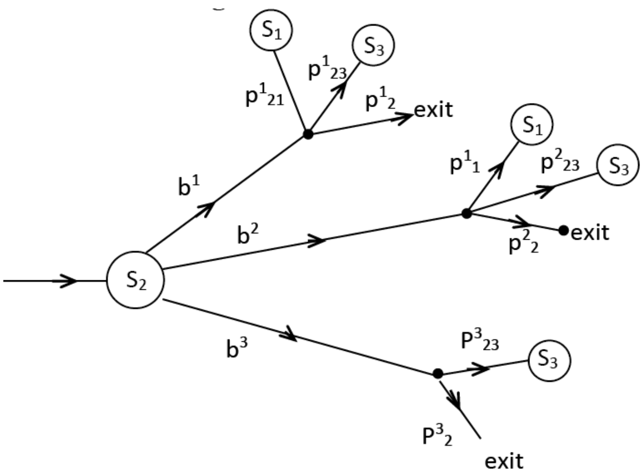


Figure 1

1. Possible states leaving S_1 server:



2. Possible states leaving S_2 server:



3. Possible States leaving S_3:

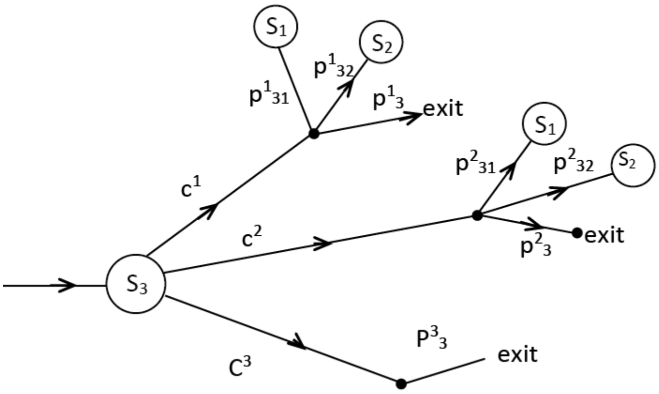


Figure 2. Transition diagram showing various states of the system

If the customer gets service from first server ith time (i=1, 2, 3), then $p_1^i + p_{12}^i = 1.$ After getting service from second server ith time (i=1, 2), we have $p_2^i + p_{21}^i + p_{23}^i = 1$ and $p_2^i + p_{23}^i = 1$ for i=3. Customer after getting service from the third server ith time (i=1, 2), we have $p_3^i + p_{31}^i + p_{32}^i = 1$ and $p_3^i + p_{32}^i = 1$ for i=3.

Let $P_{n_1,n_2,n_3}(t)$ be the probability that there are n_1, n_2, n_3 customers on 1st, 2nd and 3rd at time t. The steady–state equations are given by:

$$(\lambda+\mu_1+\mu_2+\mu_3)P_{n_1,n_2,n_3}(t) = \lambda P_{n_1-1,n_2,n_3} + A_1\mu_1 P_{n_1+1,n_2-1,n_3} + A\mu_1 P_{n_1+1,n_2,n_3} + B\mu_2 P_{n_1,n_2+1,n_3}$$
$$+ B_1\mu_2 P_{n_1,n_2+1,n_3-1} + B_2\mu_2 P_{n_1-1,n_2+1,n_3} + C\mu_3 P_{n_1,n_2,n_3+1} + C_1\mu_3 P_{n_1-1,n_2,n_3+1} + C_2\mu_3 P_{n_1,n_2-1,n_3+1} \quad (1)$$

for $n_1, n_2, n_3 > 0$.

$$(\lambda+\mu_2+\mu_3)P_{n_1,n_2,n_3}(t) = A\mu_1 P_{1,n_2-1,n_3}(t) + A_1\mu_1 P_{1,n_2,n_3} + B\mu_2 P_{0,n_2+1,n_3} + B_1\mu_2 P_{0,n_2+1,n_3-1}$$
$$+ C\mu_3 P_{0,n_2,n_3+1} + C_2\mu_3 P_{0,n_2-1,n_3+1} \quad (2)$$

for $n_1 = 0$ and $n_2, n_3 > 0$.

