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Study of Memory Effect in an Inventory Model with Linear Demand and Salvage Value

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Abstract

In this paper, we have analyzed a memory dependent inventory model with linear type demand rate and time-varying holding cost as well as salvage value. Shortage is not allowed for that model. Memory means it depends past state of the process not only current state of the process. Two type memory indexes have been established (i) differential memory index corresponding fractional order derivative (ii) integral memory index corresponding fractional order integration. The influence of memory effect corresponding differential memory index is more active compared to the integral memory index. The above statement has been justified by the numerical example. The sensitivity analysis is carried out to find out critical inventory parameters for the decision maker.

Keywords: Fractional order derivative; Classical inventory model; Salvage value; Memory dependent inventory model.

INTRODUCTION

The fractional calculus becomes an important research topic during the past three decades, owing to its application backgrounds in the different interesting fields of mathematics [1, 2, 3, 4], economics [5,6], physics[7],mechanics[8-9]. The concept of fractional calculus grew up from the question of L'Hospital to Gottfried Leibnitz about the derivative of order 1/2. The fractional derivatives are defined by the integration, so they are treated as a non- local operator. The fractional calculus is actually an extension of ordinary calculus. The fractional order derivative and fractional order integration leads to develop the whole memory dependent inventory model. The concept of memory is actively used because the order of fractional derivative and fractional integration is an index of memory. In economics, the memory was first engaged to fractional differencing and integrating by Granger and Joyeux [10] with the frame of discrete time approach. Recently fractional calculus has been used to describe the memory effect in the inventory models [3, 4].Here, an importance has also given on the salvage value of the inventory system. But our main focus is to incorporate memory effect to the inventory model with linear type demand rate and time varying holding cost as well as salvage value of the inventory.

Authors expect that inventory system is a memory affected system. For example, if an object gets its popularity in the market then its demand will increase or if it gets poor impression then its demand will gradually decrease. In some sense demand of any object depends on dealing of the shopkeeper or staff of the company with the customer i.e. the selling of any product depends on the quality as well as the shopkeeper's attitude or environment of the company or shop or public relation with the shop or company. The associated cost has been developed with fractional effect. The integral memory index comes from the associated cost.

In general, for the classical economic order quantity model, the researchers use integer order differential equation and evaluate the minimized total average cost and optimal ordering interval. Harris [11] was the first person who developed the classical economic order quantity model. From that moment, many different type concept about the inventory system was developed by different researchers [12, 13, 14, 15].Mishra et al [16], shah et al [17] developed the inventory model with considering salvage value. But we want to come out the traditional thoughts of classical inventory model. In this paper, the concept of memory has developed with considering R-L fractional order integration, evaluating minimized total average cost, optimal ordering interval.

Our analysis clear that the effect of memory is more sensitive for the differential memory index compared to the integral memory index.

REVIEW OF FRACTIONAL CALCULUS

Euler Gamma Function

Euler's gamma function is one of the best tools in fractional calculus which was proposed by the Swiss mathematicians Leonhard Euler (1707-1783).The gamma function $\Gamma(x)$is continuous extension from the factorial notation .The gamma function is denoted and defined by the formulae

$$\Gamma(x)=\int_0^\infty t^{(x-1)}e^{-t}dt \qquad x>0 \qquad (1)$$

$\Gamma(x)$is extended for all real and complex numbers and the gamma function satisfies some basic properties

$$\Gamma(x+1)=x\Gamma(x)\Rightarrow\Gamma(x)=\frac{\Gamma(x+1)}{x},\Gamma\left(\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2},\Gamma\left(-\frac{7}{6}\right)=-\frac{6}{7}\Gamma\left(\frac{1}{6}\right)$$

Numerically $x!$ can be evaluated for all positive integer values numerically but $\Gamma(x+1)$ can be evaluated for real values.

Fractional derivative on polynomial function

The idea of the fractional calculus reduced from the powerful thinking of fractional order derivative. Derivative (m^{th} order) of any continuous polynomial function $f(x)=x^n$ is derived as follows

$$\frac{d^m\left(x^n\right)}{dx^m}=n(n-1)(n-2)(n-3)(n-4)(n-5)......(n-m+1)x^{(n-m)}=\frac{m!}{(m-n)!}x^{(m-n)}\quad n\geq 0,\ n\geq m(2)$$

We know that $(n)!=\Gamma(n+1)$.Then *m!* and *(n-m)!* is replaced by the $\Gamma(m+1)$and $\Gamma(n-m+1)$in (2) and obtained as

