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Effect of Longitudinal Surface Roughness on the Performance of Rayleigh Step Bearing

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Abstract:

The purpose of this article is to analyze the performance of rough Rayleigh step bearing. Surfaces of the bearing are considered to be longitudinally rough. The roughness is characterized by a stochastic random variable with non-zero mean, variance and skewness. By using concerned boundary conditions, the Reynold's equation is solved and the pressure distribution is obtained. It is then used to get the load carrying capacity. From the results it is concluded that the load capacity of the bearing increases with the increasing value of standard deviation and it gets decreased due to $\frac{B_1}{B}$, $\bar{\alpha}$ and H_2, which is represented graphically.

Keywords: Step bearing, surface roughness, Reynolds equation, longitudinal roughness.

INTRODUCTION

Because of the characteristic of higher load capacity, Rayleigh step bearing is often used in industries. To improve load carrying capacity, there was many types of research by using an analytical method by solving the Reynolds equation. In 1918, the theory of a step bearing was firstly discussed by Lord Rayleigh [1], determining the optimum geometry with maximum load capacity per unit width for a given film thickness and bearing length. This configuration is now referred to as Rayleigh step bearing. He discovered that the best form was two parallel parts [2]. In this bearing, there are two surfaces, which are parallel to each other and divided by the lubricant film [3].Since then, the characteristics of the Rayleigh Step bearing was investigated by several investigators (Archibald [4-5], Articles et al.[6], Hong et al. [7], Zhu [8], Naduvinamani and Siddangouda [9-10], Rahmani et al. [11], Siddangouda[12], Kumar et al.[13]).

For the surface roughness, Christensen and Tonder [14-16] analyzed a stochastic method. Andharia et al. [17] studied the effect of transverse surface roughness on hydrodynamic lubrication of slider bearings with the help of a stochastic random variable having a non-zero mean(α), variance(σ), and skewness(ε). The effect of surface roughness on porous inclined stepped composite bearing is studied by Naduvinamani and Biradar [18].

Andharia et al. [19, 20, 22] and Deheri et al. [23] worked on the longitudinally rough surface roughness. Andharia et al. [19] studied the effect of longitudinal surface roughness on hydrodynamic lubrication of slider bearings. Andharia et al. [20] studied the optimum film profile of a slider bearing with longitudinal surface roughness. Deheri et al. [21] analysed longitudinally rough slider bearings with squeeze film formed by a magnetic fluid. Andharia and Patel [22] studied the effect of longitudinal roughness on the performance of a magnetic fluid based short bearing.

In this article, we have studied the effect of longitudinal surface roughness on the performance of Rayleigh step bearing.

ANALYSIS

The outlines of the Rayleigh step bearing is shown in Figure 1. There are two regions $0 < x \le B_1$, $h = h_1$ and $B_1 \le x \le B$, $h = h_2$ as shown in the figure. Among two surfaces, the lower surface is moving in the X direction and the upper surface is static. Surfaces are assumed to be longitudinally rough. There is a lubricant film between two surfaces.

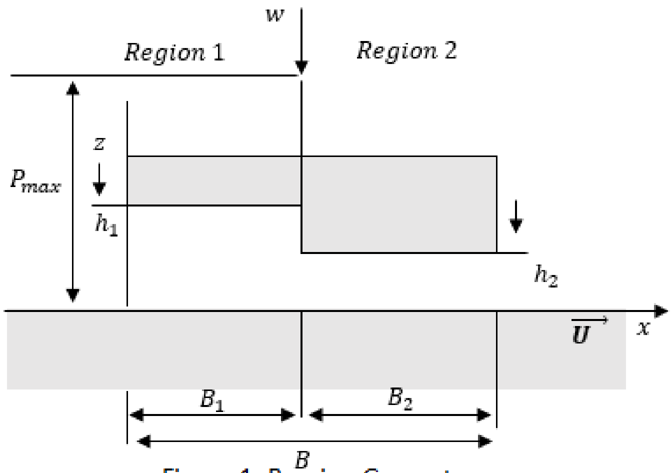

Figure 1: Bearing Geometry

The lubricant film thickness is

$$h(x) = \bar{h}(x) + \delta_s \tag{1}$$

where $\bar{h}(x)$ is the mean film thickness and δ_s is random deviation in $h(x)$. The randomness of δ_s is given by probability density function as in [19]. The relation between mean (α), standard deviation (σ) and the measure of symmetry (ε) with the random variable δ_s are also defined as in [19]. Note that α, σ and ε are all independent of x and σ is always non-negative.

