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Reliability Analysis of a System Working in High Temperature Zones with Fault-Dependent Repair during Night hours

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Abstract

In the present paper, a reliability model is developed for a system working in high- temperature zones. The system may get failed during day or night hours. The failure may be of two types i.e. major or minor. The type of failure is revealed by carrying out an inspection on failure. If it fails during night hours, then major failure is not repairable during that time but minor failure such as electric fuse repair easily possible in night hours also. In case of major failure, it is assumed that the system has to wait till the next morning when the repair facilities/components are available for repair. The working of the system depends upon the temperature, as the system may not work when temperature reaches beyond certain limit. The present study is based on the fabric manufacturing system that does not remain functional when temperature is beyond $22\,^{\circ}\mathrm{C}$ and in that case, the system is put to down mode to avoid any damage. Various measures of systems effectiveness are obtained using regenerative point technique. Profit evaluation is done and interesting results are obtained.

Keywords: Reliability analysis, High temperature zones, Minor /major failures, Day and night hours, Regenerative point technique.

INTRODUCTION

Literature on reliability models contains lot of studies on the reliability and cost-benefit analysis of various systems under various situations/assumptions. Such contributors include Tuteja and Taneja (1993), Welke et al. (1995), Teng et al. (2006), EL-Said and EL-Sherbeny (2010), Tumer and Smidts (2011), Rizwan et al. (2013), Srinivasu and Sridharan (2017), Sachdeva and Taneja (2017), Yaqoob et al. (2017), wherein various concepts such as operating rest periods, random inspection, instructions, hardware based software interaction failures, different types of recovery etc have been taken into account. All these studies have been carried out without giving any consideration to the effect of temperature on operation of the system whereas such effects may also be observed in many practical situations, viz; fabric manufacturing system.

Keeping this in view, Sheetal et al. (2018) analyzed reliability and profit of a system with effect of high temperature on operation. However, it may also be observed that repair on failure of such systems may be delayed due to non-availability of the necessary parts during night hours. Considering the concept of waiting time for repair, a reliability model for a system whose operation is affected by the temperature and repair depends on the time of failure is developed. Study is limited to working of such systems in High-temperature zones. The failure, whenever occurred, may be of two types- major and minor. The system is required to be put to down mode till the temperature is maintained to the acceptable limit. The present study is based on the fabric manufacturing system, which does not remain functional beyond $22\,^{\circ}\mathrm{C}$ and in that case the system is put to down mode as otherwise the fabric thickness may get destroyed. The system is made operative from down mode as soon as possible by using air conditioner. The system may be get failed during day or night hours. If it fails during night hours, the repair of the major failure is not possible during that time due to non-availability of heavy parts and the system has to wait till the next morning when the repair facilities/ components are available easily. However, the repair of minor failure like electric fuse replacement is possible easily even during night also.

The system is analyzed by making use of regenerative point technique. Various measures of system effectiveness such as reliability, mean time to system failure (MTSF), availability, busy period analysis, expected down time and expected number of visits of the repairman are derived. The profit incurred to the system is also evaluated and graphical study is done. On the basis of the information gathered from a fabric manufacturing plant, the estimates of rates, costs and probabilities are obtained using method of maximum likelihood. Interesting numerical results have been obtained.

OTHER ASSUMPTIONS FOR THE MODEL ARE:

1. Initial state is considered as the state of working in the high temperature zones.
2. All the random variables follow arbitrary distributions.
3. After every repair, the system becomes like a new one.

NOMENCLATURE

$w(t)$, $W(t)$:	p.d.f. and c.d.f. of wating time till the morning hours when market gets opened.
$h_1(t)$, $H_1(t)$:	p.d.f. and c.d.f. of time for increasing the temperature beyond certain limit.
$f(t)$, $F(t)$:	p.d.f. and c.d.f. of failure time.