Taking $m=\frac{1}{2},n=2,$ it reduces to $\frac{d^{\frac{1}{2}}\left(x^2\right)}{dx^{\frac{1}{2}}}=\frac{16x^{\frac{3}{2}}}{3\sqrt{\pi}}$ .Now, $\frac{d^{\frac{1}{2}}\left(x(x-1)(x-2)\right)}{dx^{\frac{1}{2}}}=\frac{1}{\sqrt{\pi}}\left(\frac{36}{15}x^{\left(\frac{5}{2}\right)}-16x^{\left(\frac{3}{2}\right)}+8x^{\left(\frac{1}{2}\right)}\right).$

Different type fractional order derivative

(i) Liouville fractional order derivative

Liouville fractional order derivative for any continuous function $f(x)$ is denoted and defined by

$$D^{\alpha}\left(f(x)\right)=\frac{1}{\Gamma(1-\alpha)}\left(\frac{d}{dx}\right)\int_{-\infty}^{x}(x-\tau)^{(-\alpha)}f(\tau)d\tau\quad\text{where } x>0 \tag{3}$$

Liouville left sided fractional derivative is denoted and defined as follows

$$D_{0^+}{}^{\alpha}\left(f(x)\right)=\frac{1}{\Gamma(m-\alpha)}\left(\frac{d^m}{dx^m}\right)\int_{0}^{x}(x-\tau)^{(-\alpha)}f(\tau)d\tau\quad\text{where } x>0 \tag{4}$$

Liouville right sided fractional derivative is denoted and defined as follows

$$D_{0^-}{}^{\alpha}\left(f(x)\right)=\frac{(-1)^m}{\Gamma(m-\alpha)}\left(\frac{d^m}{dx^m}\right)\int_{x}^{\infty}(x-\tau)^{(-\alpha)}f(\tau)d\tau\quad\text{where } x>0 \tag{5}$$

(ii)Riemann-Liouville fractional derivative(R-L)

Left Riemann-Liouville (R-L) fractional derivative of order α is denoted and defined as follows

$$_aD_x{}^{\alpha}\left(f(x)\right)=\frac{1}{\Gamma(m-\alpha)}\left(\frac{d}{dx}\right)^m\int_{a}^{x}(x-\tau)^{(m-\alpha-1)}f(\tau)d\tau\quad\text{where } x>0 \tag{6}$$

Right Riemann-Liouville(R-L) fractional derivative of order α is defined as follows

$$_xD_b{}^{\alpha}\left(f(x)\right)=\frac{1}{\Gamma(m-\alpha)}\left(-\frac{d}{dx}\right)^m\int_{x}^{b}(x-\tau)^{(m-\alpha-1)}f(\tau)d\tau\quad\text{where } x>0 \tag{7}$$

Riemann-Liouville(R-L) fractional derivative of any constant function is not equal to zero which created a distance between ordinary calculus and fractional calculus.

(iii)Caputo fractional order derivative

Left Caputo fractional derivative for the function $f(x)$ which has continuous, bounded derivatives in $[a,b]$ is denoted and defined as follows

$$ {}_a^C D_x^{\alpha}\left(f(x)\right) = \frac{1}{\Gamma(m-\alpha)}\int_a^x (x-\tau)^{(m-\alpha-1)} f^m(\tau)d\tau \quad \text{where } 0 \le m-1 < \alpha < m \tag{8} $$

Right Caputo fractional derivative for the function $f(x)$ which has continuous and bounded derivatives in $[a,b]$, is defined as follows

$$ {}_x^C D_b^{\alpha}\left(f(x)\right) = \frac{1}{\Gamma(m-\alpha)}\int_x^b (\tau-x)^{(m-\alpha-1)} f^m(\tau)d\tau \quad \text{where } 0 \le m-1 < \alpha < m \tag{9} $$

$$ {}_a^C D_x^{\alpha}(A) = o, \text{where } \text{A} = \text{constant} $$

CLASSICAL INVENTORY MODEL

The classical inventory model has been developed in this section without its formulation part.

Assumptions

In this paper, the classical and fractional order EOQ models are developed on the basis of the following assumptions.

(i) Lead time is zero.(ii)Time horizon is infinite.(iii)There is no shortage.(iv)There is no deterioration.(v) Demand rate is linear type $D(t) = (a+bt)$ for $0 \le t \le T$.

NOTATIONS:

Table-1: Used symbols and items.