$$(\lambda+\mu_1+\mu_3)P_{n_1,0,n_3}(t) = \lambda P_{n_1-1,0,n_3} + \mu_1 A P_{n_1,0,n_3} + B\mu_2 P_{n_1,1,n_3} + B_1\mu_2 P_{n_1,1,n_3-1} + B_2\mu_2 P_{n_1-1,1,n_3}$$
$$+ C\mu_3 P_{n_1,0,n_3+1} + C_1\mu_3 P_{n_1-1,0,n_3+1} \quad (3)$$

for $n_2 = 0$ and $n_1, n_3 > 0$.

$$(\lambda+\mu_1+\mu_2)P_{n_1,n_2,0}(t) = \lambda P_{n_1-1,n_2,0} + A\mu_1 P_{n_1+1,n_2-1,0} + A_1\mu_1 P_{n_1+1,n_2,0} + B\mu_2 P_{n_1,n_2+1,0}$$
$$+ B_1\mu_2 P_{n_1-1,n_2+1,0} + C\mu_3 P_{n_1,n_2,1} + C_1\mu_3 P_{n_1-1,n_2,1} + C_2\mu_3 P_{n_1,n_2-1,1} \quad (4)$$

for $n_3 = 0$ and $n_1, n_2 > 0$.

$$(\lambda+\mu_3)P_{0,0,n_3}(t) = A\mu_1 P_{1,0,n_3} + B\mu_2 P_{0,1,n_3} + B_1\mu_2 P_{0,1,n_3-1} + C\mu_3 P_{0,0,n_3+1} \quad (5)$$

for $n_1, n_2 = 0$ and $n_3 > 0$.

$$(\lambda+\mu_1)P_{n_1,0,0}(t) = \lambda P_{n_1-1,0,0} + A\mu_1 P_{n_1+1,0,0} + B\mu_2 P_{n_1,1,0} + B_2\mu_2 P_{n_1-1,1,0} + C\mu_3 P_{n_1,0,1} + C_1\mu_3 P_{n_1-1,0,1} \quad (6)$$

for $n_2, n_3 = 0$ and $n_1 > 0$.

$$(\lambda+\mu_2)P_{0,n_2,0} = A_1\mu_1 P_{1,n_2-1,0} + A\mu_1 P_{1,n_2,0} + B\mu_2 P_{0,n_2+1,0} + C\mu_3 P_{0,n_2,1} + C_2\mu_3 P_{0,n_2-1,1} \quad (7)$$

for $n_1, n_3 = 0$ and $n_2 > 0$.

$$\lambda P_{0,0,0}(t) = A\mu_1 P_{1,0,0} + B\mu_2 P_{0,1,0} + C\mu_3 P_{0,0,1} \quad (8)$$

for n_1, n_2 & $n_3 = 0$.

To find the steady–state solution of the model, we define the generating function to solve (1) to (8) as :

$$F(x, y, z) = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty} P_{n_1,n_2,n_3}(t).x^{n_1}.y^{n_2}.z^{n_3} \quad (9)$$

In order to solve the expressions, we use the following partial generating functions as:

$$\left.\begin{aligned} F_{n_2,n_3}(x) &= \sum_{n_1=0}^{\infty} P_{n_1,n_2,n_3}(t).x^{n_1} \\ G_{n_1,n_3}(y) &= \sum_{n_2=0}^{\infty} P_{n_1,n_2,n_3}(t).y^{n_2} \\ H_{n_1,n_2}(z) &= \sum_{n_3=0}^{\infty} P_{n_1,n_2,n_3}(t).z^{n_3} \end{aligned}\right\} \quad (10)$$

$$\left.\begin{aligned} I_{n_3}(x,y) &= \sum_{n_2=0}^{\infty} F_{n_2,n_3}(x).y^{n_2} = \sum_{n_1=0}^{\infty} F_{n_1,n_3}(y).x^{n_1} \\ J_{n_1}(y,z) &= \sum_{n_3=0}^{\infty} G_{n_1,n_3}(y).z^{n_3} = \sum_{n_2=0}^{\infty} H_{n_1,n_2}(z).y^{n_2} \\ K_{n_2}(x,z) &= \sum_{n_3=0}^{\infty} F_{n_2,n_3}(x).z^{n_3} = \sum_{n_1=0}^{\infty} H_{n_1,n_2}(z).x^{n_1} \end{aligned}\right\} \quad (11)$$