The Reynolds equation takes the form

$$\frac{d}{dx}\left(\frac{h^3}{\mu}\frac{dp}{dx}\right) = 6U\frac{dh}{dx} \quad (2)$$

By applying averaging process, as in [19], we get

$$\frac{d}{dx}\left(\frac{1}{\mu U}\frac{1}{q(\bar{h})}\frac{dp}{dx}\right) = 6\frac{d}{dx}\left(\frac{1}{r(\bar{h})}\right) \quad (3)$$

where

$$q(\bar{h}) = \bar{h}^{-3}[1 - 3\bar{h}^{-1}\alpha + 6\bar{h}^{-2}(\sigma^2 + \alpha^2) - 10\bar{h}^{-3}(\varepsilon + \alpha^3 + 3\sigma^2\alpha)$$

$$r(\bar{h}) = \bar{h}^{-1}[1 - \bar{h}^{-1}\alpha + \bar{h}^{-2}(\sigma^2 + \alpha^2) - \bar{h}^{-3}(\varepsilon + \alpha^3 + 3\sigma^2\alpha)$$

By integrating equation (3) with respect to x

$$\frac{1}{\mu U}\frac{1}{q(\bar{h})}\frac{dp}{dx} = 6\left[\frac{1}{r(\bar{h})} + a_m\right] \quad (4)$$

Introducing the dimensionless quantities as follow

$$H = \frac{\bar{h}}{\bar{h}_1},\ \bar{\alpha} = \frac{\alpha}{\bar{h}_1},\ \bar{\sigma} = \frac{\sigma}{\bar{h}_1},\ \bar{\varepsilon} = \frac{\varepsilon}{\bar{h}_1^{\,3}},\ P = \frac{p\bar{h}_1^{\,2}}{\mu UB},\ A_m = \frac{a_m}{\bar{h}_1},$$

$$X = \frac{x}{B},\ Q(H) = q(\bar{h}).\bar{h}_1^{\,3},\ R(H) = r(\bar{h}).\bar{h}_1, \quad (5)$$

the modified Reynold's equation of film pressure P takes the form

$$\frac{dP}{dX} = 6Q(H)\left[\frac{1}{R(H)} + A_m\right] \quad (6)$$

where $H = \begin{cases} 1, 0 \le X \le \frac{B_1}{B} \\ H_2, \frac{B_1}{B} \le X \le 1 \end{cases}$.

The film thickness remains constant in both the region. It is given by $h = h_1$ and $h = h_2$ in region – 1 and region – 2 respectively. So the right-hand side of the Reynolds equation is vanished. Hence for $i = 1, 2$

$$\frac{d}{dx}\left(\frac{h^3}{\mu}\frac{dp_i}{dx}\right) = 0 \quad (7)$$

By applying averaging process, it takes the following form

$$\frac{d}{dx}\left(\frac{1}{\mu U}\frac{1}{q(\bar{h}_i)}\frac{dp_i}{dx}\right) = 0 \quad (8)$$

By integrating equation (8) with respect to x

$$\frac{1}{\mu U}\frac{1}{q(\bar{h}_i)}\frac{dp_i}{dx} = a_i \quad (9)$$

where a_i are constants of integration.