$g_1(t),G_1(t)$:	p.d.f. and c.d.f. of minor repair time.
$g_2(t),G_2(t)$:	p.d.f. and c.d.f. of minor repair time.
$h_2(t),H_2(t)$:	p.d.f and c.d.f. of time for maintaining the temperature to acceptable range.
i(t), I(t) :	p.d.f. and c.d. f. of inspection time.
F_{r1} :	Failed unit under minor repair.
F_{r2} :	Failed unit under major repair.
F_{wr2} :	Failed unit under waiting for repair.
p_1:	probability that system fails during day hours.
q_1:	probability that system fails during night hours.
p_2:	probability of minor fault.
q_2:	probability of major fault.
(op):	The operative state.
fin:	Failed in night
fid :	Failed in day
(D):	down state of the system
A_0:	Availability
DT_0:	Expected down time
BI_0:	Busy period inspection time only
BR_0:	Busy period repair time only
$M_i(t)$:	Probability that system up initially in regenerative state i is up at time t without passing throw any other regenerative state
© :	Symbols for Laplace convolution
Ⓢ :	Symbols for Stieltjes Convolution
*:	Symbols for Laplace Transforms
**:	Symbols for Laplace Stieltjes Transforms
$Q_{ij}(t), q_{ij}(t)$:	c.d.f., p.d.f. of first passage time from a regenerative state i to a regenerative state j without visiting any other regenerative state in (0,t]
$A_i(t)$:	Probability that system is available at the instant t given that system entered regenerative state i at t=0

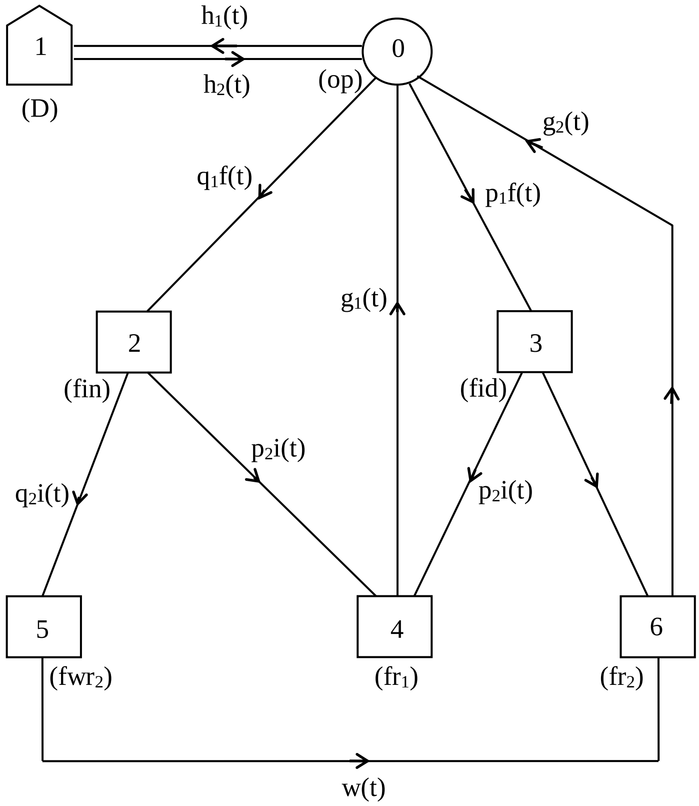

Figure 1. State Transition Diagram

Transition Probabilities and Mean Sojourn Times

The transition diagram showing various states of transition of system are shown in **Fig. 1.** The epochs of entry into the states all are regenerative states. The possible transition probabilities are given below:

$$q_{01}(t) = h_1(t)\bar{F}(t) \qquad q_{34}(t) = p_2 i(t)$$

$$q_{02}(t) = q_1 f(t)\bar{H}_1(t) \qquad q_{36}(t) = q_2 i(t)$$

$$q_{03}(t) = p_1 f(t)\bar{H}_1(t) \qquad q_{10}(t) = h_2(t)$$

$$q_{24}(t) = p_2 i(t) \qquad q_{25}(t) = q_2 i(t)$$

$$q_{40}(t) = g_1(t) \qquad q_{60}(t) = g_2(t)$$

$$q_{56}(t) = w(t)$$

The non-zero elements p_{ij} can be obtained as $p_{ij} = \lim_{s\to\infty} q_{ij}*(s)$.