$(i)D(t)$: Demand rate.	$(ii)Q$: Total order quantity.
$(iii)P$: Per unit cost.	$(iv)C_1$: Inventory holding cost per unit.
$(v)C_3$: Ordering cost or setup cost per order.	$(vi)I(t)$: Stock level or inventory level.
$(vii)T$: Ordering interval.	$(viii)HOC_{\alpha,\beta}$: Inventory holding cost per cycle for the classical inventory model.
$(ix)T^*$: Optimal ordering interval.	$(x)TOC_{\alpha,\beta}^{av}$: Total average cost during the total time interval.
$(xi)TOC_{\alpha,\beta}^{*}$: Minimized total average cost during the total time interval $[0,T]$.	$(xii)(B,.)$: Beta function.
$(xiii)T_{\alpha,\beta}^{*}$: Optimal ordering interval.	(xiv)$(\Gamma,.)$: Gamma function.
(xv)γ : Salvage value per unit.	(x $vi)SL_{\alpha,\beta}$: Salvage value for total unit with fractional effect.
(x $vii)PC_{\alpha,\beta}$: Total purchasing cost with fractional effect.	

Classical model formulation

The inventory level depletes due to demand only during the total time interval *[0, T]*.Shortages are not allowed. Inventory level reaches to zero level at *t=T*. Therefore, the classical inventory model is represented by the integer order differential equation as

$$\frac{d\left(I\left(t\right)\right)}{dt}=-\left(a+bt\right)\quad 0\le t\le T\quad \text{with}\ \ I\left(T\right)=0,\ I\left(0\right)=Q. \tag{10}$$

FRACTIONAL ORDER INVENTORY MODEL WITH MEMORY KERNEL

To study the influence of memory effects, first the differential equation (10) is written using the memory kernel function in the following form [***1***].

$$\frac{dI(t)}{dt}=-\int \mathrm{k}\left(t-t'\right)(a+bt')dt' \tag{11}$$

in which $k(t-t')$ plays the role of a time-dependent kernel. This type of kernel promises the existence of scaling features as it is often intrinsic in most natural phenomena. Thus, to generate the fractional order model we consider $k(t-t')=\frac{1}{\Gamma(1-\alpha)}(t-t')^{\alpha-2}$, where $0<\alpha\le 1$ and $\Gamma(\alpha)$ denotes the gamma function. Using the definition of fractional derivative [1], the equation (11) can be written to the form of fractional differential equations with the Caputo-type derivative in the following form as,

$$\frac{dI(t)}{dt}=-{}_0D_t^{-(\alpha-1)}\left(a+bt\right) \tag{12}$$

Now, applying fractional Caputo derivative of order $(\alpha-1)$ on both sides of (12), and using the fact that Caputo fractional order derivative and fractional integral are inverse operators, the following fractional differential equations can be obtained for the model

$${}_0^CD_t^{\alpha}\left(I\left(t\right)\right)=-(a+bt) \quad \text{or equivalently}$$

$$\frac{d^{\alpha}\left(\mathrm{I(t)}\right)}{dt^{\alpha}}=-\left(a+bt\right)\qquad,\quad 0<\alpha\le 1.0,\ \ 0\le t\le T \tag{13}$$

with boundary conditions $I(T)=0$ and $I(0)=Q$.

Long Memory effect: The strength of memory is controlled by the order of fractional derivative or fractional integration.If order of fractional derivative or fractional integrationis in (0,0.5).Then this system may be called that it has long memory effect.

Short Memory effect: If order of the fractional derivative or the fractional integration is in [0.5,1).The system is called that it has short memory effect.

Fractional order inventory model analysis

Here, keeping fixed all assumptions as classical model, the fractional order inventory model has been governed by the following fractional order differential equation as follows

$${}_0^CD_t^{\alpha}\left(I\left(t\right)\right)=-\left(a+bt\right) \tag{14}$$

$$D^{\alpha}\equiv\frac{d^{\alpha}}{dt^{\alpha}},\text{where},0<\alpha\le 1.0,\ 0\le t\le T\quad with\ I\left(T\right)=0$$

Applying Laplace transform and the corresponding inversion formula on the equation (14) we get the inventory level at time *t* is

$$I(t)=\left(Q-\frac{at^{\alpha}}{\Gamma(1+\alpha)}-\frac{bt^{\alpha+1}}{\Gamma(2+\alpha)}\right) \tag{15}$$

Using the boundary condition $I(T)=0$ on the equation (15) the total order quantity is obtained as

$$Q=\frac{aT^{\alpha}}{\Gamma(1+\alpha)}+\frac{bT^{\alpha+1}}{\Gamma(2+\alpha)} \tag{16}$$

and corresponding the inventory level at time *t* being,

$$I(t)=\left(\frac{a}{\Gamma(1+\alpha)}\left(T^{\alpha}-t^{\alpha}\right)+\frac{b}{\Gamma(2+\alpha)}\left(T^{\alpha+1}-t^{\alpha+1}\right)\right) \tag{17}$$

