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Pairwise Fuzzy σ -First Category Sets

G.Thangaraj
Professor & Head, Department of Mathematics,
Thiruvalluvar University ,Vellore-632115, Tamil Nadu, India.

A.Vinothkumar
Assistant Professor, Department of Mathematics ,
Shanmuga Industries Arts & Science College,
Tiruvannmalai-606 601, Tamil Nadu, India.

Abstract

In this paper the concepts of pairwise fuzzy σ-first category sets are studied. Several characterizations of pairwise fuzzy σ-first category sets are established.

Keywords: Pairwise fuzzy open set, pairwise fuzzy F_σ -set, pairwise fuzzy G_δ -set, pairwise fuzzy nowhere dense set, pairwise fuzzy σ-nowhere dense set, pairwise fuzzy first category set.

INTRODUCTION

In order to deal with uncertainties, the idea of fuzzy sets, fuzzy set operations was introduced by L.A.Zadeh [1] in his classical paper in the year 1965, describing fuzziness mathematically for the first time. In 1968, C.L.Chang [2] defined fuzzy topological space by using fuzzy sets introduced by Zadeh. The concept of σ-nowhere dense sets in classical topology was introduced and studied by Jiling Cao and Sina Greenwood in [3].

In 1989, A.Kandil [4] introduced and studied fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces. The concept of Baire bitopological spaces have been studied extensively in classical topology in [5] ,[6] and [7]. The concept of pairwise fuzzy σ-first category sets are defined by authors in [8]. In this paper several characterizations of pairwise fuzzy σ-first category sets are established.

PRELIMINARIES

In order to make the exposition self-contained, some basic notations and results used in the sequel are given. In this work by by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). By a fuzzy bitopological space (Kandil, 1989) we mean an ordered triple (X, T_1, T_2), where T_1 and T_2 are fuzzy topologies on the non-empty set X. Let X be a non-empty set and I the unit interval [0,1]. A fuzzy set λ in X is a mapping from X into I.

Definition 2.1. [9] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy open set if $\lambda \in T_i$ (i = 1, 2). The complement of pairwise fuzzy open set in $(X, T_1, T_2)$is called a pairwise fuzzy closed set.

Definition 2.2. [9] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy G_δ-set if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are pairwise fuzzy open sets in (X, T_1, T_2).

Definition 2.3. [9] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy F_σ -set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are pairwise fuzzy closed sets in (X, T_1, T_2).

Lemma 2.1. [10] For a family of $\{\lambda_\alpha\}$ of fuzzy sets of a fuzzy topological space (X, T), $\vee cl(\lambda_\alpha) \leq cl(\vee \lambda_\alpha)$. In case *A* is a finite set, $\vee cl(\lambda_\alpha) = cl(\vee \lambda_\alpha)$. Also $\vee int(\lambda_\alpha) \leq int(\vee\lambda_\alpha)$ in (X, T).

Definition 2.4. [11] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy dense set if $cl_{T1}\, cl_{T2}(\lambda) = cl_{T2}\, cl_{T1}(\lambda) = 1$, in (X, T_1, T_2).

Definition 2.5 [12]. A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy nowhere dense set if $int_{T1}\, cl_{T2}(\lambda) = int_{T2}\, cl_{T1}(\lambda) = 0$, in (X, T_1, T_2).

Definition 2.6. [9] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy σ-nowhere dense set if λ is a pairwise fuzzy F_σ-set in (X, T_1, T_2) such that $int_{T1} int_{T2}(\lambda) = int_{T2}\, int_{T1}(\lambda) = 0$.

Definition 2.7. [8] A fuzzy bitopological space (X, T_1, T_2) is called pairwise fuzzy σ-first category space if the fuzzy set 1_X is a pairwise fuzzy σ-first category set in (X, T_1, T_2). That is $1_X = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy σ-nowhere dense sets in (X, T_1, T_2). Otherwise (X, T_1, T_2) will be called a pairwise fuzzy σ-second category space.

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Definition 3.1.[8] Let (X,T_1,T_2) be a fuzzy bitopological space. A fuzzy set λ in (X,T_1,T_2) is called a pairwise fuzzy σ-first category set if $\lambda=\vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy σ-nowhere dense sets in (X,T_1,T_2). Any other fuzzy set in (X,T_1,T_2) is said to be a pairwise fuzzy σ-second category set in (X,T_1,T_2).