Introducing the dimensionless quantities

$$H_i = \frac{\bar{h}_i}{\bar{h}_1},\ \bar{\alpha} = \frac{\alpha}{\bar{h}_1},\ \bar{\sigma} = \frac{\sigma}{\bar{h}_1},\ \bar{\varepsilon} = \frac{\varepsilon}{\bar{h}_1^{\,3}},\ P_i = \frac{p\bar{h}_1^{\,2}}{\mu UB},\ A_i = \frac{a_i}{\bar{h}_1},$$

$$X = \frac{x}{B},\ Q(H_i) = q(\bar{h}_i).\bar{h}_1^{\,3},\ R(H_i) = r(\bar{h}_i).\bar{h}_1,$$

the modified Reynold's equation of film pressure P_i takes the form

$$\frac{dP_i}{dX} = Q(H_i)A_i \quad (10)$$

Note that $Q(H_1) = Q(1)$.

For region-1 the boundary conditions are as follow

$$P_1 = \begin{cases} 0 \quad at\ X = 0 \\ P_c\ at\ X = \frac{B_1}{B} \end{cases} \quad (11)$$

By integrating equation (10) with respect to x for region – 1

$$P_1 = \int_0^X Q(1)\,A_m.dx + d_1, \text{ where } d_1 \text{ is constant.} \quad (12)$$

Using boundary condition $P_1 = 0$ at $X = 0$ in equation (12)

$$P_1 = \int_0^X Q(1)\,A_1.dx$$

$$P_1 = Q(1)A_1X \quad (13)$$

For region – 2 the boundary conditions are as follow

$$P_2 = \begin{cases} P_c\ at\ X = \frac{B_1}{B} \\ 0\ at\ X = 1 \end{cases} \quad (14)$$

By integrating the equation (10) with respect to x for region – 2

$$P_2 = \int_{\frac{B_1}{B}}^X Q(H_2)\,A_2dx + d_2, \text{ where } d_2 \text{ is constant.} \quad (15)$$

Using $P_2 = 0$ at $X = 1$ equation (15) becomes

$$P_2 = \int_{\frac{B_1}{B}}^X Q(H_2)\,A_2dx + \int_1^{\frac{B_1}{B}} Q(H_2)\,A_2dx \quad (16)$$

Using boundary condition $P_2 = P_c$ at $X = \frac{B_1}{B}$ equation (16) becomes

$$P_2 = Q(H_2)A_2\left(\frac{B_1}{B} - 1\right) \quad (17)$$

where

$$A_1 = \frac{6}{5}\frac{\left(\frac{B_1}{B}\right)}{B}\frac{1}{Q(1)}\left(\frac{Q(H_2)}{R(H_2)} - \frac{Q(1)}{R(1)}\right) \quad (18)$$

and

$$A_2 = -\frac{6}{5}\frac{\left(\frac{B_1}{B}\right)}{B}\frac{1}{Q(H_2)}\left(\frac{Q(H_2)}{R(H_2)} - \frac{Q(1)}{R(1)}\right) \quad (19)$$

The load carrying capacity

$$W = \int_0^{\frac{B_1}{B}} P_1\,dx + \int_{\frac{B_1}{B}}^1 P_2\,dx$$

of the bearing can be obtained in dimensionless form as

$$W = \frac{3}{5}\left(\frac{Q(H_2)}{R(H_2)} - \frac{Q(1)}{R(1)}\right)\left(\frac{B_1}{B}\right)\left(1 - \frac{B_1}{B}\right) \quad (20)$$

RESULTS AND DISCUSSION

The dimensionless pressure in region 1, region 2 and load carrying capacity are presented by eq. (13), eq. (17) and eq. (20). One can observe that $\frac{B_1}{B}$, $\bar{\sigma}$, $\bar{\alpha}$, $\bar{\varepsilon}$ and H_2 are decisional parameters for pressure distribution and load carrying capacity of the bearing.

Figures 2 - 5 represent the result on load carrying capacity with respect to $\frac{B_1}{B}$ for different values of $\bar{\sigma}$, $\bar{\alpha}$, $\bar{\varepsilon}$ and H_2 respectively. From these figures, one can observe that the load carrying capacity is maximum at $\frac{B_1}{B} = 0.5$. For $\frac{B_1}{B} > 0.5$, load capacity is decreasing as the ratio $\frac{B_1}{B}$ increases.