The mean sojourn times (μ_i) in the regenerative state i is defined as the time of stay in that state before transition to any other state. If T denotes the sojourn time in the regenerative state i, then

$$\mu_i = E(T) = P_r(T>y)$$

$$\mu_0 = \int_0^\infty \overline{H}_1(t)\overline{F}(t)dt = \int_0^\infty E_1(t)\,dt \qquad \text{(say)}$$

$$\mu_1 = \int_0^\infty \overline{H}_2(t)\,dt$$

$$\mu_2 = \int_0^\infty \bar{i}(t)\,dt = -i^{*'}(o) = \int_0^\infty t\,i(t)\,dt = \mu_3$$

$$\mu_4 = \int_0^\infty \overline{G}_1(t)\,dt = -g_1^{*'}(o) = \int_0^\infty t\,g_1(t)\,dt$$

$$\mu_5 = \int_0^\infty \overline{W}(t)\,dt = -w^{*'}(o) = \int_0^\infty t\,w(t)\,dt$$

$$\mu_6 = \int_0^\infty \overline{G}_2(t)\,dt = -g_2^{*'}(o) = \int_0^\infty t\,g_2(t)\,dt$$

The unconditional mean time taken by the system to transit to any regenerative state j when time is counted from the epoch of entrance into state 'i' is mathematically stated as

$$m_{ij} = \int_0^\infty t q_{ij}(t)\,dt = -q_{ij}^{*'}(0) = \int_0^\infty t dQ_{ij}(t)\,dt$$

Thus,

$$m_{01} + m_{02} + m_{03} = \int_0^\infty t \begin{bmatrix} h_1(t)\overline{F}(t) + q_1 f(t)\overline{H}_1(t) \\ + p_1 f(t)\overline{H}_1(t) \end{bmatrix} dt = K_0(\text{say})$$

$$m_{10} = \int_0^\infty t\left[h_2(t)\right]dt = \mu_1$$

$$m_{24} + m_{25} = m_{34} + m_{35} = K_2$$

$$m_{56} = \int_0^\infty t\left[w(t)\right]dt = \mu_5$$

$$m_{60} = \int_0^\infty t\left[g_2(t)\right]dt = \mu_6$$

$$m_{40} = \int_0^\infty t\left[g_1(t)\right]dt = \mu_4$$

Reliability and Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the c.d.f of first passage time from the regenerative state i to a failed state. Regarding the failed states as absorbing states, we have the following recursive relations for $\phi_i(t)$:

$\phi_0(t) = Q_{01}(t)$ Ⓢ $\phi_1(t) + Q_{02}(t) + Q_{03}(t)$

$\phi_1(t) = Q_{10}(t)$ Ⓢ $\phi_0(t)$

Taking Laplace–Stieltjes Transforms (L.S.T.) of these relations and solving them by Cramer's rule for $\phi_0^{**}(s)$, we obtain the mean time to system failure (MTSF) as

$$\text{MTSF} = \lim_{s\to 0}\frac{1-\phi_0^{**}(s)}{s} = \frac{N}{D}$$

where $N = \mu_0 + p_{01}\mu_1$ and $D = 1 - p_{01}\,p_{10}$

Availability Analysis

Using the arguments of the theory of regenerative process, the availability $A_i(t)$ is seen to be satisfy the following recursive relation for $A_i(t)$ are given as

$A_0(t) = M_0(t) + q_{01}(t) © A_1(t) + q_{02}(t) © A_2(t) + q_{03}(t) © A_3(t)$

$A_1(t) = q_{10}(t) © A_0(t)$

$A_2(t) = q_{24}(t) © A_4(t) + q_{25}(t) © A_5(t)$

$A_3(t) = q_{34}(t) © A_4(t) + q_{35}(t) © A_5(t)$

$A_4(t) = q_{40}(t) © A_0(t)$

$A_5(t) = q_{56}(t) © A_6(t)$

$A_6(t) = q_{60}(t) © A_0(t)$

where $M_0(t)$ is the probability that the system is up initially in state '0' at that time t without visiting to any other regenerative state, we have

$$M_0(t) = \overline{H}(t)\overline{F}(t)$$

Taking Laplace transforms (L.T.) of these relations and solving them for $A_0^*(s)$, the Steady state availability of the system is given by

$$A_0 = \lim_{s\to 0}(sA_0^*(s)) = \frac{N_1}{D_1}$$

where $N_1 = \mu_0$ and $D_1 = \mu_0 + p_{01}\,\mu_1 + p_{02}\,(\mu_2 + \mu_3)$

Busy Period Analysis (Inspection Time Only)