Definition 3.2. [8] If λ is a pairwise fuzzy σ-first category set in a fuzzy bitopological space (X,T_1,T_2), then the fuzzy set $1-\lambda$ is called a pairwise fuzzy σ-residual set in (X,T_1,T_2).

Theorem. 3.3. [8] If λ is a pairwise fuzzy dense set and pairwise fuzzy G_δ-set in a fuzzy bitopological space (X,T_1,T_2), then $1-\lambda$ is a pairwise fuzzy σ-nowhere dense set in (X,T_1,T_2).

Proposition 3.4 If (λ_k)'s are pairwise fuzzy dense sets and pairwise fuzzy G_δ-sets in a fuzzy bitopological space (X,T_1,T_2), then $1-\wedge_{k=1}^{\infty}(\lambda_k)$ is a pairwise fuzzy σ-first category set in (X,T_1,T_2).
Proof. Let (λ_k)'s be pairwise fuzzy dense sets and pairwise fuzzy G_δ-sets in a fuzzy bitopological space (X,T_1,T_2). Then, by theorem 3.3, $(1-\lambda_k)$'s are pairwise fuzzy σ-nowhere dense sets in (X,T_1,T_2). This implies that $\vee_{k=1}^{\infty}(1-\lambda_k)$'s is a pairwise fuzzy σ-first category set in (X,T_1,T_2). Since $\vee_{k=1}^{\infty}(1-\lambda_k) = 1-\wedge_{k=1}^{\infty}(\lambda_k)$, $1-\wedge_{k=1}^{\infty}(\lambda_k)$ is a pairwise fuzzy σ-first category set in (X,T_1,T_2).

Proposition 3.5 If (λ_k)'s are pairwise fuzzy dense sets and pairwise fuzzy G_δ-sets in a fuzzy bitopological space (X,T_1,T_2), then $\wedge_{k=1}^{\infty}(\lambda_k)$ is a pairwise fuzzy σ-residual set in (X,T_1,T_2).
Proof. Let (λ_k)'s be pairwise fuzzy dense sets and pairwise fuzzy G_δ-sets in a fuzzy bitopological space (X,T_1,T_2). Then, by proposition 3.4, $[1-\wedge_{k=1}^{\infty}(\lambda_k)]$'s is a pairwise fuzzy σ-first category set in (X,T_1,T_2). This implies that $1 - (1-\wedge_{k=1}^{\infty}(\lambda_k)) = \wedge_{k=1}^{\infty}(\lambda_k)$ is a pairwise fuzzy σ-residual set in (X,T_1,T_2)

The following proposition gives a condition for a pairwise fuzzy first category set to become a pairwise fuzzy σ-first category set in fuzzy bitopological spaces.

Theorem 3.6 [13] If the pairwise fuzzy nowhere dense set λ is a pairwise fuzzy F_σ-set in a fuzzy bitopological space (X,T_1,T_2), then λ is a pairwise fuzzy σ-nowhere dense set in (X,T_1,T_2).

Proposition 3.7. If λ is a pairwise fuzzy first category set in a fuzzy bitopological space (X,T_1,T_2) in which each pairwise fuzzy nowhere dense set is a pairwise fuzzy F_σ-set, then λ is a pairwise fuzzy σ- first category set in (X,T_1,T_2).
Proof. Let λ be a pairwise fuzzy first category set in (X,T_1,T_2). Then $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X,T_1,T_2). By hypothesis, the pairwise fuzzy nowhere dense sets are pairwise fuzzy F_σ-sets in (X,T_1,T_2). By theorem 3.6, (λ_k)'s are pairwise fuzzy σ-nowhere dense sets in (X,T_1,T_2). Then $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$,where (λ_k)'s are pairwise fuzzy σ-nowhere dense sets in (X,T_1,T_2) implies that λ is a pairwise fuzzy σ-first category set in (X,T_1,T_2).