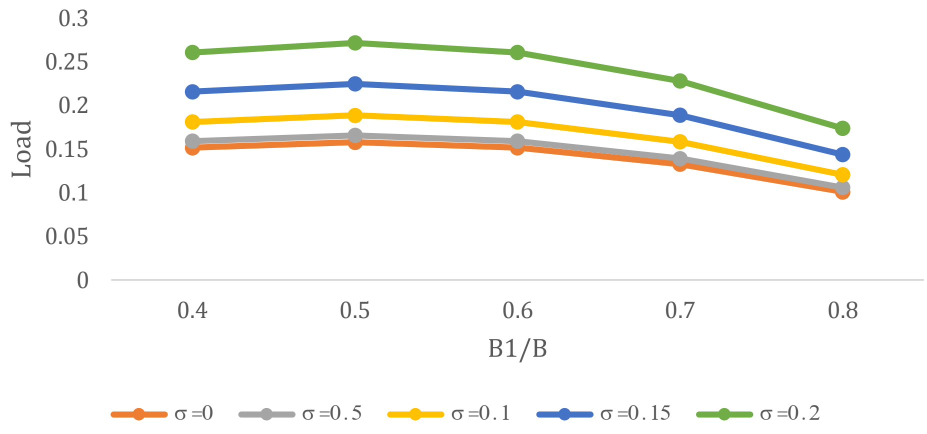

Figure – 2. Effect on load carrying capacity with respect to $\frac{B_1}{B}$ for variable $\bar{\sigma}$.

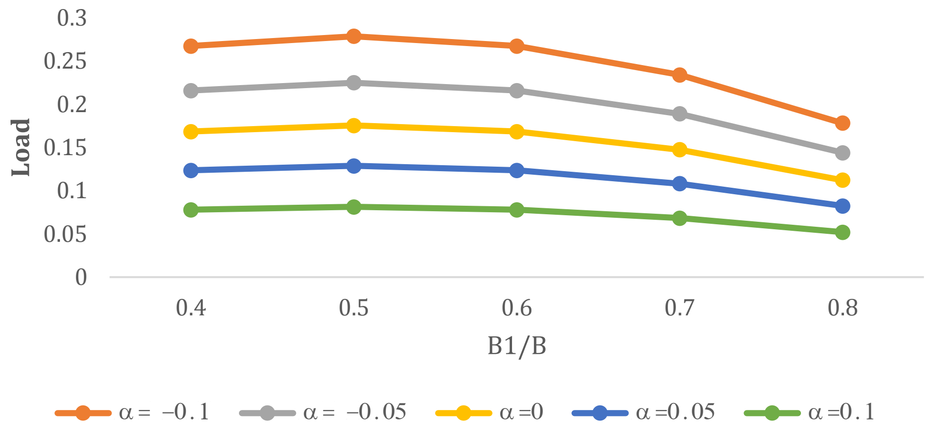

Figure – 3. Effect on load carrying capacity with respect to $\frac{B_1}{B}$ for variable $\bar{\alpha}$.

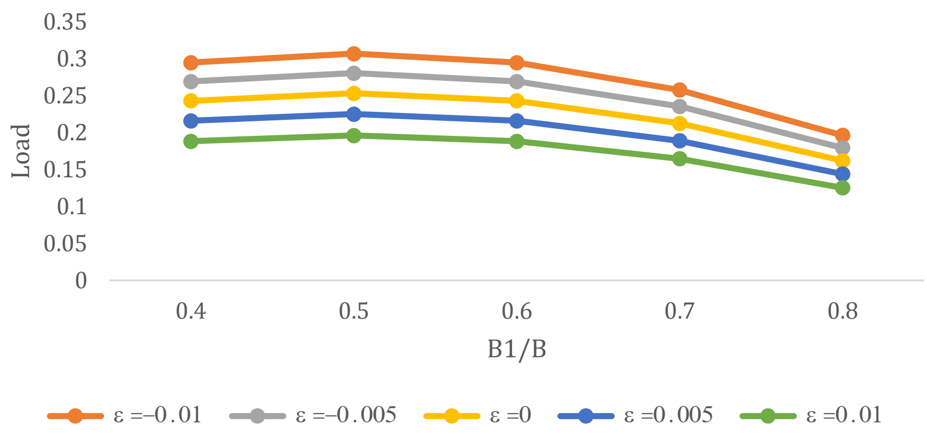

Figure – 4. Effect on load carrying capacity with respect to $\frac{B_1}{B}$ for variable$\bar{\varepsilon}$.