Inventory holding cost is assumed time dependent and the β^{th} $(0<\beta\le 1)$ order total inventory holding cost is denoted as $HOC_{\alpha,\beta}(T)$ and defined as

$$HOC_{\alpha,\beta}=C_1\left(D^{-\beta}\left(tI(t)\right)\right)$$

$$=\frac{C_1}{\Gamma(\beta)}\int_0^T t(T-t)^{(\beta-1)}\left(\begin{array}{l}\frac{a}{\Gamma(1+\alpha)}\left(T^{\alpha}-t^{\alpha}\right)\\+\frac{b}{\Gamma(2+\alpha)}\left(T^{\alpha+1}-t^{\alpha+1}\right)\end{array}\right)dt$$

$$HOC_{\alpha,\beta}(T)=\frac{C_1aT^{(\alpha+\beta+1)}}{\Gamma(\alpha+1)\Gamma(\beta)}\left(B(2,\beta)-B(\alpha+2,\beta)\right)+\frac{C_1bT^{(\alpha+\beta+2)}}{\Gamma(\beta)\Gamma(\alpha+2)}\left(B(2,\beta)-B(\alpha+3,\beta)\right) \tag{18}$$

Total purchasing cost with fractional effect

$$PC_{\alpha,\beta}=P\times Q=P\left(\frac{aT^{\alpha}}{\Gamma(1+\alpha)}+\frac{bT^{\alpha+1}}{\Gamma(2+\alpha)}\right) \tag{19}$$

Salvage quantity is

$$\left(Q-D^{-\beta}(a+bt)\right)=\left(Q-\frac{1}{\Gamma(\beta)}\int_0^T(T-t)^{\beta-1}(a+bt)dt\right)$$

$$=\left(\frac{aT^{\alpha}}{\Gamma(1+\alpha)}+\frac{bT^{\alpha+1}}{\Gamma(2+\alpha)}\right)-\left(\frac{aT^{\beta}}{\Gamma(1+\beta)}+\frac{bB(2,\beta)T^{\beta+1}}{\Gamma(\beta)}\right) \tag{20}$$

Total salvage value is given below

$$SL_{\alpha,\beta}=\gamma\left(Q-D^{-\beta}(a+bt)\right)=\gamma\left(Q-\frac{1}{\Gamma(\beta)}\int_0^T(T-t)^{\beta-1}(a+bt)dt\right)$$

$$=\gamma\left(\left(\frac{aT^{\alpha}}{\Gamma(1+\alpha)}+\frac{bT^{\alpha+1}}{\Gamma(2+\alpha)}\right)-\left(\frac{aT^{\beta}}{\Gamma(1+\beta)}+\frac{bB(2,\beta)T^{\beta+1}}{\Gamma(\beta)}\right)\right) \tag{21}$$

(here, β is considered as integral memory index).

Therefore, the total average cost per unit time per cycle is given as

$$TOC^{av}_{\alpha,\beta}(T)$$
$$=\frac{\left(PC_{\alpha,\beta}+HOC_{\alpha,\beta}(T)+C_3-SL_{\alpha,\beta}\right)}{T}$$
$$=\left(\begin{array}{l}\frac{C_1bT^{(\alpha+\beta+1)}}{\Gamma(\beta)\Gamma(\alpha+2)}\left(B(2,\beta)-B(\alpha+3,\beta)\right)+\frac{C_1aT^{(\alpha+\beta)}}{\Gamma(\alpha+1)\Gamma(\beta)}\left(B(2,\beta)-B(\alpha+2,\beta)\right)+\\ \frac{(P-\gamma)bT^{\alpha}}{\Gamma(2+\alpha)}+\frac{(P-\gamma)aT^{\alpha-1}}{\Gamma(1+\alpha)}+\\ \frac{b\gamma B(2,\beta)T^{\beta}}{\Gamma(\beta)}+\frac{a\gamma T^{\beta-1}}{\Gamma(1+\beta)}+C_3T^{-1}\end{array}\right) \quad (22)$$
$$=AT^{\alpha+\beta+1}+B_1T^{\alpha+\beta}+CT^{\alpha}+DT^{\alpha-1}+ET^{\beta}+FT^{\beta-1}+GT^{-1} \quad (23)$$

Here, four cases have been studied for the characterization of this fractional order inventory model (i)$0<\alpha\le 1.0, 0<\beta\le 1.0$, (ii)$\alpha=1.0$ and $0<\beta\le 1.0$, (iii)$\beta=1.0$ and $0<\alpha\le 1.0$, (iv) $\alpha=1.0, \beta=1.0$.

(i)Case-1: $0<\alpha\le 1.0, 0<\beta\le 1.0$.