Let $BI_i(t)$ be the probability that the system is busy due to inspection unit at an instant 't' given that system entered state i at t=0, we have the recursive relation for $BI_i(t)$ are given as:

$$BI_0(t) = q_{01}(t) © BI_1(t) + q_{02}(t) © BI_2(t) + q_{03}(t) © BI_3(t)$$

$$BI_1(t) = q_{10}(t) © BI_0(t)$$

$$BI_2(t) = W_2(t) + q_{24}(t) © BI_4(t) + q_{25}(t) © BI_5(t)$$

$$BI_3(t) = W_3(t) + q_{34}(t) © BI_4(t) + q_{36}(t) © BI_6(t)$$

$$BI_4(t) = q_{40}(t) © BI_0(t)$$

$$BI_5(t) = q_{56}(t) © BI_6(t)$$

$$BI_6(t) = q_{60}(t) © BI_0(t)$$

where $W_i(t)$ be the probability that the system is busy in state i due to inspection up to time 't' without making any transition to any other regenerative and so

$$W_3(t) = W_2(t) = \bar{I}(t)$$

Taking Laplace Transforms of the above relations and solving for $BI_0^*(s)$, the total fraction of the time for which the repairman is busy due to inspection, in steady- state, is given by:

$$BI_0 = \lim_{s\to\infty}\left(sBI_0^*(s)\right) = \frac{N_2}{D_1}$$

where $N_2 = \mu_2\ p_{02} + \mu_3\ p_{03}$ and D_1 has already mentioned

Busy Period Analysis (Repair Time Only)

Let $BR_i(t)$ be the probability that the system is busy due to inspection unit at an instant 't' given that system entered state i at t=0. The recursive relation for $BR_i(t)$ are given as

$$BR_0(t) = q_{01}(t) © BR_1(t) + q_{02}(t) © BR_2(t) + q_{03}(t) © BR_3(t)$$

$$BR_1(t) = q_{10}(t) © BR_0$$

$$BR_2(t) = q_{24}(t) © BR_4(t) + q_{25}(t) © BR_5(t)$$

$$BR_3(t) = q_{34}(t) © BR_4(t) + q_{36}(t) © BR_6(t)$$

$$BR_4(t) = q_{40}(t) © BR_0(t) + W_4(t)$$

$$BR_5(t) = q_{56}(t) © BR_6(t)$$

$$BR_6(t) = W_6 + q_{60}(t) © BR_0(t)$$

where $W_i(t)$ be the probability that the system is busy in state i due to repair up to time's' without making any transition to any other regenerative and so

$$W_4(t) = \overline{G_1}(t),\ W_6(t) = \overline{G_2}(t)$$

Taking Laplace Transforms of the above relations and solving for $BR_0^*(s)$, the total fraction of the time for which the repairman is busy for repair, in steady- state, is given by:

$$BR_0 = \lim_{s\to\infty}\left(sBR_0^*(s)\right) = \frac{N_3}{D_1}$$

where

$N_3 = \mu_4\ (p_{02}p_{24} + p_{03}p_{34}) + \mu_6\ p_{10}(p_{03}p_{36} + p_{02}p_{25})$, D_1 has already mentioned.