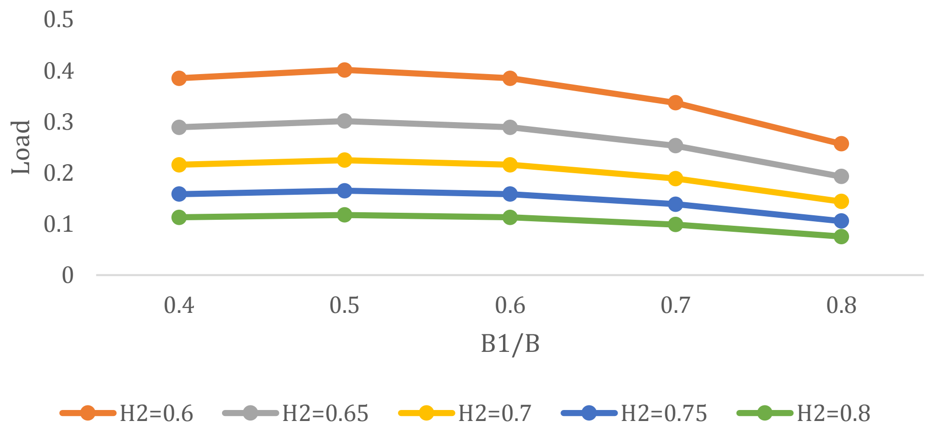

Figure – 5. Effect on load carrying capacity with respect to $\frac{B_1}{B}$ for variable $\boldsymbol{H_2}$.

The effect of a standard deviation on load carrying capacityfor different values of $\bar{\alpha}$, $\bar{\varepsilon}$ and H_2 is presented in Figures 6 – 8. It is seen that the load carrying capacity increases when the standard deviation gets increased.

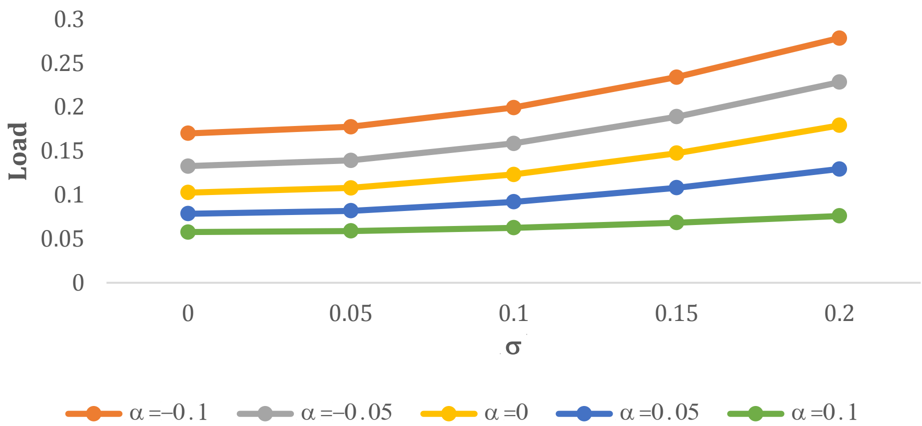

Figure – 6.Effect on load carrying capacity with respect to $\bar{\sigma}$ for variable $\bar{\alpha}$.