Multiplying (1) by x^{n_1}, summing over n_1 from 0 to ∞, using (2) and (10), we obtained:

$$(\lambda+\mu_2+\mu_3)F_{n_2,n_3}(x)+\mu_1 F_{n_2,n_3}(x)-\mu_3 P_{0,n_2,n_3}(t)$$

$$= x\lambda F_{n_2,n_3}(x)+\frac{\mu_1 A_1}{x}[F_{n_2-1,n_3}-P_{0,n_2+1,n_3}] + \frac{\mu_1 A}{x}[F_{n_2,n_3}(x)-P_{0,n_2,n_3}(t)]+B\mu_2 F_{n_2+1,n_3}(x)$$

$$+ B_1 F_{n_2+1,n_3-1}+x.B_2\mu_2 F_{n_2+1,n_3}(x)+\mu_3 cF_{n_2,n_3+1}(x) + \mu_3 x.c_1 F_{n_2,n_3+1}(x)+\mu_3 c_2 F_{n_2-1,n_3+1} \quad (12)$$

for $n_1 \geq 0$ and $n_2, n_3 > 0$.

Similarly from (3), (4), (6) by using (5), (7), (8) respectively we obtained following:

$$(\lambda+\mu_3)F_{0,n_3(x)}+\mu_1 F_{0,n_3}-\mu_1 P_{0,0,n_3}(t) = \lambda x.F_{0,n_3}(x)+\frac{\mu_1 A}{x}[F_{0,n_3}-P_{0,0,n_3}] +\mu_2 BF_{1,n_3-1}+\mu_2 B_1 F_{1,n_3-1}$$

$$+x\mu_2 B_2 F_{1,n_3}(x)+\mu_3 cF_{0,n_3+1}(x) +\mu_3 xc_1 F_{0,n_3+1}(x) \quad (13)$$

for $n_1 \geq 0$ and $n_2, n_3 > 0$.

$$(\lambda+\mu_2+\mu_1)F_{n_2,0}-\mu_1 P_{0,n_2,0} = \lambda xF_{n_2,0}(x)+\frac{\mu_1 A_1}{x}[F_{n_2-1,0}-P_{0,n_2-1,0}] + \frac{\mu_1 A}{x}[F_{n_2,0}-P_{0,n_2,0}]$$

$$+\mu_2 xB_2 F_{n_2+1,0} + \mu_3 cF_{n_2,1}+\mu_3 x.c_1 F_{n_2,1}+\mu_3 c_2 F_{n_2-1,1} \quad (14)$$

for n_1, $n_3 \geq$ and $n_2 > 0$.

$$(\lambda+\mu_1)F_{0,0}(x)-\mu_1 P_{0,0,0}(t) = \lambda xF_{0,0}(x)+\frac{\mu_1 A}{x}[F_{0,0}(x)-P_{0,0,0}(t)] +\mu_2 BF_{1,0}+\mu_2 xB_2 F_{1,0}(x)$$

$$+\mu_3 cF_{0,1}+c_1\mu_3 xF_{0,1} \quad (15)$$

for $n_1 \geq 0$ and $n_2, n_3 = 0$.

Multiplying (12) by y^{n_2}, taking sum over n_2 from 0 to ∞, using (13) and def. in (10) ,(11) we have :

$$(\lambda+\mu_1+\mu_2+\mu_3)I_{n_3}(x,y)-\mu_2 F_{0,n_3}-\mu_1 G_{0,n_3}$$

$$= x\lambda I_{n_3}(x,y)+\frac{\mu_1 yA_1}{x}[I_{n_3}(x,y)-G_{0,n_3}(y)]+\frac{\mu_1 A}{x}[I_{n_3}(x,y)-G_{0,n_3}(y)]+\frac{\mu_2 B}{y}[I_{n_3}(x,y)$$

$$-F_{0,n_3}(x)] +\frac{\mu_2 B_1}{y}[I_{n_3-1}(x,y)-F_{0,n_3-1}(x)]+ \frac{x\mu_2 B_2}{y}[I_{n_3}(x,y)-F_{0,n_3}(x)]+\mu_3 cI_{n_3+1}$$

$$+\mu_3 xc_1 I_{n_3+1}(x,y)+ \mu_3 yc_2 I_{n_3+1}(x,y) \quad (16)$$

for n_1, $n_2 \geq 0$ and $n_3 > 0$.