Proposition 3.8. If λ is a pairwise fuzzy first category set in a fuzzy bitopological space (X,T_1,T_2) in which each pairwise fuzzy nowhere dense set is a pairwise fuzzy F_σ-set, then $(1-\lambda)$ is a pairwise fuzzy σ-residual set in (X,T_1,T_2).
Proof. Let λ be a pairwise fuzzy first category set in which each pairwise fuzzy nowhere dense set is a pairwise fuzzy F_σ-set in (X,T_1,T_2). By proposition 3.7, λ is a pairwise fuzzy σ-first category set in (X,T_1,T_2). Then $(1-\lambda)$ is a pairwise fuzzy σ-residual set in (X,T_1,T_2).

Proposition 3.9. If (λ_k)'s are pairwise fuzzy G_δ-sets in a pairwise fuzzy hyperconnected space and pairwise fuzzy P-space in (X,T_1,T_2), then $1-\wedge_{k=1}^{\infty}(\lambda_k)$ is a pairwise fuzzy σ-first category set in (X,T_1,T_2).
Proof. Let (λ_k)'s be pairwise fuzzy G_δ-sets in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy P-space, the pairwise fuzzy G_δ-sets (λ_k)'s are pairwise fuzzy open sets in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy hyperconnected space, the pairwise fuzzy open sets (λ_k)'s are pairwise fuzzy dense sets in (X,T_1,T_2). Hence (λ_k)'s are pairwise dense and pairwise fuzzy G_δ-sets in (X,T_1,T_2). Then by proposition 3.4,$1-\wedge_{k=1}^{\infty}(\lambda_k)$ is a pairwise fuzzy σ-first category set in (X,T_1,T_2).

Proposition 3.10. If (λ_k)'s are pairwise fuzzy G_δ-sets in a pairwise fuzzy hyperconnected space and pairwise fuzzy P-space in (X,T_1,T_2), then $\wedge_{k=1}^{\infty}(\lambda_k)$ is a pairwise fuzzy σ-residual set in (X,T_1,T_2).
Proof. Let (λ_k)'s be pairwise fuzzy G_δ-sets in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy hyperconnected space and pairwise fuzzy P-space. Since, by proposition 3.9, $1-\wedge_{k=1}^{\infty}(\lambda_k)$ is a pairwise fuzzy σ-first category set in (X,T_1,T_2). Hence, $\wedge_{k=1}^{\infty}(\lambda_k)$ is a pairwise fuzzy σ-residual set in (X,T_1,T_2).

Proposition 3.11. If λ is a pairwise fuzzy σ-first category set in a fuzzy bitopological space (X,T_1,T_2) in which pairwise fuzzy σ-nowhere dense sets are pairwise fuzzy closed sets, then λ is a pairwise fuzzy F_σ-set in (X,T_1,T_2).
Proof. Let λ be a pairwise fuzzy σ-first category set in (X,T_1,T_2). Then $\lambda=\vee_{k=1}^{\infty}(\lambda_k)$, where, (λ_k)'s are pairwise fuzzy σ-nowhere dense sets in (X,T_1,T_2). By hypothesis, (λ_k)'s are pairwise fuzzy closed sets in (X,T_1,T_2). Now $[1-(\lambda_k)]$'s are pairwise fuzzy open sets in (X,T_1,T_2). Then $\mu=\wedge_{k=1}^{\infty}(1-(\lambda_k))$ is a pairwise fuzzy G_δ-set in (X,T_1,T_2) and $1-\mu=1-[\wedge_{k=1}^{\infty}(1-(\lambda_k))] = \vee_{k=1}^{\infty}(\lambda_k) = \lambda$. That is $\lambda = 1-\mu$ and $(1-\mu)$ is a pairwise fuzzy F_σ-set in (X,T_1,T_2). Hence, λ is a pairwise fuzzy F_σ-set in (X,T_1,T_2).

The following propositions gives a condition for a pairwise fuzzy σ-first category set to become a pairwise fuzzy first category set in fuzzy bitopological spaces.

Theorem 3.12 [8] If λ is a pairwise fuzzy σ-nowhere dense set in a pairwise fuzzy strongly irresolvable space (X,T_1,T_2), then λ is a pairwise fuzzy nowhere dense set and pairwise fuzzy F_σ-set in (X,T_1,T_2).