Expected Duration of Time during Which Temperature Raised Beyond Acceptable Limit or Expected Down Time

Let $DT_i(t)$ be the probability that the system is in down state at instant't' given that the system entered regenerative state i at t=0. The recursive relation for $DT_i(t)$ are given as

$$DT_0(t) = q_{01}(t) © DT_1(t) + q_{02}(t) © DT_2(t) + q_{03}(t) © DT_3(t)$$

$$DT_1(t) = q_{10}(t) © DT_0(t) + M_1$$

$$DT_2(t) = q_{24}(t) © DT_4(t) + q_{25}(t) © DT_5(t)$$

$$DT_3(t) = q_{35}(t) © DT_5(t) + q_{34}(t) © DT_4(t)$$

$$DT_4(t) = q_{40}(t) © DT_0(t)$$

$$DT_5(t) = q_{56}(t) © DT_6(t)$$

$$DT_6(t) = q_{60}(t) © DT_0(t)$$

where $M_1(t)$ be the probability that the system is down due to temperature rises beyond the limit up to time't' without making any transition to any other regenerative state and so

$$M_1(t) = \overline{H_2}(t)$$

Taking Laplace Transforms of $DT_i(t)$ relations and solving for $DT_0^{**}(s)$, the total fraction of the time for which the system is in down state, in steady- state, is given by

$$DT_0 = \lim_{s\to\infty}\left(sDT_0^{**}(s)\right) = \frac{N_4}{D_1}$$

where $N_4 = \mu_1\ p_{01}$ and D_1 has already mentioned.

Expected Number of Visits by the Repairman

Let $V_i(t)$ be the expected number of visit by the repairman of the unit by server in time interval (0, t] given that the system entered the regenerative state i at t=0. The recursive relation for $V_i(t)$ are given as:

$$V_0(t) = Q_{01}(t)\ Ⓢ\ V_1(t) + Q_{02}(t)\ Ⓢ\ [1+V_2(t)] + Q_{03}(t)\ Ⓢ\ [1+V_3(t)]$$

$$V_1(t) = Q_{10}(t)\ Ⓢ\ V_0(t)$$

$V_2(t) = Q_{25}(t)$ Ⓢ $V_5(t) + Q_{24}(t)$ Ⓢ $V_4(t)$

$V_3(t) = Q_{35}(t)$ Ⓢ $V_5(t) + Q_{34}(t)$ Ⓢ $V_4(t)$

$V_4(t) = Q_{40}(t)$ Ⓢ $V_0(t)$

$V_5(t) = Q_{56}(t)$ Ⓢ $V_6(t)$

$V_6(t) = Q_{60}(t)$ Ⓢ $V_0(t)$

Taking Laplace Transforms of $V_i(t)$ relations and solving for $V_0^{**}(s)$, the total number of visits by the repairman, in steady- state, is given by:

$$V_0 = \lim_{s\to\infty}\left(sV_0^{**}(s)\right) = \frac{N_5}{D_1}$$

where $N_5 = p_{02} + p_{03}$ and D_1 has already mentioned.

Expected Number of times the Temperature is Maintained whenever Reaches beyond Requisite Limit

Let TM_i (t) be the expected number of time the temperature is maintained whenever it reaches beyond certain limit of the unit by server in time interval (0,t], given that the system entered the regenerative state 'i' at t=0. The recursive relation for TM_i (t) is given by:

$TM_0(t) = Q_{01}(t)$ Ⓢ$[1+ TM_1(t)] + Q_{02}(t)$ Ⓢ$TM_2(t) + Q_{03}(t)$ Ⓢ $TM_3(t)$

$TM_1(t) = Q_{10}(t)$ Ⓢ $TM_0(t)$

$TM_2(t) = Q_{24}(t)$ Ⓢ $TM_4(t) + Q_{25}(t)$ Ⓢ $TM_5(t)$

$TM_3(t) = Q_{35}(t)$ Ⓢ $TM_5(t) + Q_{34}(t)$ Ⓢ $TM_4(t)$

$TM_4(t) = Q_{40}(t)$ Ⓢ $TM_0(t)$

$TM_5(t) = Q_{56}(t)$ Ⓢ $TM_6(t)$

$TM_6(t) = Q_{60}(t)$ Ⓢ $TM_0(t)$

Taking Laplace Transforms of $TM_i(t)$ relations and solving for $TM_0^{**}(s)$, the expected number of times the Temperature is maintained whenever reaches beyond the requisite limit is given by

$$TM_0 = \lim_{s\to\infty}\left(sTM_0^{**}(s)\right) = \frac{N_6}{D_1},$$

where $N_6 = p_{01}$ and D_1 has already mentioned.