In this case, the total average cost becomes

$$TOC^{av}_{\alpha,\beta}(T)=AT^{\alpha+\beta+1}+B_1T^{\alpha+\beta}+CT^{\alpha}+DT^{\alpha-1}+ET^{\beta}+FT^{\beta-1}+GT^{-1}$$
$$\left(A=\frac{C_1b}{\Gamma(\beta)\Gamma(\alpha+2)}\left(B(2,\beta)-B(\alpha+3,\beta)\right), B_1=\frac{C_1a}{\Gamma(\beta)\Gamma(\alpha+1)}\left(B(2,\beta)-B(\alpha+2,\beta)\right)\right),$$
$$C=\frac{(P-\gamma)b}{\Gamma(\alpha+2)}, D=\frac{(P-\gamma)a}{\Gamma(\alpha+1)}, \mathrm{E}=\frac{\gamma a}{\Gamma(\beta+1)}, \mathrm{F}=\frac{bB(2,\beta)\gamma}{\Gamma(\beta)}, \mathrm{G}=\mathrm{C}_3$$

Therefore, the inventory model can be written as

$$\begin{cases}\mathrm{Min}\,TOC^{av}_{\alpha,\beta}(T)=AT^{\alpha+\beta+1}+B_1T^{\alpha+\beta}+CT^{\alpha}+DT^{\alpha-1}+ET^{\beta}+FT^{\beta-1}+GT^{-1}\\ \text{Subject to}\, T\ge 0\end{cases} \quad (24)$$

(A) Primal Geometric programming method

The above inventory model (24) is solved by primal geometric programming method [3, 4].

The dual form of the above primal (24) model is as,

$$\mathrm{Max}\,d(w)=\left(\frac{A}{w_1}\right)^{w_1}\left(\frac{B_1}{w_2}\right)^{w_2}\left(\frac{C}{w_3}\right)^{w_3}\left(\frac{D}{w_4}\right)^{w_4}\left(\frac{E}{w_5}\right)^{w_5}\left(\frac{F}{w_6}\right)^{w_6}\left(\frac{G}{w_7}\right)^{w_7} \quad (25)$$

Orthogonal condition is as

$$(\alpha+\beta+1)w_1+(\alpha+\beta)w_2+(\alpha)w_3+(\alpha-1)w_4+\beta w_5+(\beta-1)w_6-w_7=0 \quad (26)$$

Normalized condition is as

$$w_1+w_2+w_3+w_4+w_5+w_6+w_7=1 \quad (27)$$

Primal –dual relations are given below

$$\left(\begin{array}{l}AT^{\alpha+\beta+1}=w_1d(w), B_1T^{\alpha+\beta}=w_2d(w), CT^{\alpha}=w_3d(w)\\ DT^{\alpha-1}=w_4d(w), ET^{\beta}=w_5d(w), FT^{\beta-1}=w_6d(w), GT^{-1}=w_7d(w)\end{array}\right)$$

From the above primal- dual relations we get,

$$\begin{pmatrix} \left(\frac{B_1 w_1}{A w_2}\right)^{(\beta)} = \left(\frac{C w_2}{B_1 w_3}\right), \\ \left(\frac{B_1 w_1}{A w_2}\right) = \left(\frac{D w_3}{C w_4}\right), \left(\frac{B_1 w_1}{A w_2}\right)^{(\alpha-\beta-1)} = \left(\frac{E w_4}{D w_5}\right) \\ \left(\frac{B_1 w_1}{A w_2}\right) = \left(\frac{F w_5}{E w_6}\right), \left(\frac{B_1 w_1}{A w_2}\right)^{(\beta)} = \left(\frac{G w_6}{F w_7}\right) \end{pmatrix} \tag{28}$$

along with,

$$T = \left(\frac{B_1 w_1}{A w_2}\right) \tag{29}$$

The non-linear equations (26, 27 and 28) for $w_1, w_2, w_3, w_4, w_5, w_6, w_7$ can be solved to obtain $w_1^*, w_2^*, w_3^*, w_4^*, w_5^*, w_6^*, w_7^*$ .Optimal ordering interval and minimized total average cost can be solved from (29) and (24) analytically.

(ii)Case-2: $\alpha = 1.0,\ 0<\beta \le 1.0.$

I n this case, the total average cost becomes as

$$TOC_{1,\beta}^{av}(T) = AT^{\beta+2} + B_1 T^{1+\beta} + CT^{\beta} + DT^{\beta-1} + ET + FT^0 + GT^{-1}$$

$$\begin{pmatrix} A = \frac{C_1 b}{\Gamma(\beta)\Gamma(3)}\left(B(2,\beta) - B(4,\beta)\right), B_1 = \frac{C_1 a}{\Gamma(\beta)\Gamma(2)}\left(B(2,\beta) - B(3,\beta)\right), \\ C = \frac{a\gamma}{\Gamma(\beta+1)}, D = \frac{bB(2,\beta)\gamma}{\Gamma(\beta)}, E = \frac{(P-\gamma)b}{\Gamma(3)}, F = \frac{(P-\gamma)a}{\Gamma(2)}, \mathrm{G} = \mathrm{C}_3 \end{pmatrix}$$

In this case, the inventory model can be written as follows

$$\begin{cases} \mathrm{Min}\,TOC_{1,\beta}^{av}(T) = AT^{\beta+2} + B_1 T^{1+\beta} + CT^{\beta} + DT^{\beta-1} + ET + FT^0 + GT^{-1} \\ \text{Subject to } T \ge 0 \end{cases} \tag{30}$$

In similar way of case-1, the minimized total average cost and optimal ordering interval has evaluated from (30).