Proposition 3.13 If λ is a pairwise fuzzy σ-first category set in a pairwise strongly irresolvable space (X,T_1,T_2), then λ is a pairwise fuzzy first category set in (X,T_1,T_2).
Proof. Let λ be a pairwise fuzzy σ-first category set in (X,T_1,T_2). Then $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy σ-first category set in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise strongly irresolvable space, by theorem 3.12, the pairwise fuzzy σ-nowhere dense sets (λ_k)'s are pairwise fuzzy nowhere dense sets in (X,T_1,T_2). Hence $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X,T_1,T_2), implies that λ is a pairwise fuzzy first category set in (X,T_1,T_2).

Proposition 3.14. If λ is a pairwise fuzzy σ-first category set such that $int_{T_i}(\lambda) = 0$ in a fuzzy bitopological space (X,T_1,T_2) in which pairwise fuzzy σ-nowhere dense sets are pairwise fuzzy closed sets, then λ is a pairwise fuzzy first category set in (X,T_1,T_2).

Proof. Let λ be a pairwise fuzzy σ-first category set in (X,T_1,T_2). Then $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where, (λ_k)'s are pairwise fuzzy σ-nowhere dense sets in (X,T_1,T_2). By hypothesis, the pairwise fuzzy σ-nowhere dense sets (λ_k)'s are pairwise fuzzy closed sets in (X,T_1,T_2). By proposition 3.11, λ is a pairwise fuzzy F_σ-set in (X,T_1,T_2). Now $int_{T_i}(\lambda) = 0$ implies that $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$. But, from lemma 2.1, $\vee_{k=1}^{\infty}int_{T_i}(\lambda_k) \leq int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k))$ (i=1,2). This implies that $\vee_{k=1}^{\infty}int_{T_i}(\lambda_k) \leq 0$. That is $\vee_{k=1}^{\infty}int_{T_i}(\lambda_k) = 0$. Hence, $int_{T_i}(\lambda_k) = 0$ for each k. Since (λ_k)'s are pairwise fuzzy closed, $cl_{T_i}(\lambda_k) = \lambda_k$. Then $int_{T_i}cl_{T_i}(\lambda_k) = int_{T_i}(\lambda_k) = 0$. That is, $int_{T_i}cl_{T_i}(\lambda_k) = 0$ for each k. Hence (λ_k)'s are pairwise fuzzy nowhere dense sets in (X,T_1,T_2). Therefore $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy nowhere dense sets, implies that λ is a pairwise fuzzy first category set in (X,T_1,T_2).

Proposition 3.15 If the fuzzy bitopological space (X,T_1,T_2) is a pairwise fuzzy submaximal space and if λ is pairwise fuzzy σ-first category set in (X,T_1,T_2), then λ is a pairwise fuzzy first category set in (X,T_1,T_2).

Proof. Let $\lambda=\vee_{k=1}^{\infty}(\lambda_k)$ be a pairwise fuzzy σ-first category set in (X,T_1,T_2), where the fuzzy sets (λ_k)'s are pairwise fuzzy σ-nowhere dense set in (X,T_1,T_2). Then we have $int_{T_i}(\lambda_k)=0$ and (λ_k)'s are pairwise fuzzy F_σ- sets in (X,T_1,T_2). Now $int_{T_i}(\lambda_k)=0$, implies that $1- int_{T_i}(\lambda_k) = 1- 0 = 1$ and hence $cl_{T_i}(1-\lambda_k)=1$. Since (X,T_1,T_2) is a pairwise fuzzy submaximal space, the pairwise fuzzy dense sets $(1-\lambda_k)$'s are pairwise fuzzy open sets in (X,T_1,T_2) and hence (λ_k)'s are pairwise fuzzy closed sets in (X,T_1,T_2). Then $cl_{T_i}(\lambda_k)= (\lambda_k)$ and $int_{T_i}(\lambda_k)=0$ implies that $int_{T_i}cl_{T_i}(\lambda_k)= int_{T_i}(\lambda_k)=0$. That is (λ_k)'s are pairwise fuzzy nowhere dense set in (X,T_1,T_2). Therefore $\lambda=\vee_{k=1}^{\infty}(\lambda_k)$ be a pairwise fuzzy first category set in (X,T_1,T_2).

CONCLUSION

In this paper, several characterization of pairwise fuzzy σ-first category sets are established. The conditions for a pairwise fuzzy first category set to become a pairwise fuzzy σ-first category sets are established. Also the conditions under which pairwise fuzzy σ-first category sets to become a pairwise fuzzy first category sets are also established.

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