Profit Analysis

In steady state, the expected profit per unit time incurred to the system is given by

Profit $(P_0) = C_0A_0 - C_1(DT_0) - C_2(BR_0) - C_2(BI_0) - C_3V_0 - C_4(TM_0)$

C_0: Revenue per up time

C_1: Lose per unit up time during the system remains in down

C_2: Cost per unit time for the repairman is busy for repair

C_3: Cost per visit of the repair man

C_4: Cost per maintenance

NUMERICAL RESULTS AND DISCUSSION

Let us assume all the distributions involved as exponential i.e.

$$f(t) = \lambda e^{-\lambda t}, h_1(t) = \alpha_1 e^{-\alpha_1 t}, h_2(t) = \alpha_2 e^{-\alpha_2 t},$$
$$w(t) = \beta e^{-\beta t}, g_2(t) = \beta_2 e^{-\beta_2 t}, g_1(t) = \beta_1 e^{-\beta_1 t},$$
$$i(t) = \delta e^{-\delta t}.$$

Using the following values of various parameters estimated from the data collected from a fabric manufacturing plant, i.e. λ=0.04167, α=1, α_1= 0.0455, α_2=0.75, β=0.93, C_0=5000, C_1=2500, C_2=2000, C_3=2000 and C_4=1000; the numerical results of various measures of system effectiveness are obtained as:

- Mean Time to System Failure (MTSF) = 25.46966 hour
- Availability (A_0) =0.919729
- Expected Busy Period Inspection time only (BI_0) per hour = 0. 022996
- Expected Busy Period Repair time only (BR_0) per hour = 0. 0562996
- Expected Fraction of Down time (DT_0) =0.140751
- Expected number of Visits of repairman (V_0) per hour = 0. 056138
- Expected number of times per hour the temperature is maintained whenever it reaches beyond the acceptable limit (TM_0)=0.021352
- Profit incurred per hour to the system(P)=2831.241

Nature of MTSF and Availability with regard to failure rate, repair rate, etc is shown in **Figs 2 and 3,** which reveals that:

i. MTSF and Availability both get decreased with increase in the values of failure rate(λ).

However, they have higher values for higher values of repair rate(α).

ii. MTSF get increases with increases in the down time.

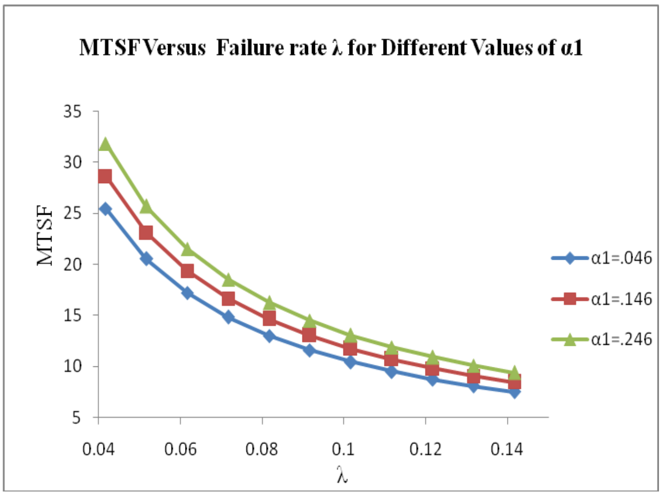

Figure 2.

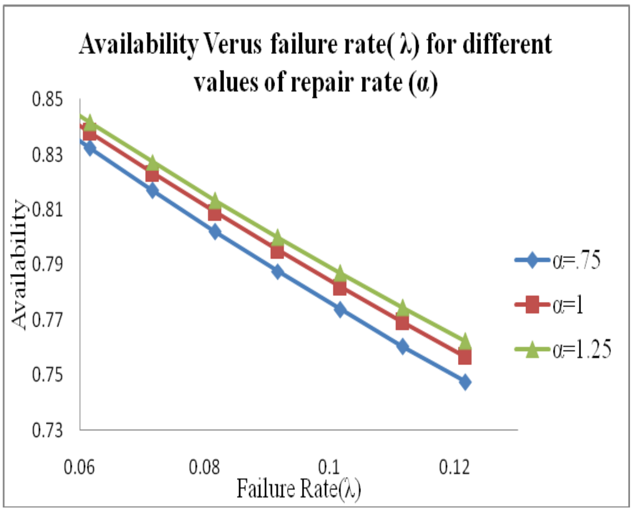

Figure 3.