(iii)Case-3: $\beta = 1.0,\ 0<\alpha \le 1.0.$

The total average cost is

$$TOC_{\alpha,1}^{av}(T) = AT^{\alpha+2} + B_1 T^{1+\alpha} + CT^{\alpha} + DT^{\alpha-1} + ET + FT^0 + GT^{-1}$$

$$\begin{pmatrix} A = \frac{C_1 b}{\Gamma(\alpha+2)}\left(B(2,1) - B(\alpha+3,1)\right), B_1 = \frac{C_1 a}{\Gamma(\alpha+1)\Gamma(1)}\left(B(2,1) - B(\alpha+2,1)\right), \\ C = \frac{(P-\gamma)b}{\Gamma(\alpha+2)}, D = \frac{(P-\gamma)a}{\Gamma(\alpha+1)}, E = \frac{a\gamma}{\Gamma(2)}, \mathrm{F} = \frac{bB(2,1)\gamma}{\Gamma(1)}, \mathrm{G} = \mathrm{C}_3 \end{pmatrix}$$

In this case, the inventory model is

$$\begin{cases} \mathrm{Min}\,TOC_{\alpha,1}^{av}(T) = AT^{\alpha+2} + B_1 T^{1+\alpha} + CT^{\alpha} + DT^{\alpha-1} + ET + FT^0 + GT^{-1} \\ \text{Subject to } T \ge 0 \end{cases} \tag{31}$$

In similar way of case-1, the minimized total average cost and optimal ordering interval has been evaluated from (31).

(iv) Case-4: $\alpha = 1.0, \beta = 1.0$

I n this case, the total average cost is obtained

$$TOC_{1,1}^{av}(T) = AT^3 + B_1T^2 + CT + DT^0 + ET^{-1}$$

$$\left(\begin{array}{l} A = \frac{C_1 b}{\Gamma(3)}\left(B(2,1) - B(4,1)\right), B_1 = \frac{C_1 a}{\Gamma(2)\Gamma(1)}\left(B(2,1) - B(3,1)\right), \\ C = \frac{(P-\gamma)b}{\Gamma(3)} + \frac{a\gamma}{\Gamma(2)}, D = \frac{(P-\gamma)a}{\Gamma(2)} + \frac{bB(2,1)\gamma}{\Gamma(1)}, E = C_3 \end{array} \right)$$

In this case, the inventory model can be written as follows

$$\begin{cases} \text{Min} TOC_{1,1}^{av}(T) = AT^3 + B_1T^2 + CT + DT^0 + ET^{-1} \\ \text{Subject to } T \geq 0 \end{cases} \tag{32}$$

In similar way of case-1, the minimized total average cost and optimal ordering interval has been formulated from (32).

NUMERICAL EXAMPLE

Here we have calculated the minimized total average cost and the optimal ordering interval with assuming example.

Example: Let $a = 10, b = 6, C_1 = 7, C_3 = 20, P = 50, \gamma = 0.1$ in appropriate units.

Table-2: Minimized total average cost and optimal ordering interval for $0 < \alpha \leq 1.0, \beta = 1.0$.

(here↑ uses for increasing value and ↓ uses for decreasing value)

α	β	$T^*_{\alpha,\beta}$	$TOC^*_{\alpha,\beta}$
0.1	1.0	2.9560	600.36722
0.2	1.0	2.6096	660.8729
0.3	1.0	2.2811	713.3738
0.4	1.0	1.9652	754.7137
0.5	1.0	1.6580	781.6777↓(decreasing)(above)
0.6	1.0	1.3571	**791.1120(maximum)**
0.7	1.0	1.0672	780.0738↓(decreasing)(below)
0.8↑ (growing memory effect)	1.0	0.7811	746.1518
0.9	1.0	0.5323	688.5363
1.0	1.0	0.3533	611.2919

Corresponding differential memory index α, the minimized total average cost becomes maximum at $\alpha = 0.6$ then gradually decreases below and above. Hence, at the critical memory effect, profit is low compared to the other moment. Some bad memory started to work at that moment then again it is able to recover.