Similarly from (14) by using (15) we have:

$$(\lambda+\mu_1+\mu_2)I_0(x,y)-\mu_2F_{0,0}-\mu_1G_{0,0}$$

$$=\lambda xI_0(x,y)+\frac{\mu_1yA_1}{x}[I_0(x,y)-G_{0,0}(y)]+\frac{\mu_1A}{x}[I_0(x,y)-G_{0,0}]+\frac{\mu_2xB_2}{y}[I_0(x,y)-F_{0,0}(x)]$$

$$+\frac{\mu_2B}{y}[I_0(x,y)-F_{0,0}(x)]+\mu_3cI_1(x,y)+\mu_3xc_1I_1(x,y)+\mu_3yc_2I_1(x,y) \quad (17)$$

Multiplying (16) by Z^{n_3}, taking sum over n_3 from 0 to ∞, using (17) and def. of generating function in (9), we have :

$$F(x,y,z)=\frac{\mu_3I_0(x,y)[1-\frac{1}{Z}(c+xc_1+yc_2)]+\mu_2K_0(x,z)\left[1-\frac{1}{y}(B+ZB_1+xB_2)\right]+\mu_1I_0(y,z)[1-\frac{1}{x}(yA_1+A)]}{\lambda(1-x)+\mu_1[1-\frac{1}{x}(yA_1+A)]+\mu_2[1-\frac{1}{y}(B+ZB_1+B_2x)]+\mu_3[1-\frac{1}{Z}(c+xc_1+yc_2)]} \quad (18)$$

rom (18) we have following:

$$-\mu_3C_1I_0(1,1)-\mu_2B_2K_0(1,1)+\mu_1J_0(1,1)=\lambda+\mu_1-\mu_2B_2-\mu_2C_1 \quad (19)$$

$$-\mu_3C_2I_0(1,1)+\mu_2K_0(1,1)+\mu_1A_1J_0(1,1)=\mu_2-\mu_3C_2-\mu_1A_1 \quad (20)$$

$$\mu_3I_0(1,1)-\mu_2B_1K_0(1,1)=\mu_3-\mu_2B_1 \quad (21)$$

Solving (19), (20) and (21), we get:

$$K_0(1,1)=1+\frac{\lambda A_1}{\mu_2[-1+B_2A_1+B_1(A_1C_2+C_2)]} \quad (22)$$

$$J_0(1,1)=1+\frac{\lambda}{\mu_1}\left[\frac{1-B_1C_2}{-1+A_1B_2+B_1(A_1C_1+C_2)}\right] \quad (23)$$

$$I_0(1,1)=1+\frac{\lambda AB_1}{\mu_3(-1+A_1B_2+B_1(A_1C_1+C_2))} \quad (24)$$

Let us denote $F(x,y,z)=\dfrac{f(x,y,z)}{g(x,y,z)}$

where

$$f(x,y,z)=\mu_3I_0(x,y)[1-\frac{1}{Z}(c+xc_1+yc_2)]+\mu_2K_0(x,z)[1-\frac{1}{y}(B+ZB+xB_2)]$$

$$+\mu_1J_0(y,z)[1-\frac{1}{x}(yA_1+A)] \quad (25)$$

and $g(x,y,z)=\lambda(1-x)+\mu_1[1-\frac{1}{x}(yA_1+A)]+\mu_2[1-\frac{1}{y}(B+ZB_1+xB_2)]$

$$+\mu_3[1-\frac{1}{Z}(C+xC_1+yC_2)] \quad (26)$$

Let Lq_1 = Marginal mean queue length at first server.