The profitability aspect has been studied graphically with respect to various parameters and using the expressions for various measures of system effectiveness as shown in **Figs 4 and 5.**

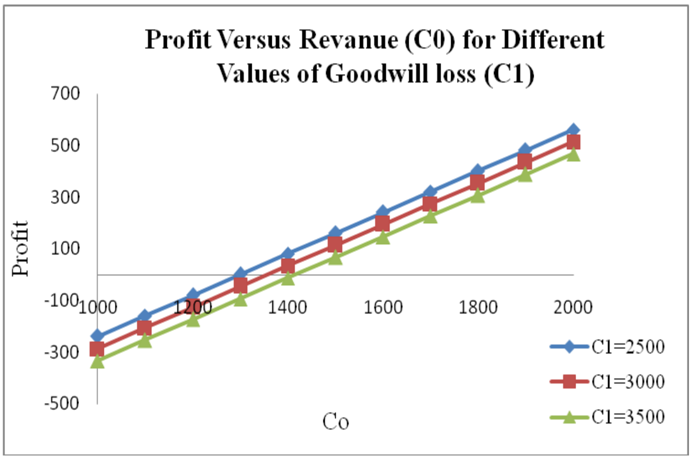

Figure 4.

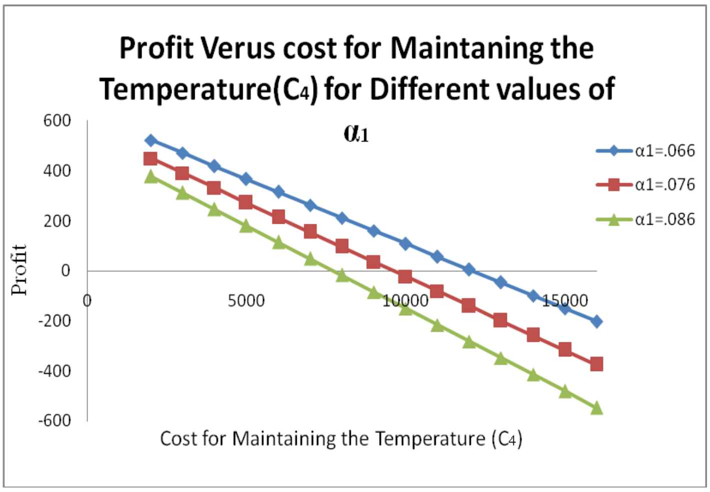

Figure 5.

The interpretations drawn from the above graphs are tabulated as follows:

Figure	Assumed value of the Parameter for which the information by the plant was not provided	Profit		For	Profit ≥ 0 if
		Increses	Decreases		
4	C_{02}=5000, C_2=2000, C_3=1000, C_4=1000, λ= 0.04167, α=0.75	With increase in C_{01}	With increase in C_1	C_1=2500	$C_0 \geq 1298.901$
				C_1=3000	$C_0 \geq 1361.901$
				C_1=3500	$C_0 \geq 1421.901$
5	C_{01}=5000,C_2=1000, C_3=1000, C_4=1000, λ= 0.04167, α=0.75	With decrease in α_1	With increase in C_4	α_1=0.066	$C_4 \leq 12013.91$
				α_1=0.076	$C_4 \leq 10106.49$
				α_1=0.086	$C_4 \leq 8136.316$

CONCLUDING REMARKS

A reliability model based on a fabric manufacturing system has been developed in which repair is affected by the time and operation is affected by the high temperature. The expressions for various performance measures have been obtained. The results obtained for a particular case highlight the importance of study as the cut-off points obtained for various parameters help in taking valuable/economical decisions.

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