Table-3: The minimized total average cost and optimal ordering interval $0 < \beta \leq 1.0, \alpha = 1.0$.

α	β	$T^*_{\alpha,\beta}$	$TOC^*_{\alpha,\beta}$
1.0	0.1	0.3611	610.4477
1.0	0.2	0.3577	611.3749
1.0	0.3	0.3554	611.9330
1.0	0.4	0.3537	612.2153↓(**decreasing**)(above)
1.0	**0.5**	0.3528	**612.2954(maximum)**
1.0	0.6	0.3523	612.2315↓(**decreasing**)(below)
1.0	0.7	0.3521	612.0685
1.0	0.8	0.3523	611.8411
1.0	0.9	0.3527	611.5757
1.0	1.0	0.3533	611.2919

It is clear from the table-3 that in long memory effect corresponding the memory index β, the minimized total average cost is low compared to the short memory effect. There is no significant difference among the numerical values of the minimized total average cost for different values for gradually increasing memory effect of the integral memory index β.

Sensitivity analysis

We will now study the effects of changes in the values of the parameters $a, b, C_1, C_3, P, \gamma$ on the minimized total average cost and the optimal ordering interval using the above numerical example corresponding long memory effect and memory less system.

Table-4: Sensitivity analysis for $\alpha = 0.1$.

parameter	Parameter Change (%)	$T_{\alpha,1}{}^{*}$	$TOC^{*}{}_{\alpha,1}(T)$	parameter	Parameter Change (%)	$T_{\alpha,1}{}^{*}$	$TOC^{*}{}_{\alpha,1}(T)$
a	+50%	3.4076	696.4028	C_1	+50%	2.6005	633.1830
	+10%↑	3.0574	520.5336		+10%↑	2.8687	607.6571↑
	-10%	2.8473↑	579.6003		-10%	3.0111↓	595.9995
	-50%	2.3005	487.8951		-50%	3.6657	554.4134
b	+50%	2.5372	788.2989	C_3	+50%	2.9766	603.7434
	+10%↑	2.8535	638.9674		+10%↑	2.9602	601.0483
	-10%	3.0725	561.1169↑		-10%	2.9519↑	599.6951↑
	-50%	3.7865↓	394.4174		-50%	2.9352	596.9773
γ	+50%	2.9524	600.7976	P	+50%	3.3422	854.6165
	+10%↑	2.9553	600.4573		+10%↑	3.0425	651.9240
	-10%	2.9567↓	600.2870↑		-10%	2.8633↑	548.3764↑
	-50%	2.9596	599.9461		-50%	2.4008	334.2686

(i) The minimized total average cost increases when b,γ,C_1,C_3 increases and hence, profit decreases with increasing value of b,γ,C_1,C_3. The changes of the minimized total average cost does not maintain any sequence.

(ii) The optimal ordering interval decreases with gradually increasing value of b,γ,C_1. The optimal ordering interval increases with gradually increasing value of a,P,C_3.

(iii) For changing of γ there is no sensitive effect compared to the other parameter.

In long memory effect, the critical memory parameters are b,P for the decision maker.

Table-5: The Sensitivity analysis for $\alpha = 1.0$.

parameter	Parameter Change (%)	$T_{\alpha,1}{}^{*}$	$TOC^{*}{}_{\alpha,1}(T)$	paramet er	Parameter Change (%)	$T_{\alpha,1}{}^{*}$	$TOC^{*}{}_{\alpha,1}(T)$
a	+50%	0.3491	862.0113	C_1	+50%	0.3481	612.1225
	+10%↑	0.3525	661.4372		+10%↑	0.3522	611.4601
	-10%	0.3542	561.1459↑		-10%	0.3544	611.1225↑
	-50%	0.3578	360.5544		-50%	0.3590	610.4336
b	+50%	0.2924	635.3838	C_3	+50%	0.4291	636.8417
	+10%↑	0.3382	616.4974		+10%↑	0.3699	616.8226
	-10%	0.3706	605.8404		-10%	0.3358↑	505.4877↑
	-50%	0.4770	580.6431↑		-50%	0.2525	578.3073
γ	+50%	0.3533	611.2919	P	+50%	0.2929	885.3180
	+10%↑	0.3533	611.2919		+10%↑	0.3384	666.4771
	-10%	0.3533	611.2919		-10%	0.3704	555.8671↑
	-50%	0.3533	611.2919		-50%	0.4726	330.9240

(i)The minimized total average cost increases when a,b,C_1,C_3,P increases and hence, profit decreases with increasing value of a,b,C_1,C_3,P.

(ii)There is no effect on the minimized total average cost and optimal ordering interval for changing salvage value γ.