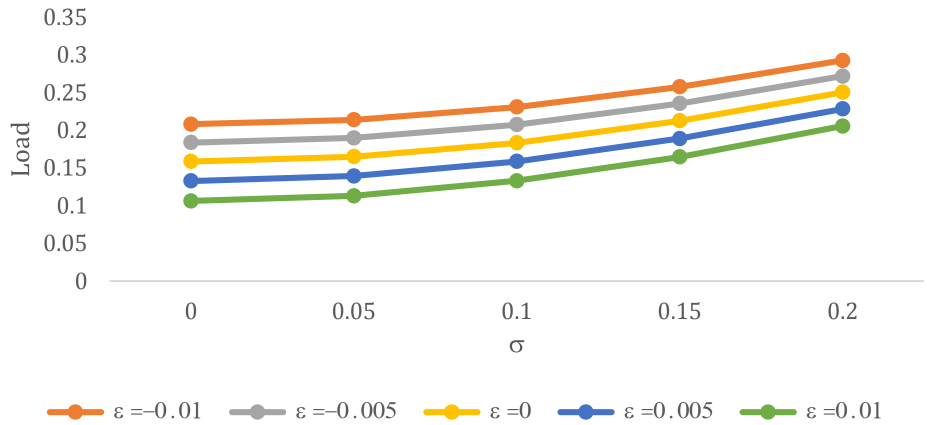

Figure – 7. Effect on load carrying capacity with respect to $\boldsymbol{\bar{\sigma}}$ for variable $\boldsymbol{\bar{\varepsilon}}$.

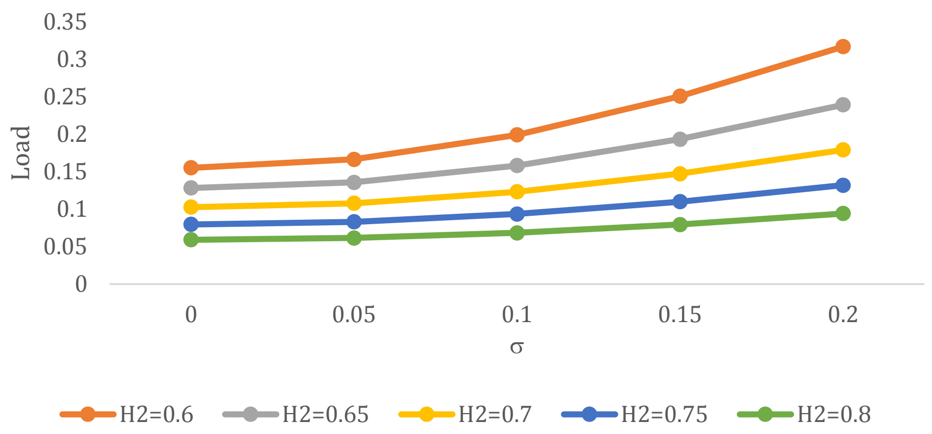

Figure – 8. Effect on load carrying capacity with respect to $\bar{\sigma}$ for variable H_2.

Figures (9 – 10) presents the variation of the load carrying capacity with respect to H_2 for different values of $\bar{\alpha}$ and $\bar{\varepsilon}$. It is clear that the load carrying capacity is decreasing sharply due to H_2.

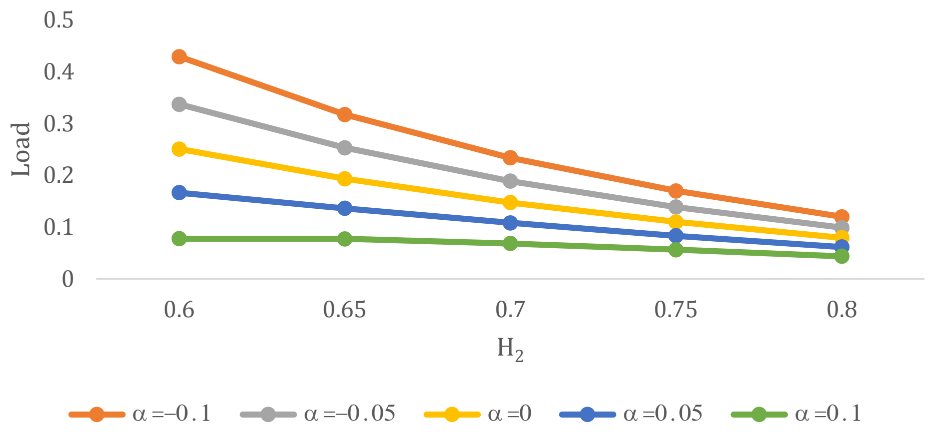

Figure – 9. Effect on load carrying capacity with respect to H_2 for variable $\bar{\alpha}$.