$$= \frac{\left(-\frac{\partial f}{\partial x}\right)_{(1,1,1)}\left(\frac{\partial^2 g}{\partial x^2}\right)_{(1,1,1)} + \left(\frac{\partial g}{\partial x}\right)_{(1,1,1)}\left(\frac{\partial^2 f}{\partial x^2}\right)_{(1,1,1)}}{2\left[\left(\frac{\partial g}{\partial x}\right)_{(1,1,1)}\right]^2}$$

$$= \frac{-\lambda[1-B_1C_2]}{[-1+A_1B_2+B_1(A_1C_1+C_2)[-\lambda+\mu_1-\mu_2B_2-\mu_2C_1]}$$

$$= \frac{-\lambda[1-B_1C_2]}{R[-\lambda+\mu_1-\mu_2B_2-\mu_3C_1]} \quad (27)$$

where $R = -1 + A_1B_2 + B_1 (A_1C_1 + C_2)$

Lq_2 = Marginal mean queue length at second server.

$$= \frac{\left(-\frac{\partial f}{\partial y}\right)_{(1,1,1)}\left(\frac{\partial^2 g}{\partial y^2}\right)_{(1,1,1)} + \left(\frac{\partial g}{\partial y}\right)_{(1,1,1)}\left(\frac{\partial^2 f}{\partial y^2}\right)_{(1,1,1)}}{2\left[\left(\frac{\partial g}{\partial y}\right)_{(1,1,1)}\right]^2}$$

$$= \frac{-\lambda A_1}{R[-\mu_1A_1+\mu_2-\mu_3C_2]} \quad (28)$$

Lq_3 = Marginal mean queue length at third server.

$$= \frac{\left(-\frac{\partial f}{\partial z}\right)_{(1,1,1)}\left(\frac{\partial^2 g}{\partial z^2}\right)_{(1,1,1)} + \left(\frac{\partial g}{\partial z}\right)_{(1,1,1)}\left(\frac{\partial^2 f}{\partial z^2}\right)_{(1,1,1)}}{2\left[\left(\frac{\partial g}{\partial z}\right)_{(1,1,1)}\right]^2}$$

$$= \frac{-\lambda A_1B_1}{R[-\mu_2B_1+\mu_3]} \quad (29)$$

Let Lq = Mean queue length of the entire system

$$= Lq_1 + Lq_2 + Lq_3$$

$$= \left(-\frac{\lambda}{R}\right)\left[\left\{\frac{(1+B_1C_2)}{(-\lambda+\mu_1-\mu_2B_2-\mu_3C_1}\right\}+\left\{\frac{A_1}{(\mu_1A_1+\mu_2-\mu_3C_2)}\right\}+\left\{\frac{AB_1}{(-\mu_2B_1+\mu_3}\right\}\right] \quad (30)$$

RESULTS AND DISCUSSION

We discuss particular cases varying the values of n, i.e. number of times the service by the same server is allowed.

Case 1: when n=2

$$Lq = \frac{\lambda}{R}\left[\frac{\left\{1 - c^1 p^1_{32}\left(b^1 p^1{}_{23} + b^2 p^2{}_{23}\right)\right\}}{\lambda - \mu_1 + b^1 p^1_{21}\mu_2 + c^1 p^1_{31}\mu_3} + \frac{\left(a^1 p^1_{12} + a^2 p^2_{12}\right)}{\left[\mu_1(a^1 p^1_{12} + a^2 p^2_{12}) - \mu_2 + \mu_3 c^1 p^1_{32}\right]} + \frac{\left(a^1 p^1_{12} + a^2 p^2_{12}\right)(b^1 p^1_{23} + b^2 p^2_{23})}{\left[\mu_2(b^1 p^1_{23} + b^2 p^2_{23}) - \mu_3\right]}\right]$$

Replacing a^1 by a, a^2 by a', b^1 by b, b^2 by b', c^1 by c and c^2 by c' in above result for Lq, we obtain the same result as was obtained by Kumar and Taneja (2017) wherein the authors discussed the case of revisit of customer to the servers atmost once for service.

Case 2: When n=3

1. Behaviour of the mean queue length of the entire system with respect to arrival rate (λ) is depicted in **Table 1**. The probability of leaving the first server first time (a^1) has been varied whereas the value of other parameters has been kept fixed.

Table 1: Mean Queue Length with respect to λ and a^1.