In memory less system, here , the critical memory parameters are a,P for the decision maker.

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GRAPHICAL PRESENTATION

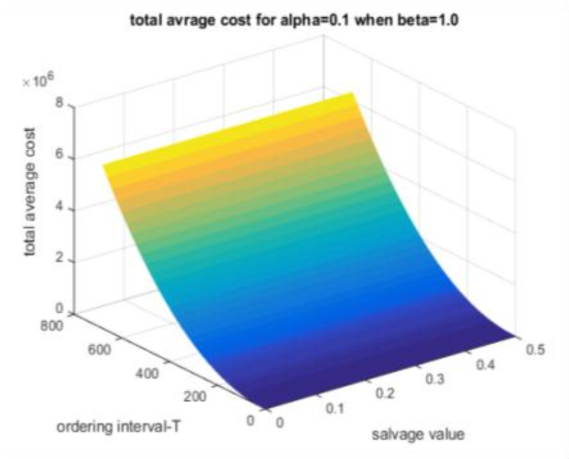

Fig-1

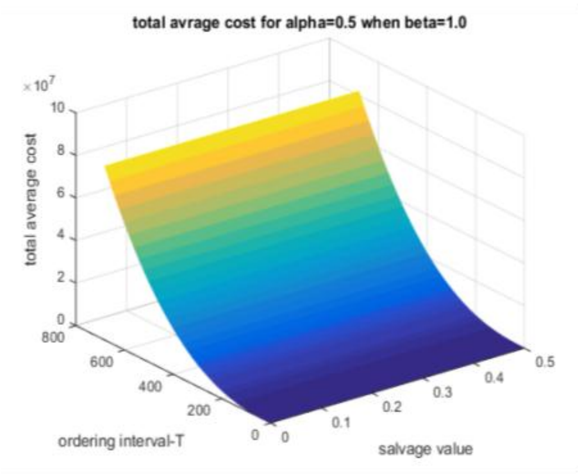

Fig-2

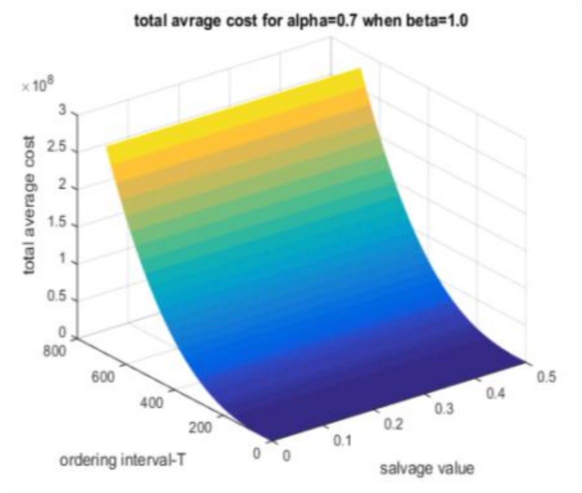

Fig-3

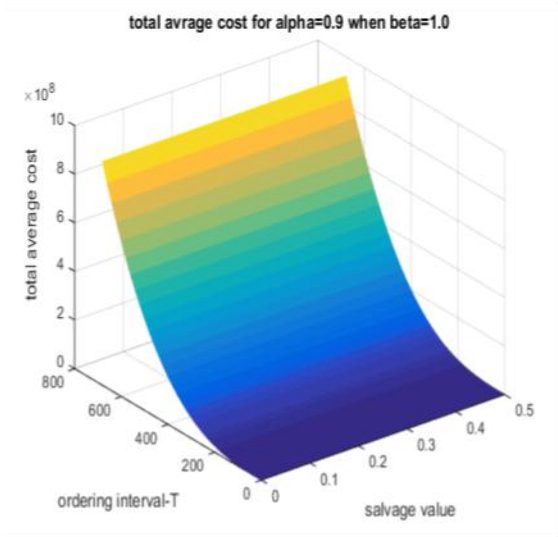

Fig-4

CONCLUSION

In the present paper, we want to establish memory dependent inventory model for linear type demand rate with time varying holding cost as well as salvage value. The importance of memory effect on the inventory model gives a new direction in the inventory management. For the differential memory index, the minimized total average cost becomes maximum at $\alpha = 0.6$ and then gradually decreases below and above. In long memory effect corresponding differential memory index, profit is high compared to the short memory effect. But corresponding integral memory index, there is no sensitive memory effect on the minimized total average cost and the optimal ordering interval. Here, we also have studied about the three dimensional structure of the minimized total average cost with respect to salvage value and ordering interval –*T*. In long memory effect and memory less system, *P* is the critical memory parameter for the decision maker. The sensitivity analysis shows that sufficient care should not be taken to estimate the parameter γ in market studies.

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