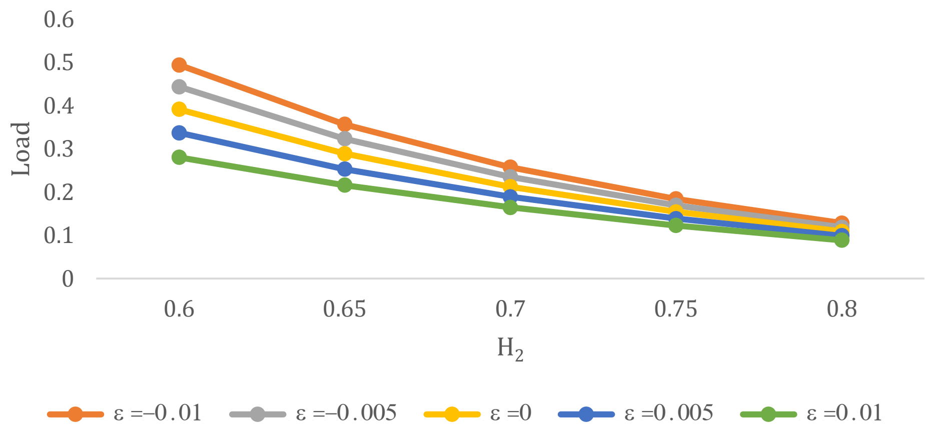

Figure – 10. Effect on load carrying capacity with respect to H_2 for variable $\bar{\varepsilon}$.

The effect of $\bar{\alpha}$ and $\bar{\varepsilon}$ on the load carrying capacity is shown in Figure 11. It is observed that the load carrying capacity decreases due to $\bar{\alpha}$ and $\bar{\varepsilon}$.

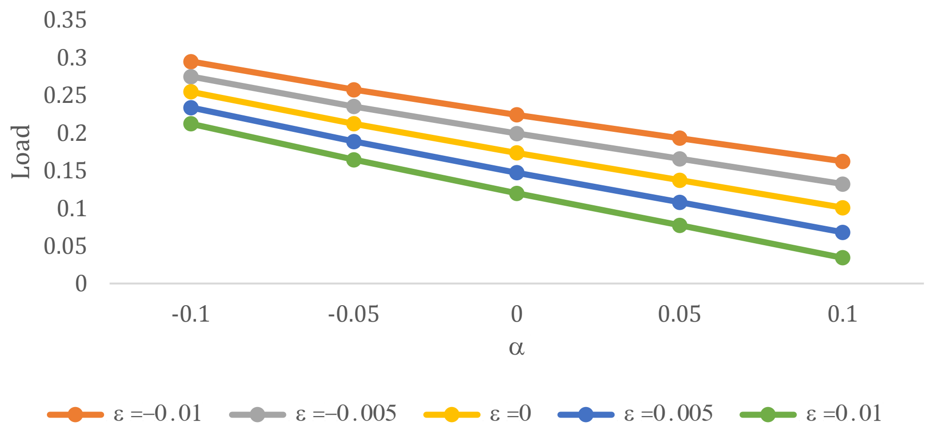

Figure – 11. Effect on load carrying capacity with respect to $\bar{\boldsymbol{\alpha}}$ for variable $\bar{\boldsymbol{\varepsilon}}$.

CONCLUSION

It is concluded that the aspect ratio $(\frac{B_1}{B})$, variance $(\bar{\alpha})$ and height ratio (H_2) are the parameters which affects negatively on the performance of step bearing with longitudinal surface roughness. However standard deviation improves the load capacity of the bearing unlike in the case of transverse surface roughness. These points must be considered for making a good bearing design.

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