$\mu_1=40,\mu_2=35,\mu_3=38,p_{12}{}^1=0.4,\ ,p_{12}{}^2=0.3,p_{12}{}^3=0.6,p_1{}^1=0.6,\ p_1{}^2=0.7,\ p_1{}^3=0.4,b^1=0.5,b^2=0.3,b^3=0.2,p_{21}{}^1=0.2,\ p_{21}{}^2=0.15,\ p_{21}{}^3=0.1,p_2{}^1=0.5,\ p_2{}^2=0.6,\ p_2{}^3=0.7,p_{23}{}^1=0.3,\ p_{23}{}^2=0.25,\ p_{23}{}^3=0.2,c^1=0.7,c^2=0.2,c^3=0.1,p_3{}^1=0.65,\ p_3{}^2=0.6,p_3{}^3=0.5,p_{31}{}^1=0.2,p_{31}{}^2=0.15,p_{31}{}^3=0.1,p_{32}{}^1=0.15,p_{32}{}^2=0.25,p_{32}{}^3=0.4$												
	$a^1=0.5,\ a^2=0.3,\ a^3=0.2$				$a^1=0.6,\ a^2=0.25,\ a^3=0.15$				$a^1=0.7,\ a^2=0.2,\ a^3=0.1$			
λ	Lq_1	Lq_2	Lq_3	L	Lq_1	Lq_2	Lq_3	L	Lq_1	Lq_2	Lq_3	L
5	123	26.6	18.1	168	123	26.8	17.9	168	123	26.9	17.6	167
6	141	32	21.7	195	141	32.1	21.5	195	141	32.2	21.2	194
7	157	37.3	25.4	220	157	37.5	25	219	157	37.6	24.7	219
8	171	42.6	29	242	171	42.8	28.6	242	170	43	28.2	242
9	182	48	32.6	263	182	48.2	32.2	262	182	48.3	31.7	262
10	191	53.3	36.2	281	191	53.5	35.8	280	191	53.7	35.3	280
11	198	58.6	39.9	297	198	58.9	39.3	296	198	59.1	38.8	296
12	203	63.9	43.5	311	203	64.2	42.9	310	203	64.4	42.3	310
13	206	69.3	47.1	322	206	69.6	46.5	322	205	69.8	45.9	321
14	206	74.6	50.7	332	206	74.9	50.1	331	206	75.2	49.4	330
15	205	79.9	54.4	339	204	80.3	53.6	338	204	80.6	52.9	338

Graph for Mean Queue Length of the System (Lq) has been plotted with respect to arrival rate λ for different values of a^1 as shown in the **Fig. 3**.

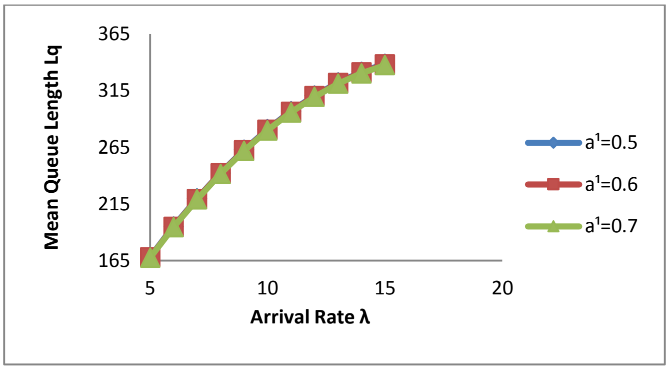


Figure 3

Following can be interpreted from **Table 1** as well as **Fig. 3**

Mean queue length (Lq) of the system increases with the increase in mean arrival rate (λ) whereas it decreases with the increase in probability (a^1). However there is no significant difference between mean queue lengths with respect to (a^1).

2. Behaviour of mean queue length for each server and the entire system with respect to service rate of first server (μ_1) is depicted in **Table 2**. The service rate of second server (μ_2) has also been varied whereas the values of other parameters have been kept fixed.

Table 2: Mean queue lengths with respect to μ_1 for different μ_2.

$\mu_3=19, \lambda=10, a^1=0.5, a^2=0.3, a^3=0.2, p_{12}^1=0.4, ,p_{12}^2=0.3, p_{12}^3=0.6, p^1{}_1=0.6, p^2{}_1=0.7, p^3{}_1=0.4, b^1=0.5, b^2=0.3, b^3=0.2, p_{21}^1=0.2, p_{21}^2=0.15, p_{21}^3=0.1, p_2^1=0.5, p_2^2=0.6, p_2^3=0.7, p_{23}^1=0.3, p_{23}^2=0.25, p_{23}^3=0.2, c^1=0.7, c^2=0.2, c^3=0.1, p_3^1=0.65, p_3^2=0.6, p_3^3=0.5, p_{31}^1=0.2, p_{31}^2=0.15, p_{31}^3=0.1, p_{32}^1=0.15, p_{32}^2=0.25, p_{32}^3=0.4$												
	$\mu_2=20$				$\mu_2=22$				$\mu_2=24$			
μ_1	Lq_1	Lq_2	Lq_3	L	Lq_1	Lq_2	Lq_3	L	Lq_1	Lq_2	Lq_3	L
25	90.57	23.03	13.83	127.4	87.66	30.64	13.29	131.6	84.75	38.26	12.76	135.8
26	99.38	21.46	13.83	134.7	96.47	29.08	13.29	138.8	93.56	36.7	12.76	143
27	108.2	19.9	13.83	141.9	105.3	27.52	13.29	146.1	102.4	35.14	12.76	150.3
28	117	18.34	13.83	149.2	114.1	25.96	13.29	153.3	111.2	33.58	12.76	157.5
29	125.8	16.78	13.83	156.4	122.9	24.4	13.29	160.6	120	32.01	12.76	164.8
30	134.6	15.22	13.83	163.7	131.7	22.83	13.29	167.8	128.8	30.45	12.76	172
31	143.4	13.66	13.83	170.9	140.5	21.27	13.29	175.1	137.6	28.89	12.76	179.3
32	152.2	12.09	13.83	178.2	149.3	19.71	13.29	182.3	146.4	27.33	12.76	186.5
33	161	10.53	13.83	185.4	158.1	18.15	13.29	189.6	155.2	25.77	12.76	193.8
34	169.9	8.97	13.83	192.7	167	16.59	13.29	196.8	164	24.21	12.76	201
35	178.7	7.409	13.83	199.9	175.8	15.03	13.29	204.1	172.9	22.64	12.76	208.3

Graph for Mean Queue Length of the System (Lq) has been plotted with respect to service rate of first server (μ_1) for different values of μ_2 as shown in the **Fig. 4.**

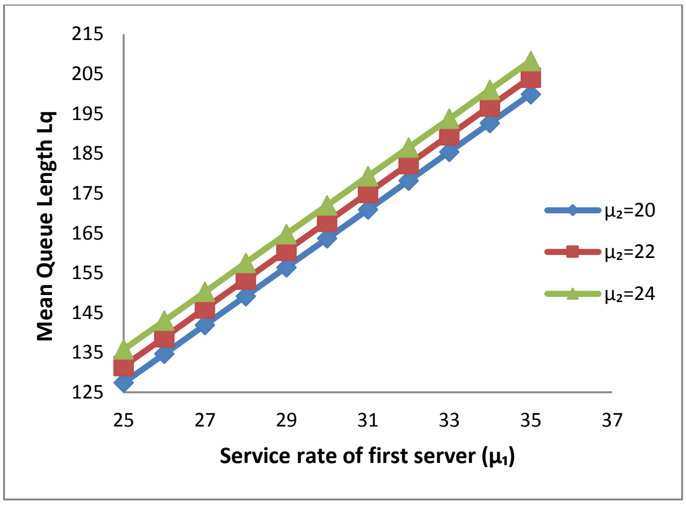


Figure 4

Following can be interpreted from **Table 2** as well as **Fig. 4.**

Mean queue length of the system (Lq) increases with the increase in the value of service rate of first server (μ_1) as well as increase in service rate of second server (μ_2).

CONCLUSION

A queuing model has been developed on a practical situation which may be observed in various fields of life such as hospitals, administration and manufacturing. A customer/ product is served/dealt by three servers to whom the customer can revisit but at most twice. Expected queue length has been obtained for the model and its nature has been depicted through numerical results and graphs with respect to arrival/service rates taking particular cases which reveal that mean queue length get increased with increase in the value of arrival rate, service rate of first server and service rate of second server. However the mean queue length decreases with increase in the probability of leaving the first server by the customer. The results derived in the present model have also been deduced by putting n=2 to show that these results are the same as were obtained by Kumar and Taneja (2017).

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